PHYS 1441 – Section 002
Lecture #12

Wednesday, Mar. 11, 2009

Dr. Jaehoon Yu

- Newton's Law of Universal Gravitation
- Satellite Motion
- Motion in Resistive Force
- Work done by a constant force

Announcements

- Reading Assignments
  - CH 5.4, 5.5 and 5.9
- Spring break next week
  - Mar. 16 – Mar. 20
  - Have a safe break!
- Mid-term exam
  - Comprehensive exam
    - Covers CH1.1 – what we finish Monday, Mar. 23 (CH6.4?) + Appendix A
  - Date: Wednesday, Mar. 25
  - Time: 1 – 2:20pm
  - In class – SH103
- Quiz
  - Monday, Mar. 23
  - Beginning of the class
  - CH 4.1 to what we finish today
- No colloquium today

Special Project Reminder

- Using the fact that $g=9.80m/s^2$ on the Earth's surface, find the average density of the Earth.
  - Use the following information only
    - the gravitational constant is
    - The radius of the Earth is

$$G = 6.67\times10^{-11}\ N\cdot m^2/kg^2$$

$$R_E = 6.37\times10^3 km$$

- 20 point extra credit
- Due: Monday, Mar. 30
- You must show your OWN, detailed work to obtain any credit!! Copying what is in this lecture note is not acceptable!!

Newton's Law of Universal Gravitation

People have been very curious about the stars in the sky, making observations for a long time. The data people collected, however, have not been explained until Newton has discovered the law of gravitation.

Every particle in the Universe attracts every other particle with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.

How would you write this law mathematically?

$$F_g \propto \frac{m_1 m_2}{r_{12}^2}$$

With G

$$F_g = G \frac{m_1 m_2}{r_{12}^2}$$

G is the universal gravitational constant, and its value is

$$G = 6.673 \times 10^{-11}$$

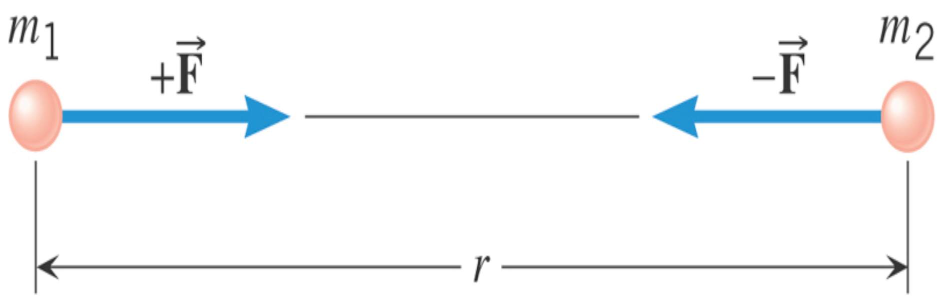
Unit? $N \cdot m^2 / kg^2$

This constant is not given by the theory but must be measured by experiments.

This form of forces is known as the inverse-square law, because the magnitude of the force is inversely proportional to the square of the distances between the objects.

Ex. Gravitational Attraction

What is the magnitude of the gravitational force that acts on each particle in the figure, assuming m_1=12kg, m_2=25kg, and r=1.2m?

