PHYS 1441 – Section 002
Lecture #6

Monday, Feb. 16, 2009

Dr. Jaehoon Yu

- Motion in Two Dimensions
  - Projectile Motion
  - Maximum ranges and heights
- Newton's Laws of Motion
  - Force
  - Newton's first law: Inertia & Mass

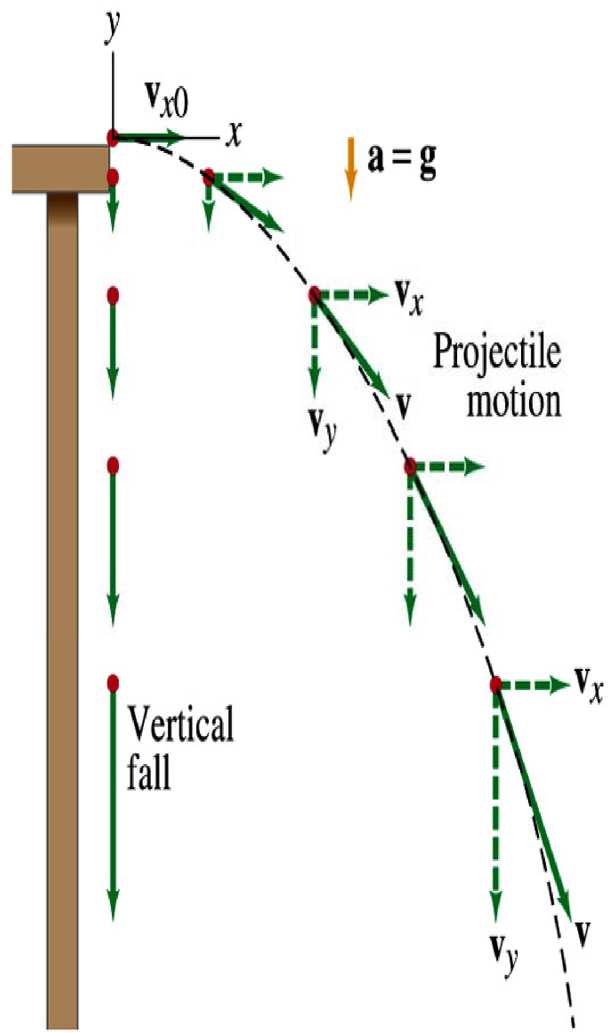
Today's homework is homework #4, due 9pm, Tuesday, Feb. 25!!

Announcements

- First term exam
  - 1 – 2:20pm, Wednesday, Feb. 18
  - Covers: CH1.1 – what we complete on Monday, Feb. 16 + appendix A1 – A8
  - Do not miss the exam

Projectile Motion

- A 2-dim motion of an object under the gravitational acceleration with the following assumptions
  - Free fall acceleration, g, is constant over the range of the motion
    - $\vec{g} = -9.8\vec{j}\left(m/s^2\right)$
    - $a_x = 0\,m/s^2$ and $a_y = -9.8\,m/s^2$
  - Air resistance and other effects are negligible
- A motion under constant acceleration!!!! ➔ Superposition of two motions
  - Horizontal motion with constant velocity (<u>no acceleration</u>) $v_{xf} = v_{x0}$
  - Vertical motion under constant acceleration (g) $v_{yf} = v_{y0} + a_y t = v_{y0} + (-9.8)t$



Kinematic Equations in 2-Dim

x-component

y-component

$$v_x = v_{xo} + a_x t$$

$$v_y = v_{yo} + a_y t$$

$$x = \tfrac{1}{2}\left(v_{xo} + v_x\right)t$$

$$y = \tfrac{1}{2}\left(v_{yo} + v_y\right)t$$

$$v_x^2 = v_{xo}^2 + 2a_x x$$

$$v_y^2 = v_{yo}^2 + 2a_y y$$

$$x = v_{xo}t + \tfrac{1}{2}a_x t^2$$

$$y = v_{yo}t + \tfrac{1}{2}a_y t^2$$

Show that a projectile motion is a parabola!!!

x-component $v_{xi} = v_i \cos\theta_i$

y-component $v_{yi} = v_i \sin\theta_i$

$$\vec{a} = a_x\vec{i} + a_y\vec{j} = -g\vec{j}$$

In a projectile motion, the only acceleration is gravitational one whose direction is always toward the center of the earth (downward).

