PHYS 1441 – Section 002
Lecture #11

Monday, Mar. 9, 2009

Dr. Jaehoon Yu

- Force of Friction
  - Motion with friction
- Uniform Circular Motion
- Centripetal Acceleration and Force
- Banked and Unbanked Road

Today's homework is homework #6, due 9pm, Tuesday, Mar. 24!!

Announcements

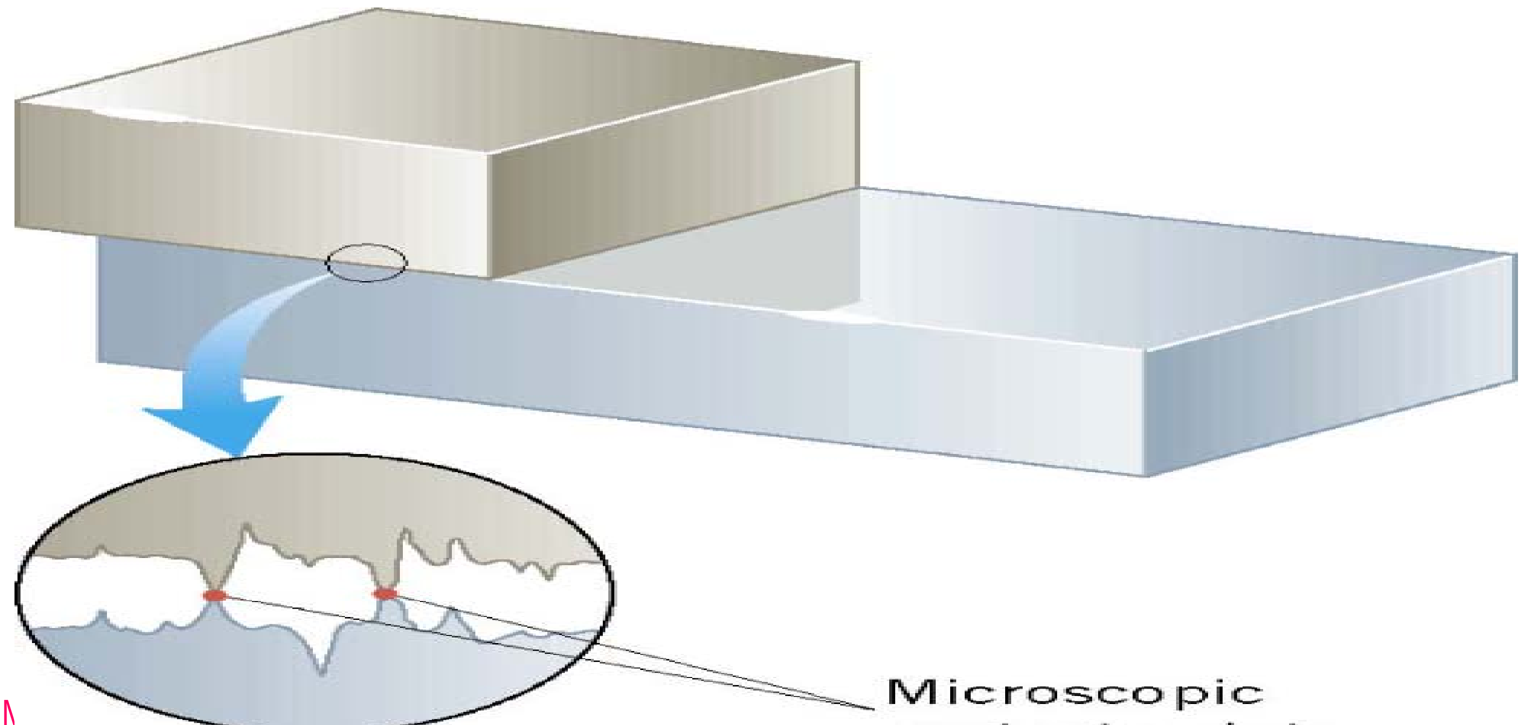
- Mid-term exam
  - Comprehensive exam
    - Covers CH1.1 – what we finish Monday, Mar. 23 + Appendix A
  - Date: Wednesday, Mar. 25
  - Time: 1 – 2:20pm
  - In class – SH103
- Quiz
  - Monday, Mar. 23
  - Beginning of the class
  - CH 4.1 to what we finish this Wednesday, Mar. 11

Special Project

- Using the fact that $g=9.80m/s^2$ on the Earth's surface, find the average density of the Earth.
- 20 point extra credit
- Due: Monday, Mar. 30
- You must show your OWN, detailed work to obtain any credit!!

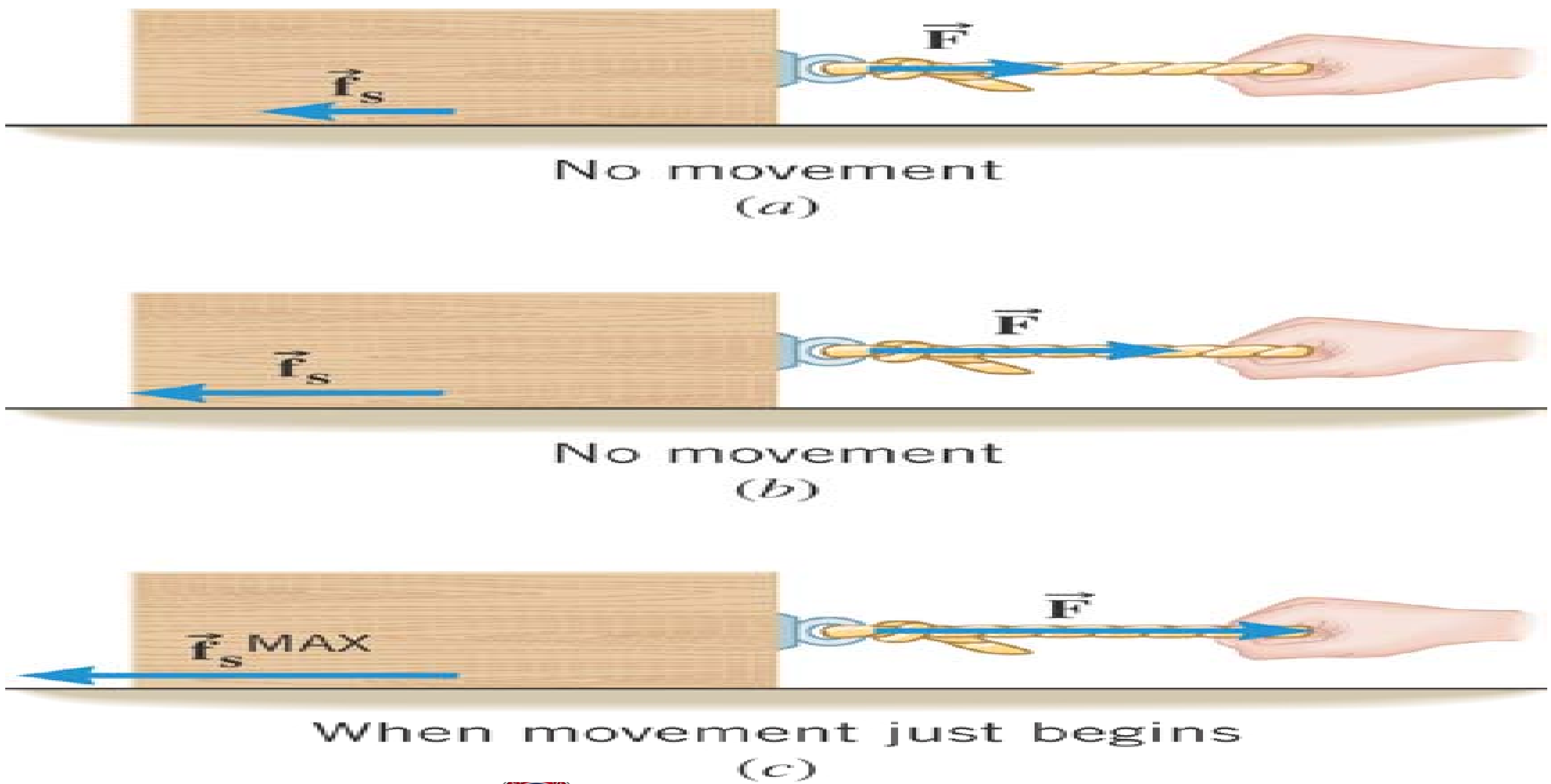
Friction Force

When an object is in contact with a surface there is a force acting on that object. The component of this force that is parallel to the surface is called the ***friction force***. <u>*The resistive force exerted on a moving object due to viscosity or other types of frictional property of the medium in or surface on which the object moves. Always opposite to the movement!!*</u>



Static Friction

When the two surfaces are not sliding across one another the friction is called ***static friction***. ***The resistive force exerted on the object up to the time just before the object starts moving.***



Magnitude of Static Friction

The magnitude of the static friction force can have any value from zero up to the maximum value.

$$f_s \le f_s^{MAX}$$

$$f_s^{MAX} = \mu_s F_N$$

$0 < \mu_s < 1$ is called the <u>coefficient of static friction</u>.

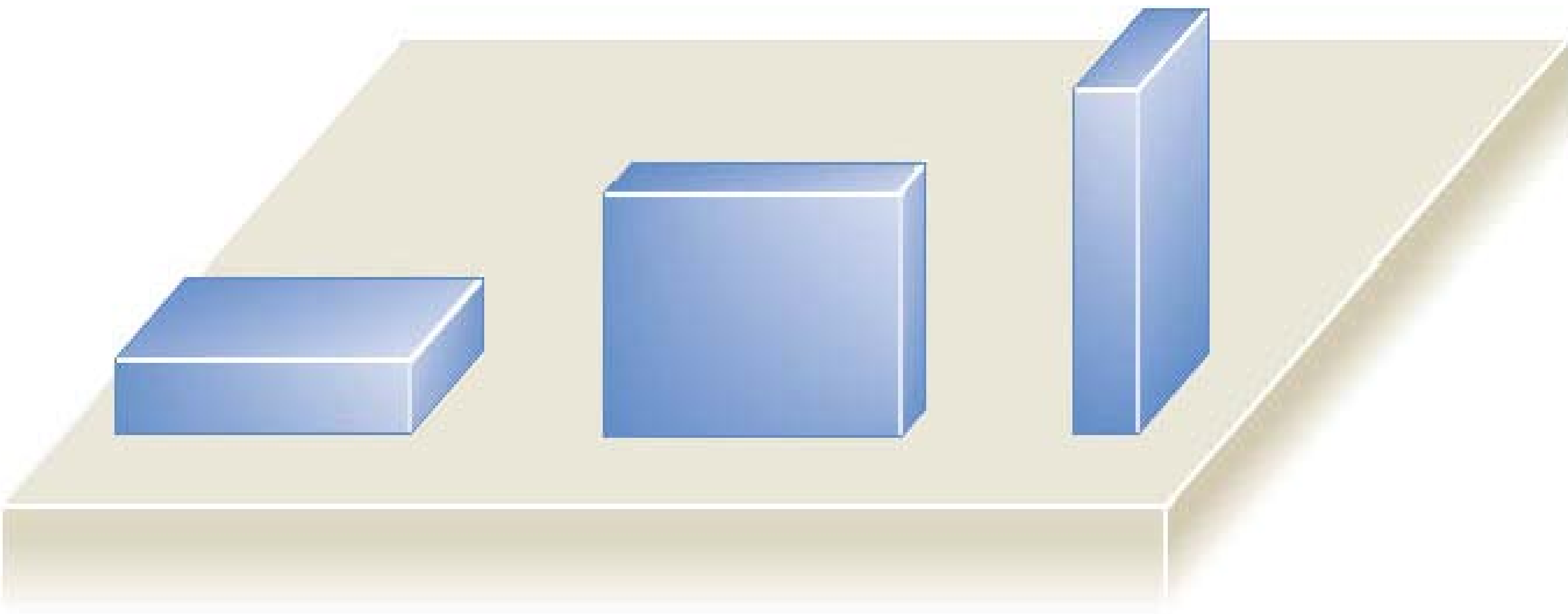
What is the unit? None

Once the object starts moving, there is <u>NO MORE</u> static friction!!