$a_x=0$

$$x_f = v_{xi}t = v_i \cos\theta_i t$$

$$t = \frac{x_f}{v_i \cos\theta_i}$$

$$y_f = v_{yi}t + \frac{1}{2}(-g)t^2 = v_i \sin\theta_i t - \frac{1}{2}gt^2$$

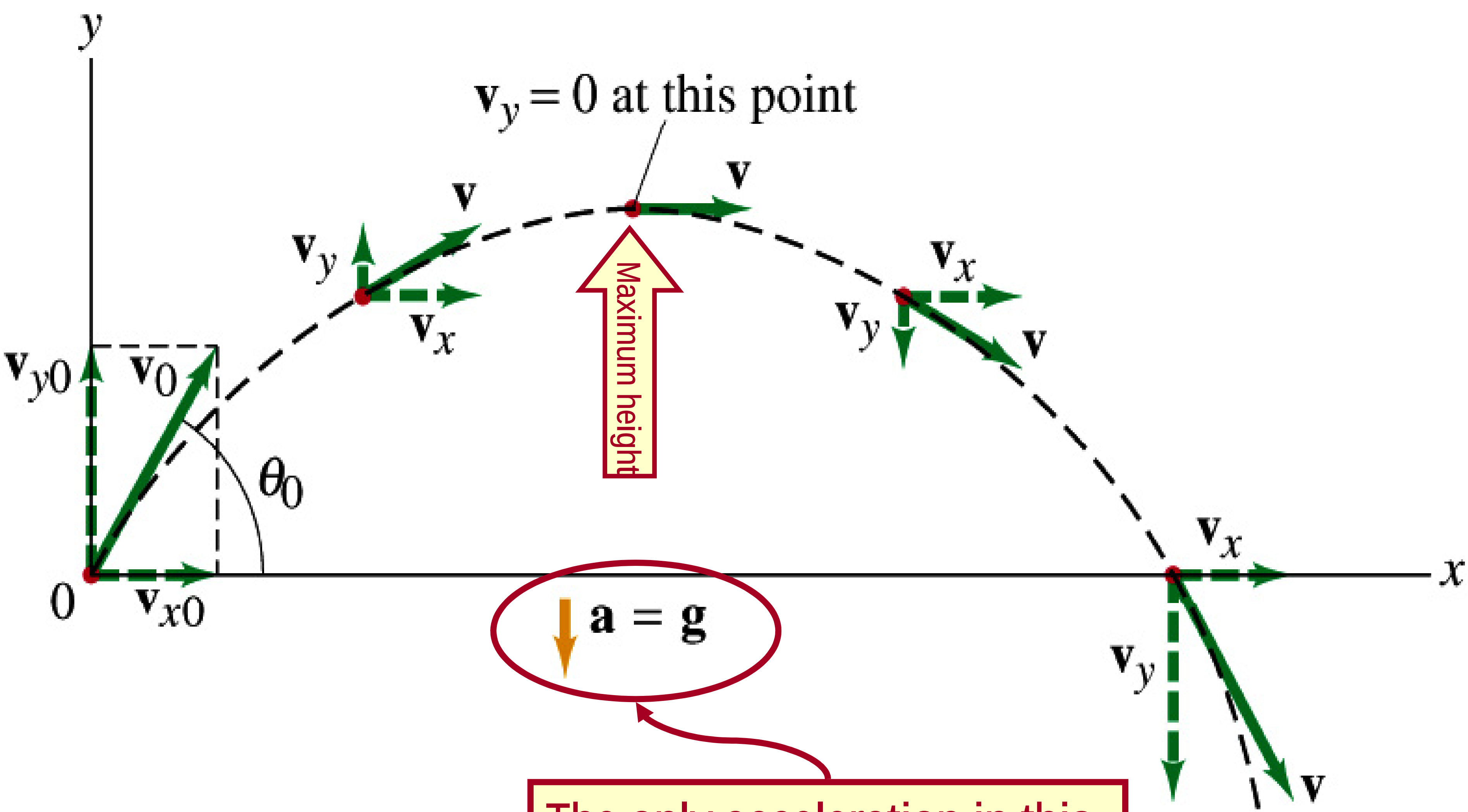
Plug t into the above

$$y_f = v_i \sin\theta_i \left(\frac{x_f}{v_i \cos\theta_i}\right) - \frac{1}{2}g\left(\frac{x_f}{v_i \cos\theta_i}\right)^2$$

$$y_f = x_f \tan\theta_i - \left(\frac{g}{2v_i^2 \cos^2\theta_i}\right)x_f^2$$

What kind of parabola is this?

Projectile Motion



The only acceleration in this motion. It is a constant!!

Example for Projectile Motion

A ball is thrown with an initial velocity **v**=(20**i**+40**j**)m/s. Estimate the time of flight and the distance the ball is from the original position when landed.

Which component determines the flight time and the distance?

Flight time is determined by the y component, because the ball stops moving when it is on the ground after the flight.