Kinetic friction applies during the move!!

Note that the magnitude of the frictional force <u>does not depend on the contact area of the surfaces</u>.

$$f_s^{MAX} = \mu_s F_N$$



Kinetic Friction

Static friction opposes the *impending* relative motion between two objects.

Kinetic friction opposes the relative sliding motions that actually does occur. ***The resistive force exerted on the object during its movement.***

$$f_k = \mu_k F_N$$

$0 < \mu_k < 1$ is called the coefficient of kinetic friction.

What is the direction of friction forces? opposite to the movement

Coefficient of Friction

Approximate Values of the Coefficients of Friction for Various Surfaces*

Materials	Coefficient of Static Friction, μ_s	Coefficient of Kinetic Friction, μ_k
Glass on glass (dry)	0.94	0.4
Ice on ice (clean, 0 °C)	0.1	0.02
Rubber on dry concrete	1.0	0.8
Rubber on wet concrete	0.7	0.5
Steel on ice	0.1	0.05
Steel on steel (dry hard steel)	0.78	0.42
Teflon on Teflon	0.04	0.04
Wood on wood	0.35	0.3

*The last column gives the coefficients of kinetic friction, a concept that will be discussed shortly.

What are these?

Forces of Friction Summary

Resistive force exerted on a moving object due to viscosity or other types frictional property of the medium in or surface on which the object moves.

These forces are either proportional to the velocity or the normal force.

Force of static friction, f_s: *The resistive force exerted on the object until just before the beginning of its movement*

Empirical Formula

$$\left|\vec{f}_s\right| \le \mu_s \left|\vec{F}_N\right|$$

What does this formula tell you?

Frictional force increases till it reaches the limit!!

Beyond the limit, the object moves, and there is <u>NO MORE</u> static friction but kinetic friction takes it over.

Force of kinetic friction, f_k

The resistive force exerted on the object during its movement

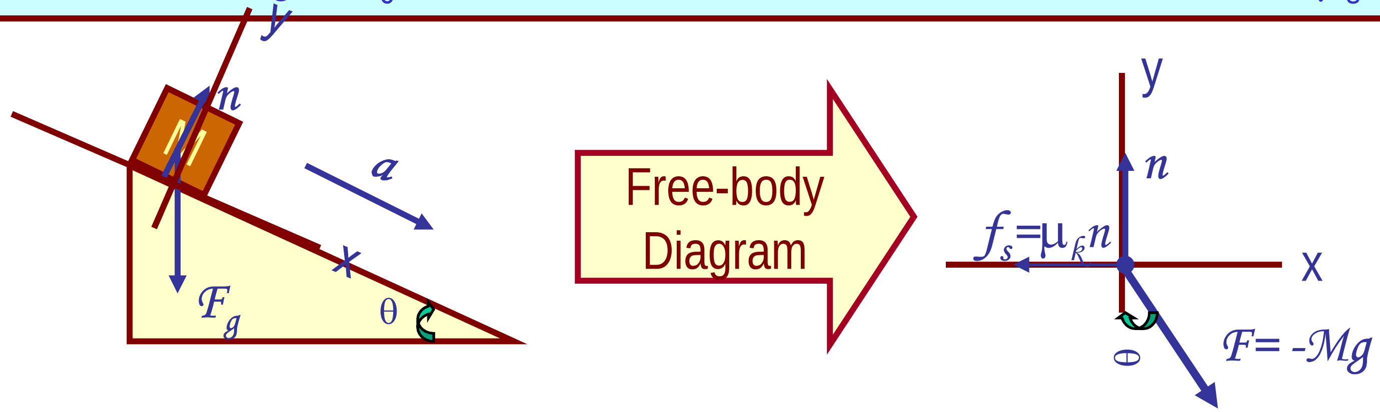
$$\left|\vec{f}_k\right| = \mu_k \left|\vec{F}_N\right|$$

Which direction does kinetic friction apply?

Opposite to the motion!

Example w/ Friction

Suppose a block is placed on a rough surface inclined relative to the horizontal. The inclination angle is increased till the block starts to move. Show that by measuring this critical angle, θ_c, one can determine coefficient of static friction, μ_s.



Net force $$\vec{F} = M\vec{a} = \vec{F}_g + \vec{n} + \vec{f}_s$$

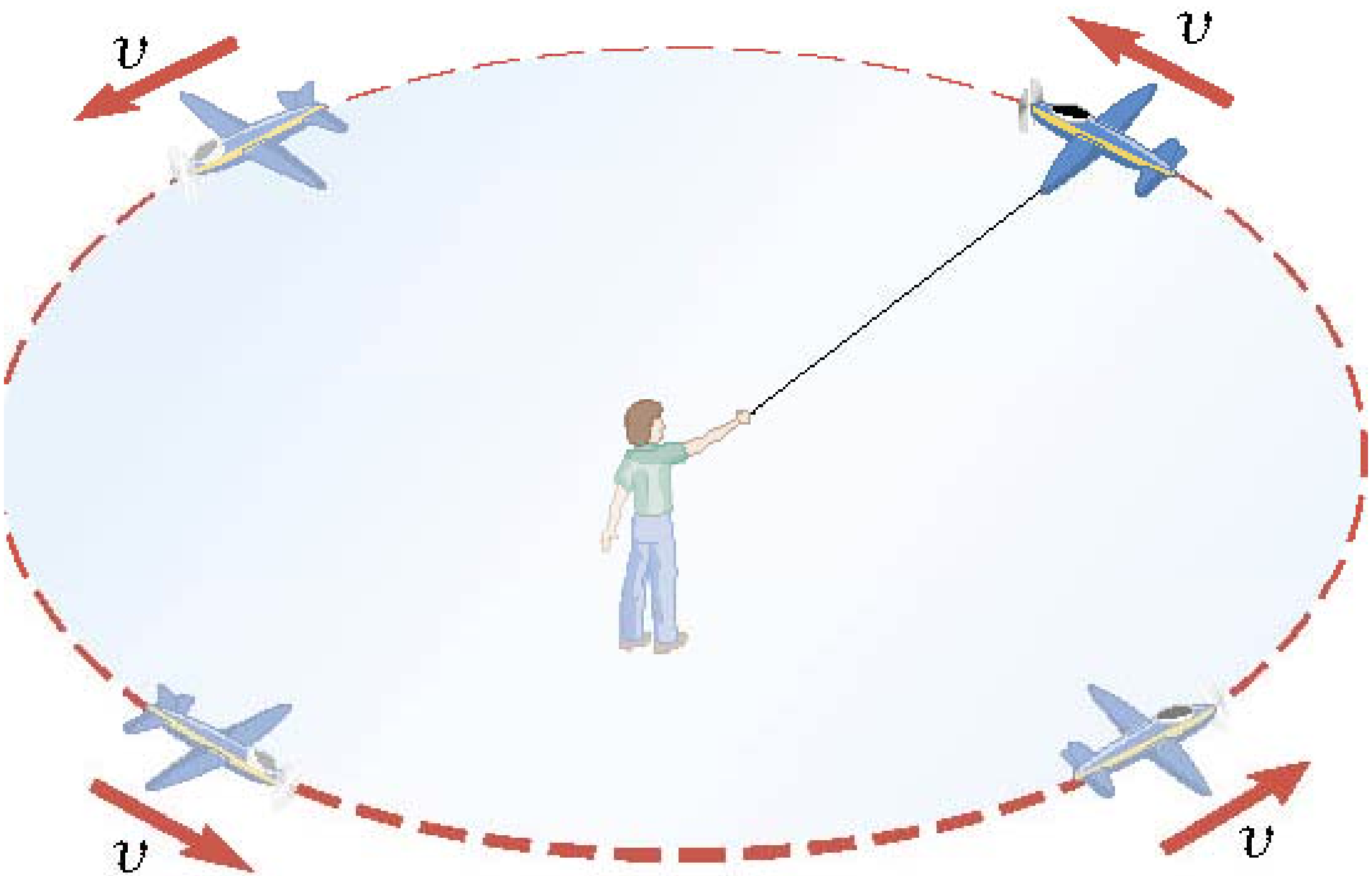
x comp. $$F_x = F_{gx} - f_s = Mg\sin\theta - f_s = 0 \qquad f_s = \mu_s n = Mg\sin\theta_c$$

y comp. $$F_y = Ma_y = n - F_{gy} = n - Mg\cos\theta_c = 0 \qquad n = F_{gy} = Mg\cos\theta_c$$

$$\mu_s = \frac{Mg\sin\theta_c}{n} = \frac{Mg\sin\theta_c}{Mg\cos\theta_c} = \tan\theta_c$$

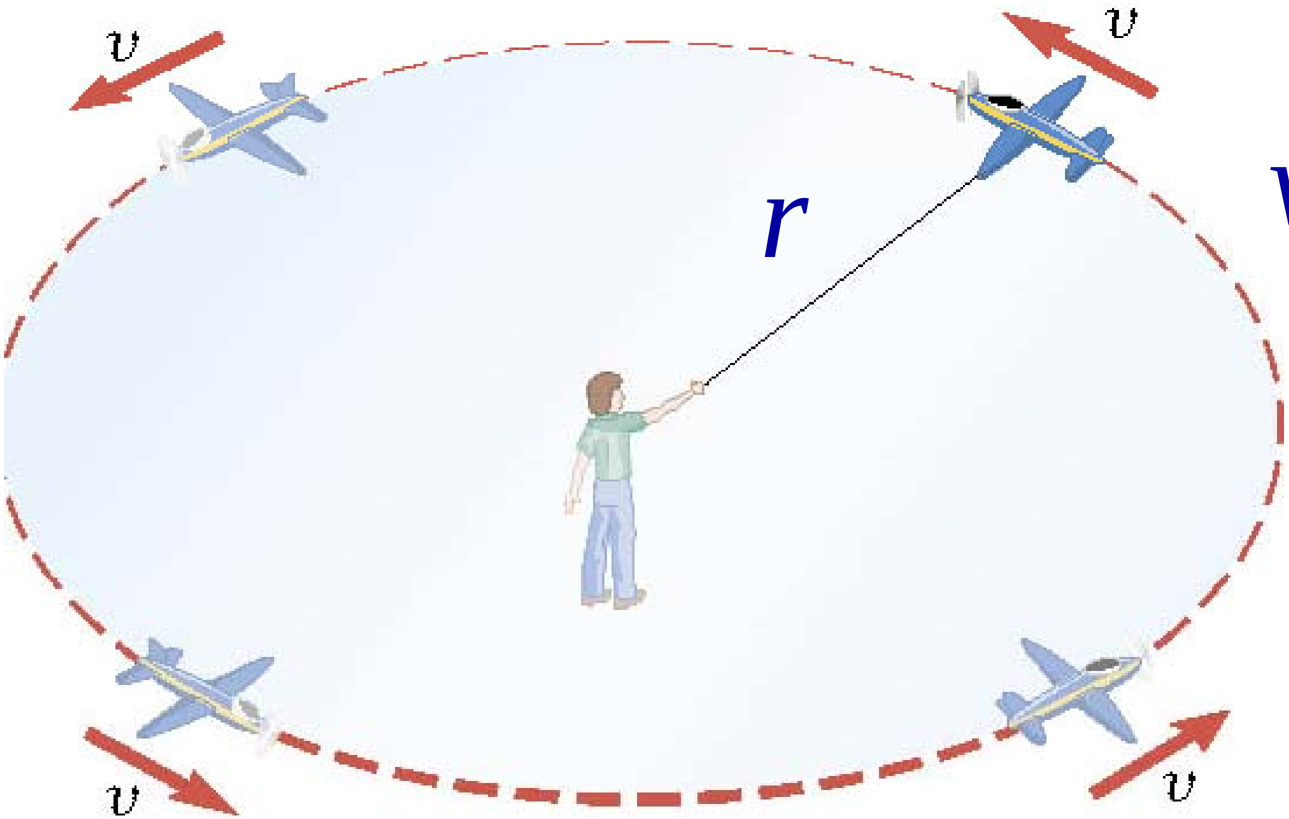
Definition of the Uniform Circular Motion

Uniform circular motion is the motion of an object traveling at a constant speed on a circular path.