$$y_f = 40t + \frac{1}{2}(-g)t^2 = 0m$$

$$t(80 - gt) = 0$$

So the possible solutions are…

$$\therefore t = 0 \text{ or } t = \frac{80}{g} \approx 8\sec$$

$$\therefore\ t \approx 8\sec$$

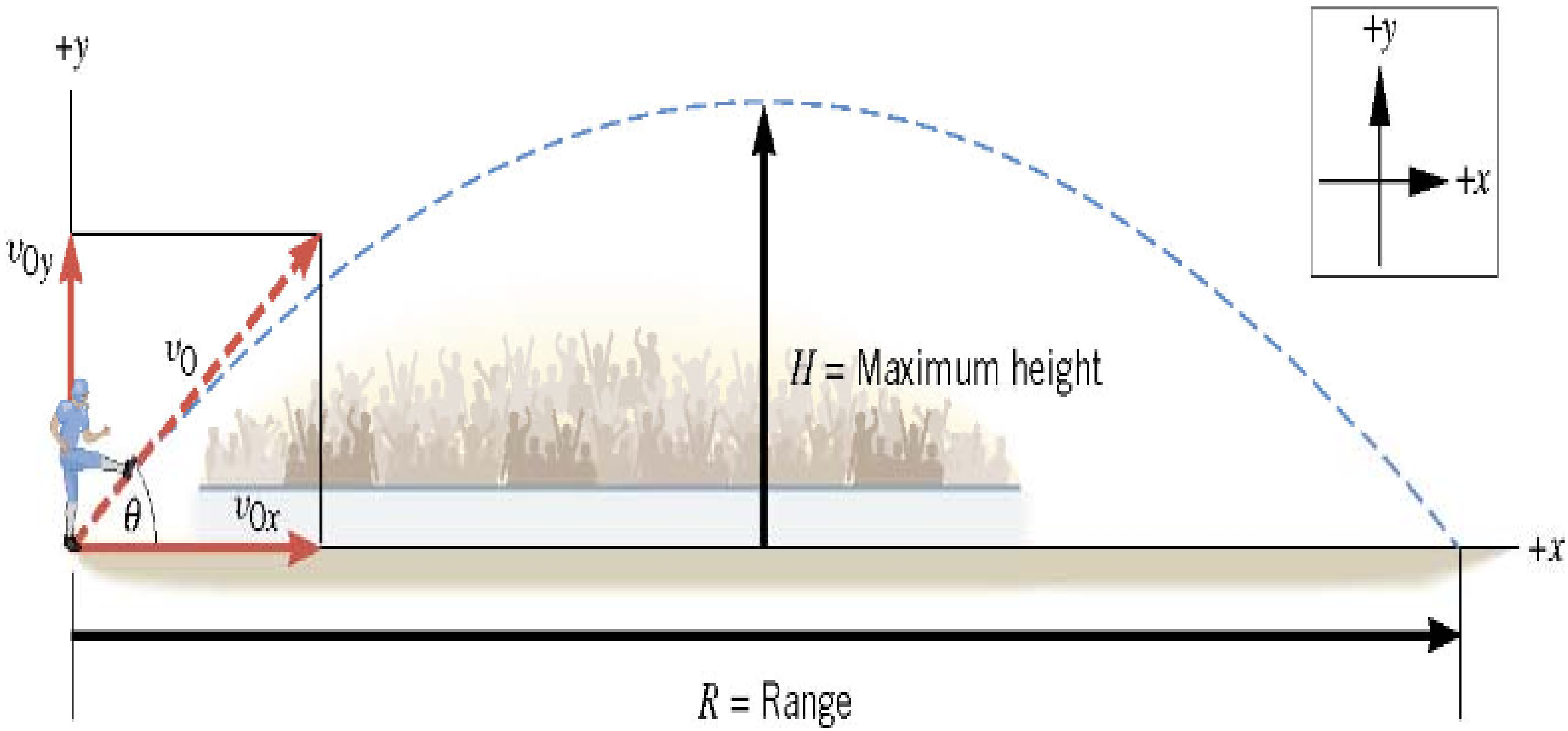
Why isn't 0 the solution?

Distance is determined by the x component in 2-dim, because the ball is at $y=0$ position when it completed it's flight.

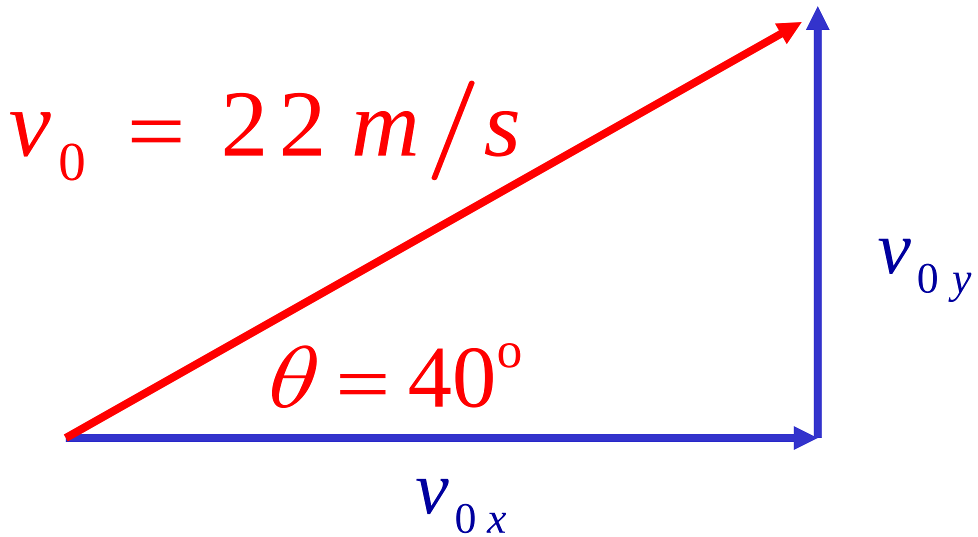
$$x_f = v_{xi}t = 20 \times 8 = 160(m)$$

Ex. The Height of a Kickoff

A placekicker kicks a football at and angle of 40.0 degrees and the initial speed of the ball is 22 m/s. Ignoring air resistance, determine the maximum height that the ball attains.



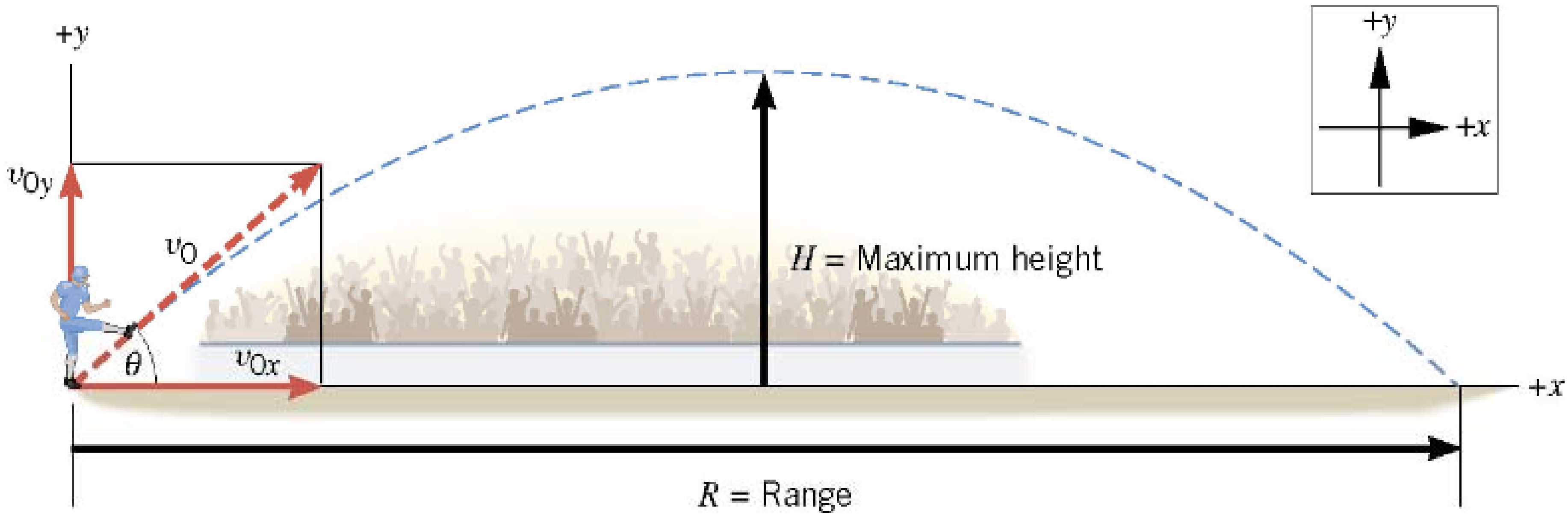
First, the initial velocity components



$$v_{ox} = v_o \cos\theta = \left(22\,\mathrm{m/s}\right)\cos 40^{o} = 17\,\mathrm{m/s}$$

$$v_{oy} = v_o \sin\theta = \left(22\,\mathrm{m/s}\right)\sin 40^{o} = 14\,\mathrm{m/s}$$

Motion in y-direction is of the interest..



y	a_y	v_y	v_{oy}	t
?	-9.8 m/s^2	0 m/s	+14 m/s	

Now the nitty, gritty calculations…

y	a_y	v_y	v_{oy}	t
?	-9.80 m/s^2	0	14 m/s	

What happens at the maximum height?

The ball's velocity in y-direction becomes 0!!

And the ball's velocity in x-direction? Stays the same!! Why?

Because there is no acceleration in x-direction!!

Which kinematic formula would you like to use?