Speed of a uniform circular motion?

Let T be the period of this motion, the time it takes for the object to travel once around the complete circle whose radius is r.



$$v = \frac{\text{distance}}{\text{time}}$$

$$= \frac{2\pi r}{T}$$

Ex. : A Tire-Balancing Machine

The wheel of a car has a radius of 0.29m and is being rotated at 830 revolutions per minute on a tire-balancing machine. Determine the speed at which the outer edge of the wheel is moving.

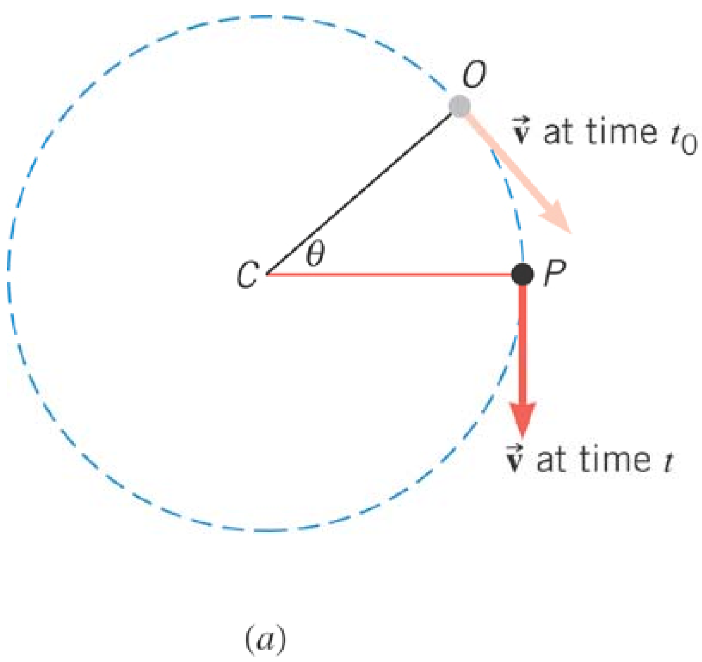
$$\frac{1}{830\,\text{revolutions/min}} = 1.2\times10^{-3}\ \text{min/revolution}$$

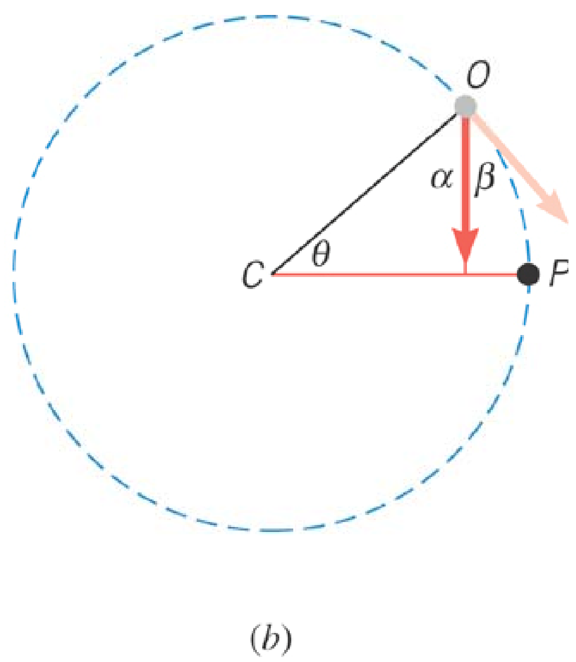
$$T = 1.2\times10^{-3}\ \text{min} = 0.072\ \text{s}$$

$$v = \frac{2\pi r}{T} = \frac{2\pi(0.29\ \text{m})}{0.072\ \text{s}} = 25\,\text{m/s}$$

Centripetal Acceleration

In uniform circular motion, the speed is constant, but the direction of the velocity vector is *not constant.*