$$v_y^2 = v_{oy}^2 + 2a_y y$$

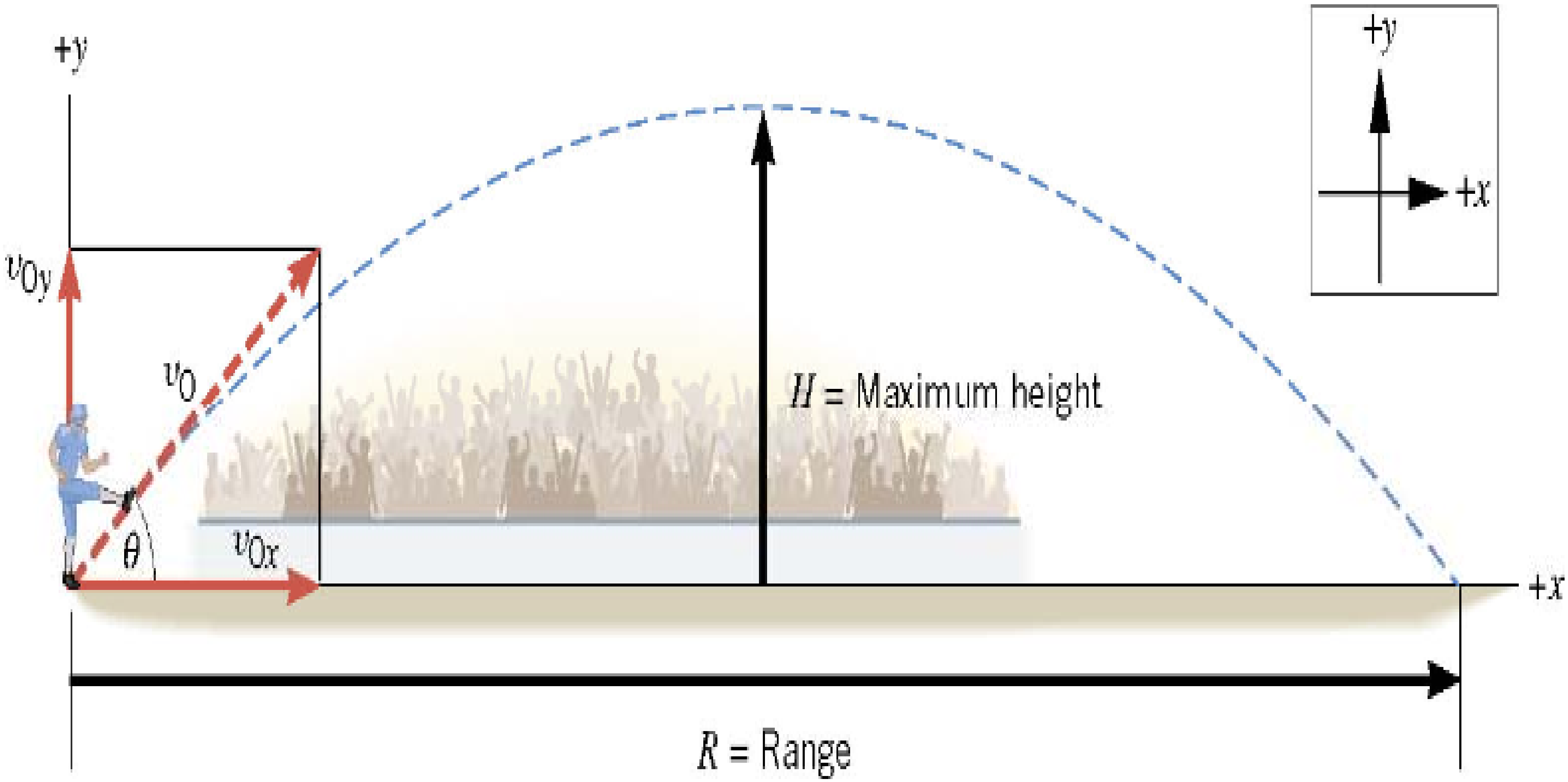
Solve for y

$$y = \frac{v_y^2 - v_{oy}^2}{2a_y}$$

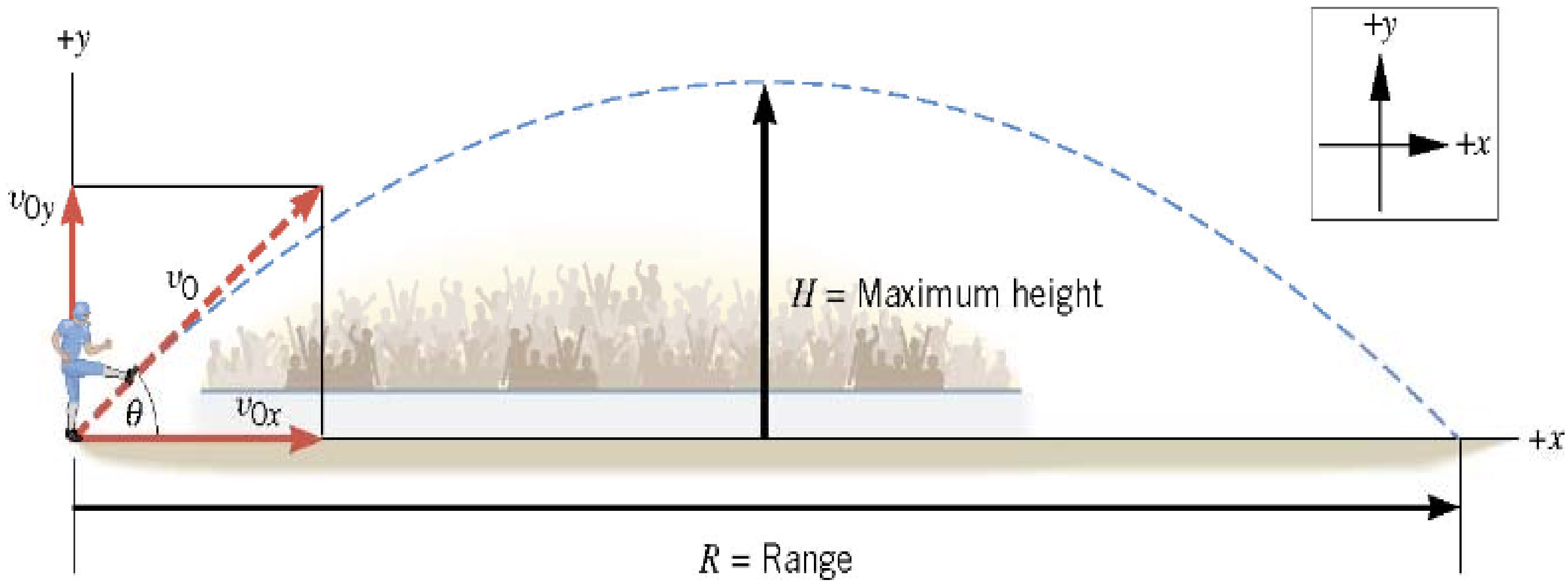
$$y = \frac{0-(14\,\mathrm{m/s})^2}{2(-9.8\,\mathrm{m/s^2})} = +10\ \mathrm{m}$$

Ex. The Time of Flight of a Kickoff

What is the time of flight between kickoff and landing?



What is y when it reached the max range?



y	a_y	v_y	v_{oy}	t
0 m	-9.80 m/s^2		14 m/s	?

Now solve the kinematic equations in y direction!!

y	a_y	v_y	v_{oy}	t
0	-9.80 m/s^2		14 m/s	?