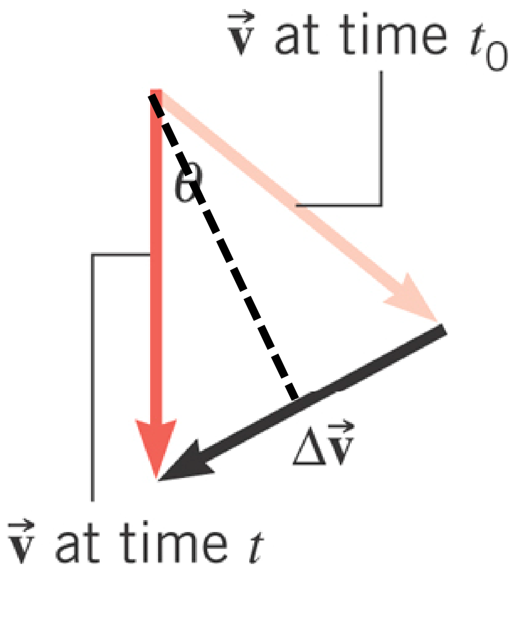
$$\alpha + \beta = 90^{\circ}$$

$$\alpha + \theta = 90^{\circ}$$

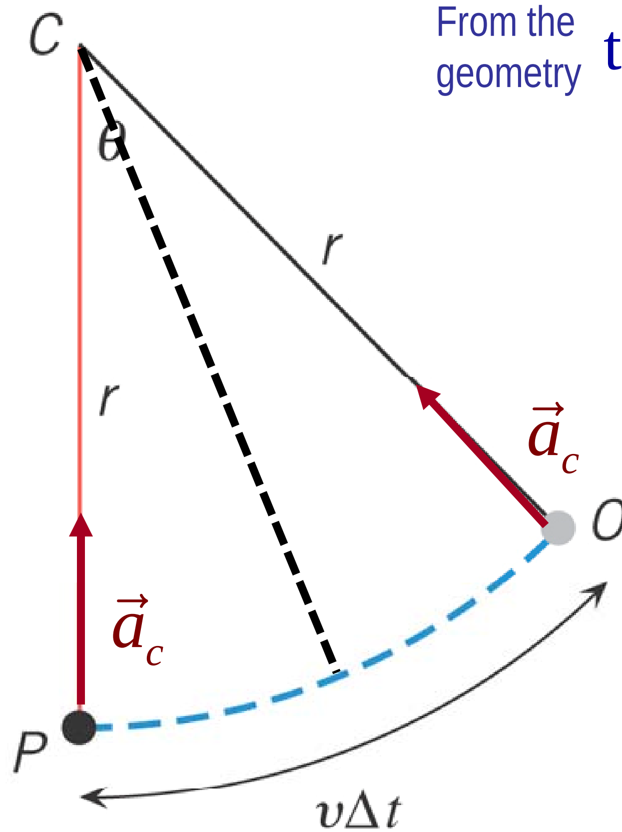
$$\beta - \theta = 0$$

$$\beta = \theta$$

Centripetal Acceleration



(a)

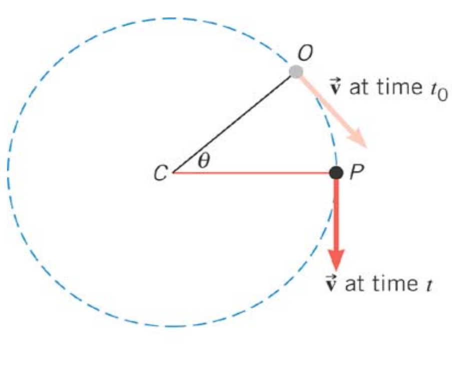


From the geometry

$$\tan\theta/2 = \frac{\Delta v}{\cancel{2}v} = \frac{v\Delta t}{\cancel{2}r}$$

$$\frac{\Delta v}{\Delta t} = \frac{v^2}{r}$$

$$\boxed{a_c = \frac{v^2}{r}}$$

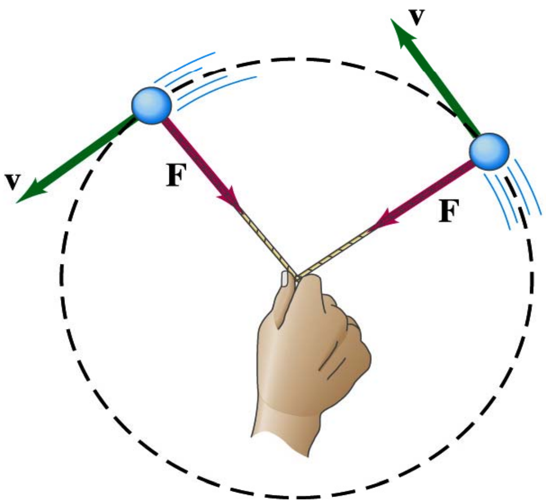


What is the direction of a_c?

Always toward the center of circle!

Centripetal Acceleration

Newton's Second Law & Uniform Circular Motion



The centripetal acceleration is always perpendicular to the velocity vector, v, and points to the center of the axis (radial direction) in a uniform circular motion.*

$$\boxed{a_c = \frac{v^2}{r}}$$

Are there forces in this motion? If so, what do they do?

The force that causes the centripetal acceleration acts toward the center of the circular path and causes the change in the direction of the velocity vector. This force is called the centripetal force.

$$\sum F_c = ma_c = m\frac{v^2}{r}$$

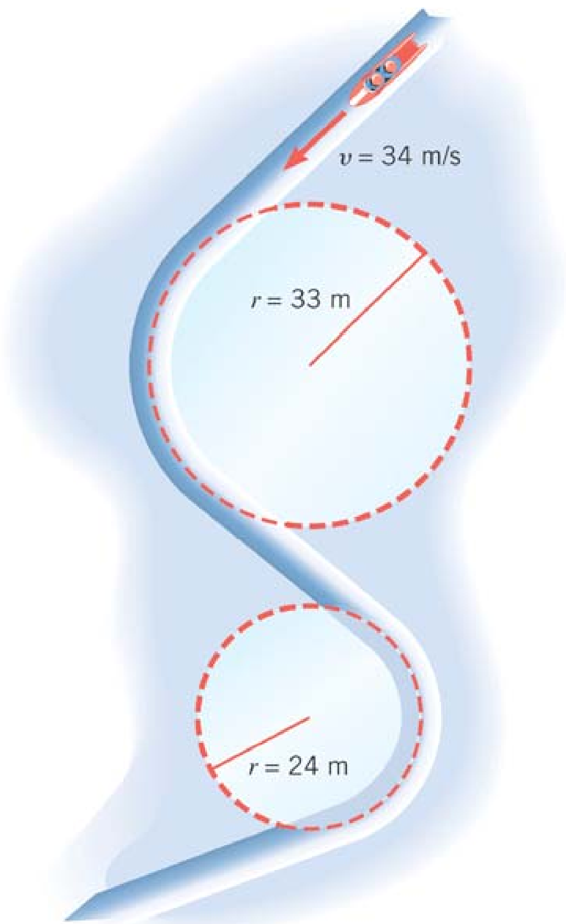
What do you think will happen to the ball if the string that holds the ball breaks?

The external force no longer exist. Therefore, based on Newton's 1st law, the ball will continue its motion without changing its velocity and will fly away following the tangential direction to the circle.

*Mirriam Webster: Proceeding or acting in a direction toward a center or axis

Ex. Effect of Radius on Centripetal Acceleration

The bobsled track at the 1994 Olympics in Lillehammer, Norway, contain turns with radii of 33m and 23m. Find the centripetal acceleration at each turn for a speed of 34m/s, a speed that was achieved in the two –man event. Express answers as multiples of $g=9.8m/s^2$.



Centripetal acceleration:

$$a_r = \frac{v^2}{r}$$

R=33m

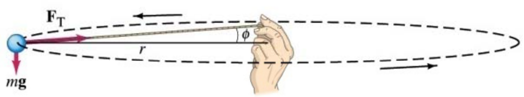
$$a_{r=33m} = \frac{(34)^2}{33} = 35\,m/s^2 = 3.6g$$

R=24m

$$a_{r=24m} = \frac{(34)^2}{24} = 48\,m/s^2 = 4.9g$$

Example of Uniform Circular Motion

A ball of mass 0.500kg is attached to the end of a 1.50m long cord. The ball is moving in a horizontal circle. If the string can withstand maximum tension of 50.0 N, what is the maximum speed the ball can attain before the cord breaks?



Centripetal acceleration:

$$a_r = \frac{v^2}{r}$$

When does the string break?

$$\sum F_r = ma_r = m\frac{v^2}{r} > T$$

when the required centripetal force is greater than the sustainable tension.

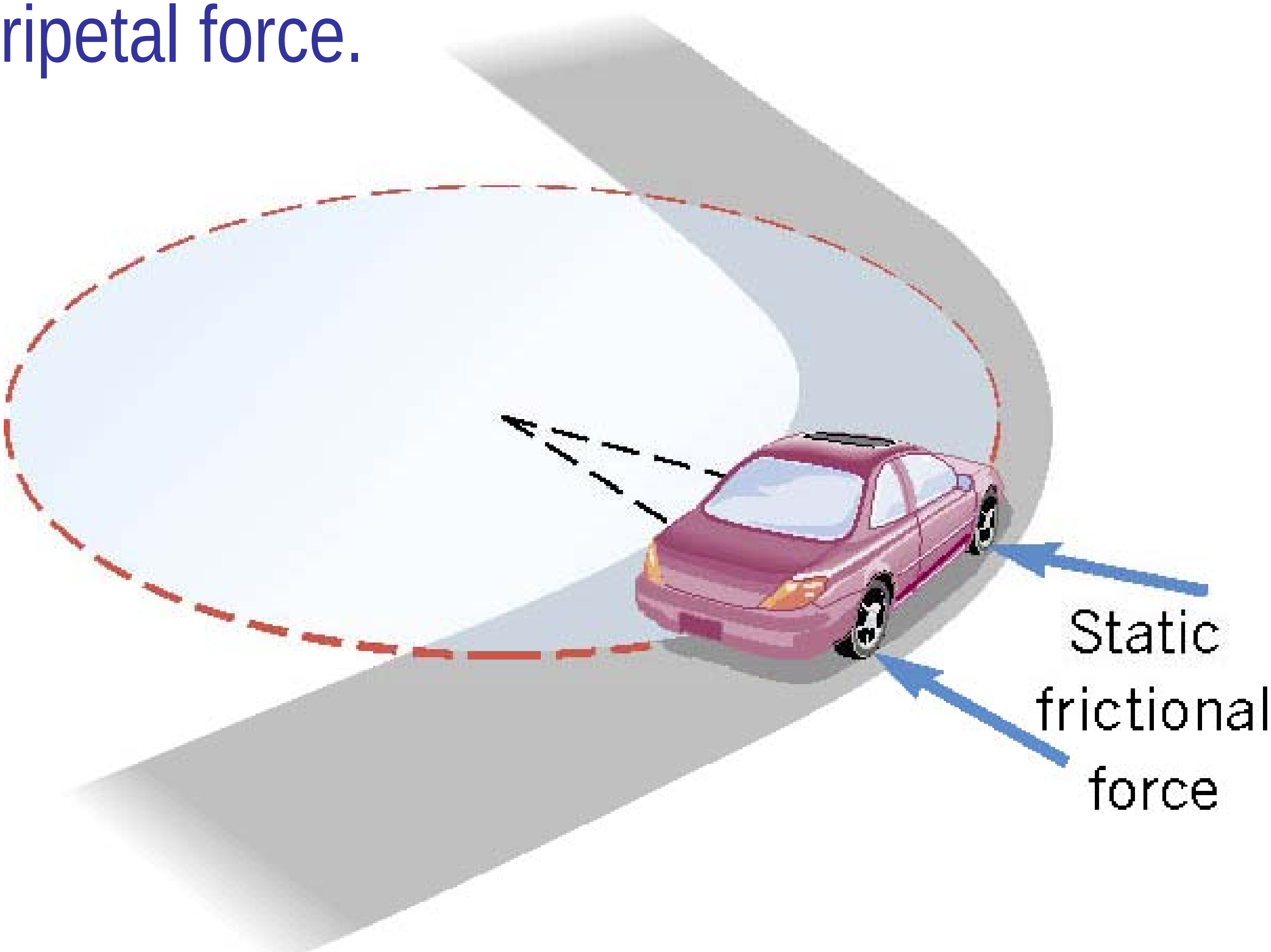
$$m\frac{v^2}{r} = T \qquad v = \sqrt{\frac{Tr}{m}} = \sqrt{\frac{50.0 \times 1.5}{0.500}} = 12.2\,(m/s)$$

Calculate the tension of the cord when speed of the ball is 5.00m/s.