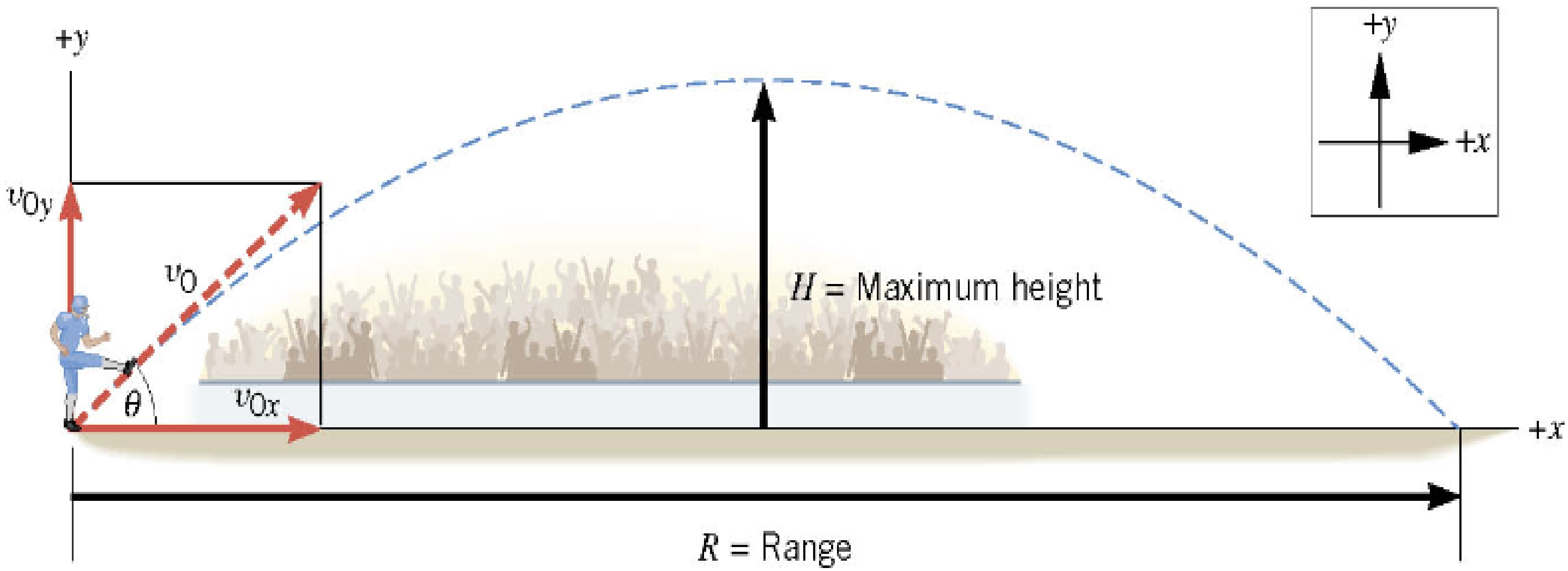
$$y = v_{oy}t + \tfrac{1}{2}a_y t^2 \xrightarrow{\text{Since } y=0} 0 = v_{oy}t + \tfrac{1}{2}a_y t^2 = t\left(v_{oy} + \tfrac{1}{2}a_y t\right)$$

Two soultions $t = 0$ or

$$v_{oy} + \tfrac{1}{2}a_y t = 0 \xrightarrow{\text{Solve for t}} t = \frac{-v_{oy}}{\frac{1}{2}a_y} = \frac{-2v_{oy}}{a_y} = \frac{-2\cdot 14}{-9.8} = 2.9s$$

Ex. The Range of a Kickoff

Calculate the range R of the projectile.



$$x = v_{ox}t + \tfrac{1}{2}a_x t^2 = v_{ox}t = (17\,\text{m/s})(2.9\text{ s}) = +49\text{ m}$$

Example for a Projectile Motion

- A stone was thrown upward from the top of a cliff at an angle of 37° to horizontal with initial speed of 65.0m/s. If the height of the cliff is 125.0m, how long is it before the stone hits the ground?

$$v_{xi} = v_i \cos\theta_i = 65.0 \times \cos 37^o = 51.9 m/s$$

$$v_{yi} = v_i \sin\theta_i = 65.0 \times \sin 37^o = 39.1 m/s$$

$$y_f = -125.0 = v_{yi} t - \frac{1}{2} g t^2$$

Becomes

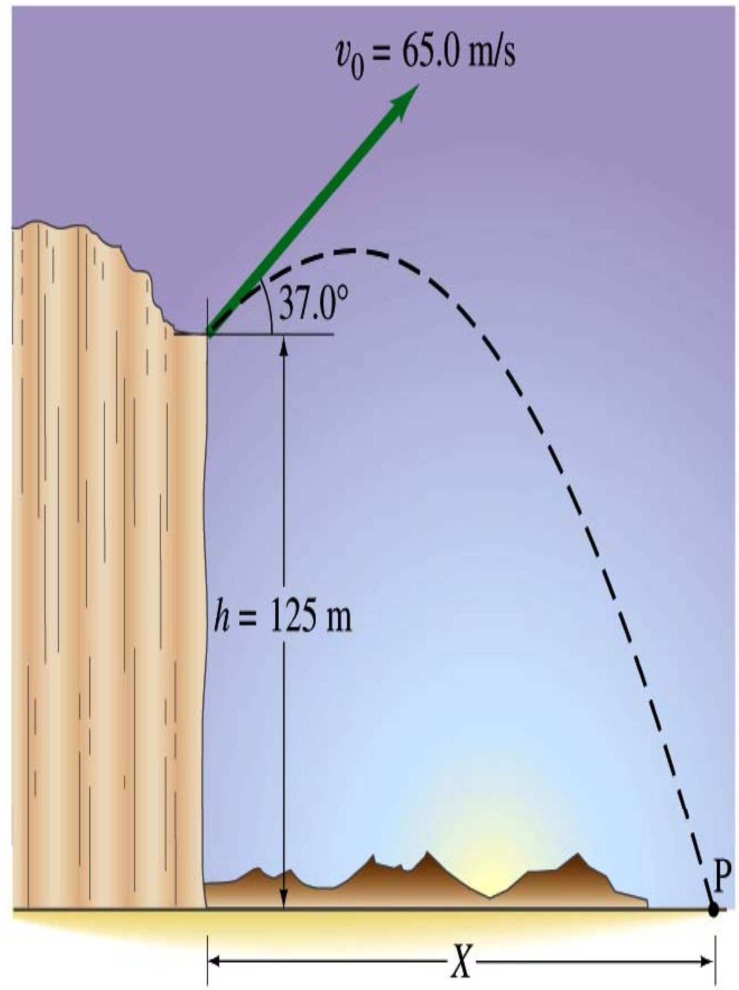
$$g t^2 - 78.2t - 250 = 9.80t^2 - 78.2t - 250 = 0$$

$$t = \frac{78.2 \pm \sqrt{(-78.2)^2 - 4 \times 9.80 \times (-250)}}{2 \times 9.80}$$

$$t = -2.43 s \quad \text{or} \quad t = 10.4 s$$

$$\boxed{t = 10.4 s}$$

Since negative time does not exist.



Example cont'd

- What is the speed of the stone just before it hits the ground?

$$v_{xf} = v_{xi} = v_i \cos\theta_i = 65.0 \times \cos 37^o = 51.9 m/s$$

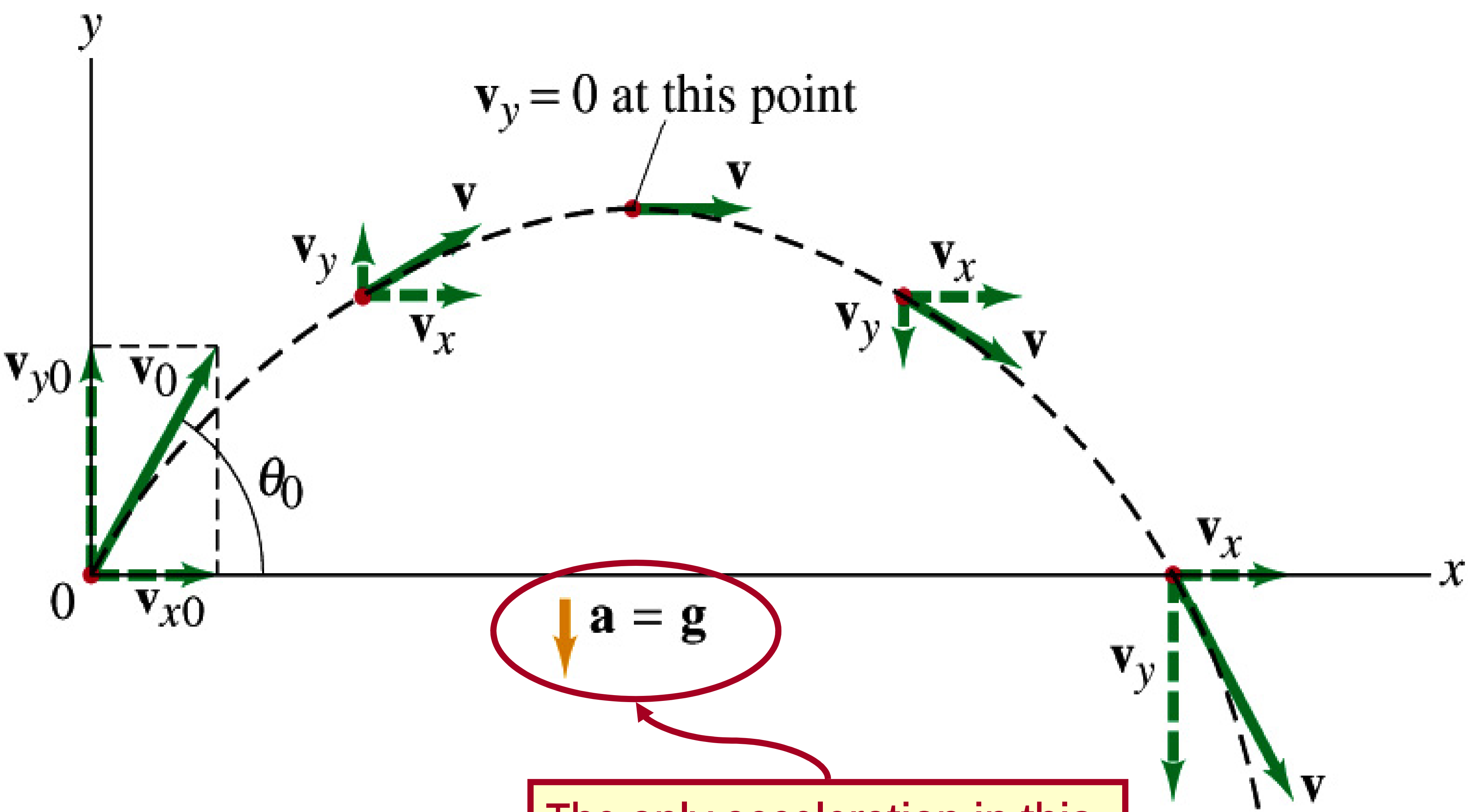
$$v_{yf} = v_{yi} - gt = v_i \sin\theta_i - gt = 39.1 - 9.80 \times 10.4 = -62.8 m/s$$

$$|v| = \sqrt{v_{xf}^{\ 2} + v_{yf}^{\ 2}} = \sqrt{51.9^2 + (-62.8)^2} = 81.5 m/s$$

- What are the maximum height and the maximum range of the stone?

Do these yourselves at home for fun!!!

Projectile Motion



The only acceleration in this motion. It is a constant!!

Horizontal Range and Max Height

- Based on what we have learned in the previous lecture, one can analyze a projectile motion in more detail
  - Maximum height an object can reach
  - Maximum range

What happens at the maximum height?

At the maximum height the object's vertical motion stops to turn around!!

$$v_{yf} = v_{0y} + a_y t = v_0 \sin\theta_0 - g t_A = 0$$

Solve for t_A

$$\boxed{\therefore t_A = \frac{v_0 \sin\theta_0}{g}}$$

Time to reach to the maximum height!!