$$T = m\frac{v^2}{r} = 0.500 \times \frac{(5.00)^2}{1.5} = 8.33\,(N)$$

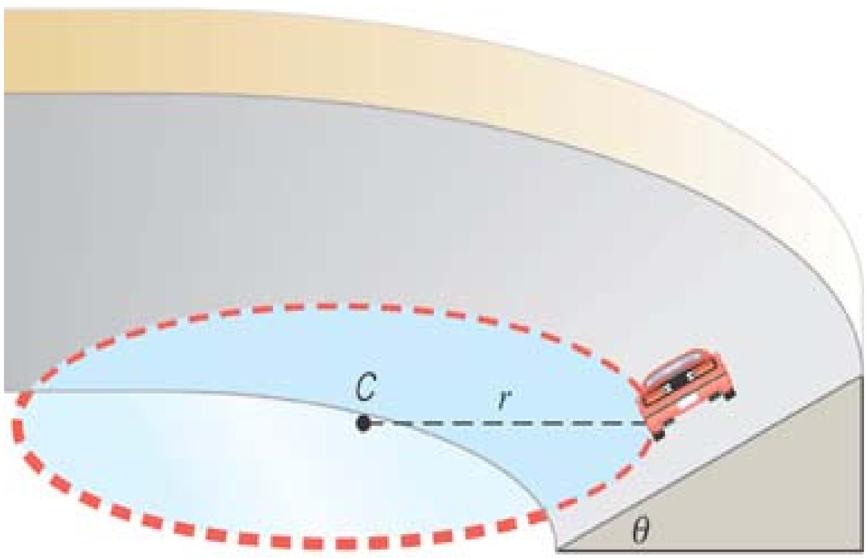
Unbanked Curve and Centripetal Force

On an unbanked curve, the static frictional force provides the centripetal force.



Banked Curves

On a frictionless banked curve, the centripetal force is the horizontal component of the normal force. The vertical component of the normal force balances the car's weight.



(a)

Ex. The Daytona 500

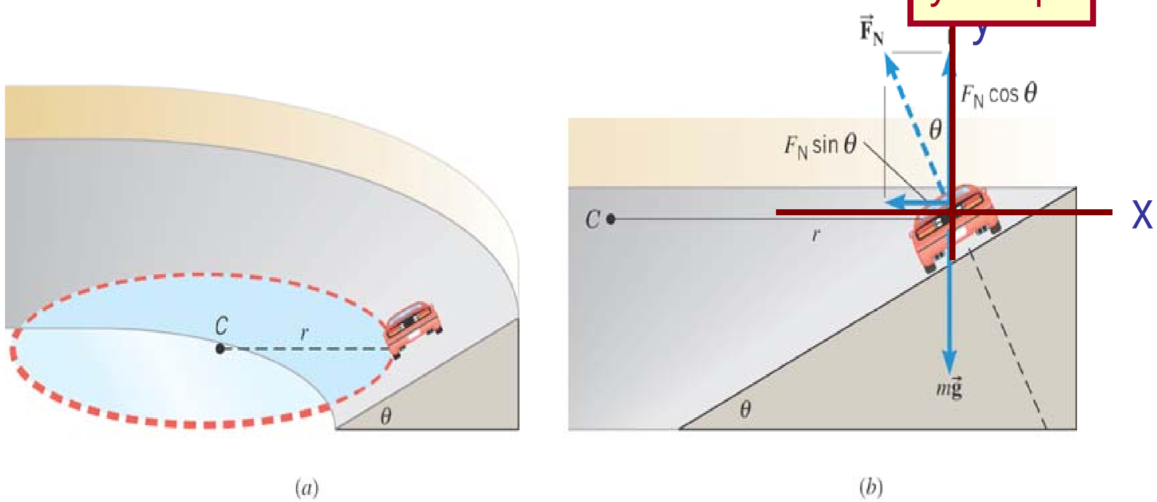
The Daytona 500 is the major event of the NASCAR season. It is held at the Daytona International Speedway in Daytona, Florida. The turns in this oval track have a maximum radius (at the top) of r=-316m and are banked steeply, with θ=31°. Suppose these maximum radius turns were frictionless. At what speed would the cars have to travel around them?

x comp.

$$\sum F_x = F_N \sin\theta - m\frac{v^2}{r} = 0$$

y comp.

$$\sum F_y = F_N \cos\theta - mg = 0$$



$$\tan\theta = \frac{\cancel{m}v^2}{\cancel{m}gr} = \frac{v^2}{gr}$$

$$v^2 = gr\tan\theta$$

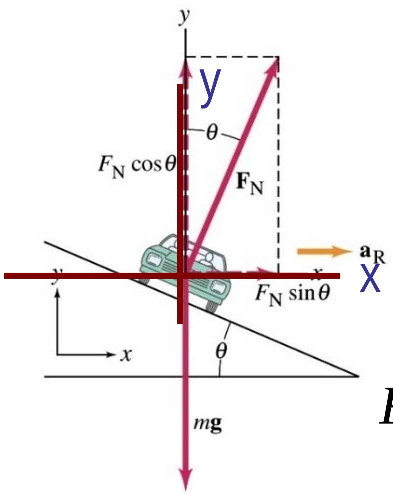
$$v = \sqrt{gr\tan\theta} =$$

$$\sqrt{9.8\cdot 316\tan\left(31^{o}\right)} = 43\,m/s =$$

$$96\,mi/hr$$

Ex. 5 – 7 Bank Angle

(a) For a car traveling with speed v around a curve of radius r, determine the formula for the angle at which the road should be banked so that no friction is required to keep the car from skidding.



x comp. $\sum F_x = F_N \sin\theta - ma_r = F_N \sin\theta - \frac{mv^2}{r} = 0$

$$F_N \sin\theta = \frac{mv^2}{r}$$

y comp. $\sum F_y = F_N \cos\theta - mg = 0 \qquad F_N \cos\theta = mg$

$$F_N = \frac{mg}{\cos\theta} \qquad F_N \sin\theta = \frac{mg\sin\theta}{\cos\theta} = mg\tan\theta = \frac{mv^2}{r} \qquad \boxed{\tan\theta = \frac{v^2}{gr}}$$

(b) What is this angle for an expressway off-ramp curve of radius 50m at a design speed of 50km/h?

$$v = 50\,km/hr = 14\,m/s \qquad \tan\theta = \frac{(14)^2}{50\times 9.8} = 0.4 \qquad \theta = \tan^{-1}(0.4) = 22^\circ$$