Horizontal Range and Max Height

Since no acceleration is in x direction, it still flies even if v_y=0.

$$R = v_{0x}t = v_{0x}(2t_A) = 2v_0\cos\theta_0\left(\frac{v_0\sin\theta_0}{g}\right)$$

Range

$$\boxed{R = \left(\frac{v_0^2\sin 2\theta_0}{g}\right)}$$

$$y_f = h = v_{0y}t + \frac{1}{2}(-g)t^2 = v_0\sin\theta_0\left(\frac{v_0\sin\theta_0}{g}\right) - \frac{1}{2}g\left(\frac{v_0\sin\theta_0}{g}\right)^2$$

Height

$$\boxed{y_f = h = \left(\frac{v_0^2\sin^2\theta_0}{2g}\right)}$$

Maximum Range and Height

- What are the conditions that give maximum height and range of a projectile motion?

$$h = \left(\frac{v_0^{\ 2} \sin^2 \theta_0}{2g} \right)$$

This formula tells us that the maximum height can be achieved when $\theta_i = 90^o$!!!

$$R = \left(\frac{v_0^{\ 2} \sin 2\theta_0}{g} \right)$$

This formula tells us that the maximum range can be achieved when $2\theta_i = 90^o$, i.e., $\theta_i = 45^o$!!!

Force

We've been learning kinematics; describing motion without understanding what the cause of the motion is. Now we are going to learn dynamics!!

FORCE is what causes an object to move.

Can someone tell me what FORCE is?

The above statement is not entirely correct. Why?

Because when an object is moving with a constant velocity no force is exerted on the object!!!

FORCEs are what cause any changes to the velocity of an object!!

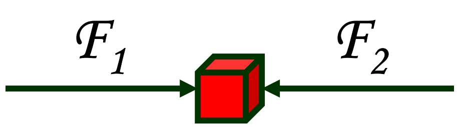
What does this statement mean?

When there is force, there is change of velocity!!

What does force cause? It causes an acceleration.!!

What happens if there are several forces being exerted on an object?

Forces are vector quantities, so vector sum of all forces, the NET FORCE, determines the direction of the acceleration of the object.



NET FORCE, $F = F_1 + F_2$

When the net force on an object is 0, it has constant velocity and is at its equilibrium!!

More Force

There are various classes of forces

Contact Forces: Forces exerted by physical contact of objects

Examples of Contact Forces: Baseball hit by a bat, Car collisions

Field Forces: Forces exerted without physical contact of objects

Examples of Field Forces: Gravitational Force, Electro-magnetic force

What are possible ways to measure strength of the force?

A calibrated spring whose length changes linearly with the force exerted .

Forces are vector quantities, so the addition of multiple forces must be done following the rules of vector additions.

Newton's First Law and Inertial Frames

Aristotle (384-322BC): *A natural state of a body is rest. Thus force is required to move an object. To move faster, ones needs larger forces.*

Galileo's statement on natural states of matter: *Any velocity once imparted to a moving body will be rigidly maintained as long as the external causes of retardation are removed!!*

Galileo's statement is formulated by Newton into the 1st law of motion (Law of Inertia): *In the absence of external forces, an object at rest remains at rest and an object in motion continues in motion with a constant velocity.*

What does this statement tell us?

- When no force is exerted on an object, the acceleration of the object is 0.
- Any isolated object, the object that do not interact with its surroundings, is either at rest or moving at a constant velocity.
- Objects would like to keep its current state of motion, as long as there are no forces that interfere with the motion. This tendency is called the Inertia.

A frame of reference that is moving at a constant velocity is called the *Inertial Frame*

Is a frame of reference with an acceleration an *Inertial Frame?*

NO!

Mass

Mass: *A measure of the inertia of a body* or *quantity of matter*

- Independent of the object's surroundings: The same no matter where you go.
- Independent of the method of measurement: The same no matter how you measure it.

The heavier the object, the bigger the inertia !!

It is harder to make changes of motion of a heavier object than a lighter one.

The same forces applied to two different masses result in different acceleration depending on the mass.

$$\boxed{\frac{m_1}{m_2} \equiv \frac{a_2}{a_1}}$$

Note that the mass and the weight of an object are two different quantities!!

Weight of an object is the magnitude of the gravitational force exerted on the object.

Not an inherent property of an object!!!

Weight will change if you measure on the Earth or on the moon but the mass won't!!

Unit of mass? kg

Newton's Second Law of Motion

The acceleration of an object is directly proportional to the net force exerted on it and is inversely proportional to the object's mass.

How do we write the above statement in a mathematical expression?

$$\vec{a} = \frac{\sum_i \vec{F_i}}{m}$$

From this we obtain

$$\boxed{\sum_i \vec{F_i} = m\vec{a}}$$

Newton's 2nd Law of Motion

Since it's a vector expression, each component must also satisfy:

$$\boxed{\sum_i F_{ix} = ma_x} \quad \boxed{\sum_i F_{iy} = ma_y} \quad \boxed{\sum_i F_{iz} = ma_z}$$

Unit of the Force

From the vector expression in the previous page, what do you conclude the dimension and the unit of the force are?

$$\boxed{\sum_i \overrightarrow{F_i} = m\vec{a}}$$

The dimension of force is $[m][a] = [M][LT^{-2}]$

The unit of force in SI is $[Force] = [m][a] = [M][LT^{-2}] = (\mathrm{kg})\left(\frac{\mathrm{m}}{\mathrm{s}^2}\right) = kg \cdot m/s^2$

For ease of use, we define a new derived unit called, Newton (N)

$$\boxed{1N \equiv 1kg \cdot m/s^2 \approx \frac{1}{4} lbs}$$