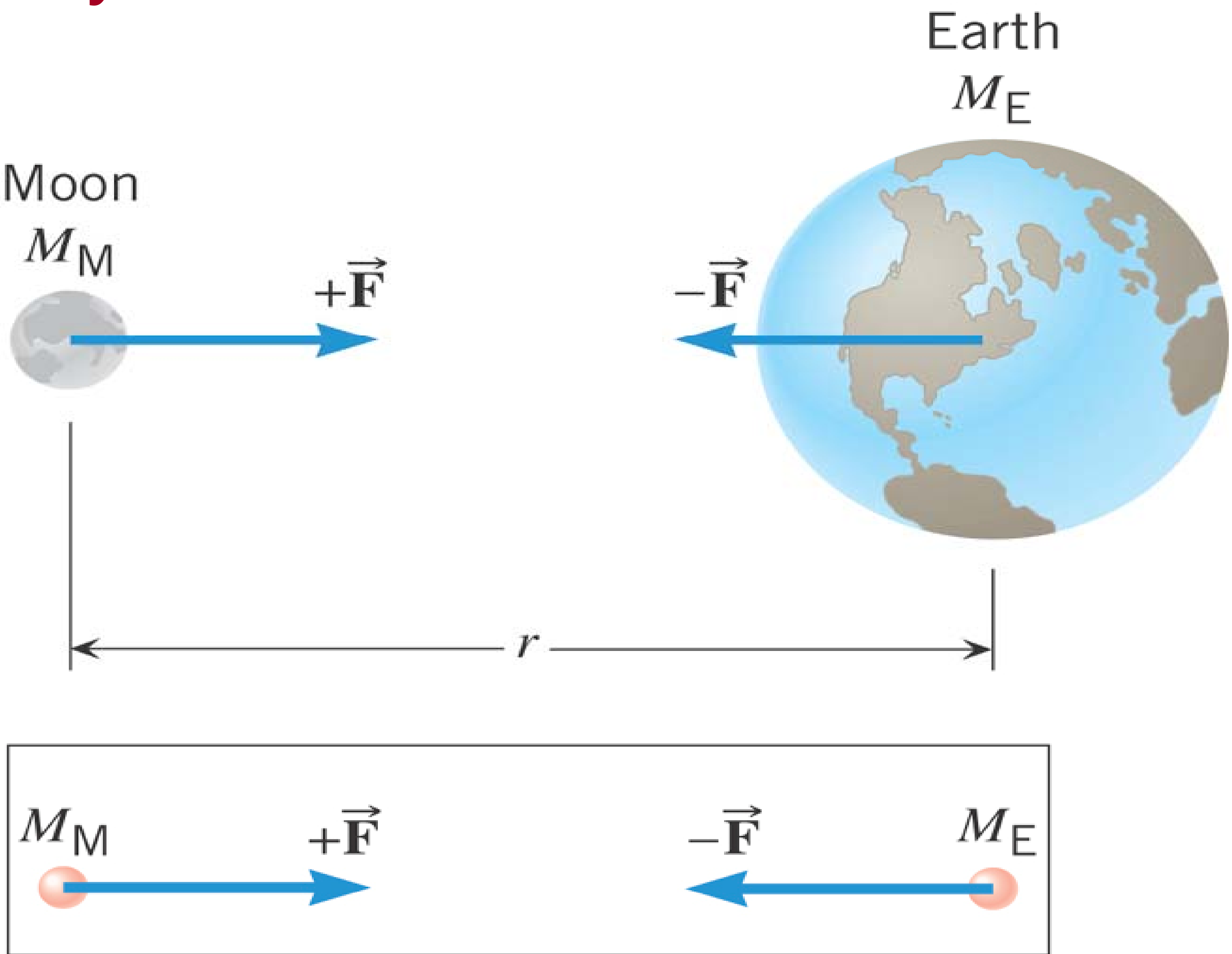


$$F = G\frac{m_1 m_2}{r^2}$$

$$= \left(6.67\times 10^{-11}\ \mathrm{N}\cdot\mathrm{m}^2/\mathrm{kg}^2\right)\frac{(12\ \mathrm{kg})(25\ \mathrm{kg})}{(1.2\ \mathrm{m})^2}$$

$$= 1.4\times 10^{-8}\ \mathrm{N}$$

Why does the Moon orbit the Earth?



Gravitational Force and Weight

Gravitational Force, F_g

The attractive force exerted on an object by the Earth

$$\vec{F}_G = m\vec{a} = m\vec{g}$$

Weight of an object with mass M is

$$W = \left|\vec{F}_G\right| = M\left|\vec{g}\right| = Mg$$

What is the SI unit of weight?

N

Since weight depends on the magnitude of gravitational acceleration, g, it varies depending on geographical location.

By measuring the forces one can determine masses. This is why you can measure mass using the spring scale.

Gravitational Acceleration

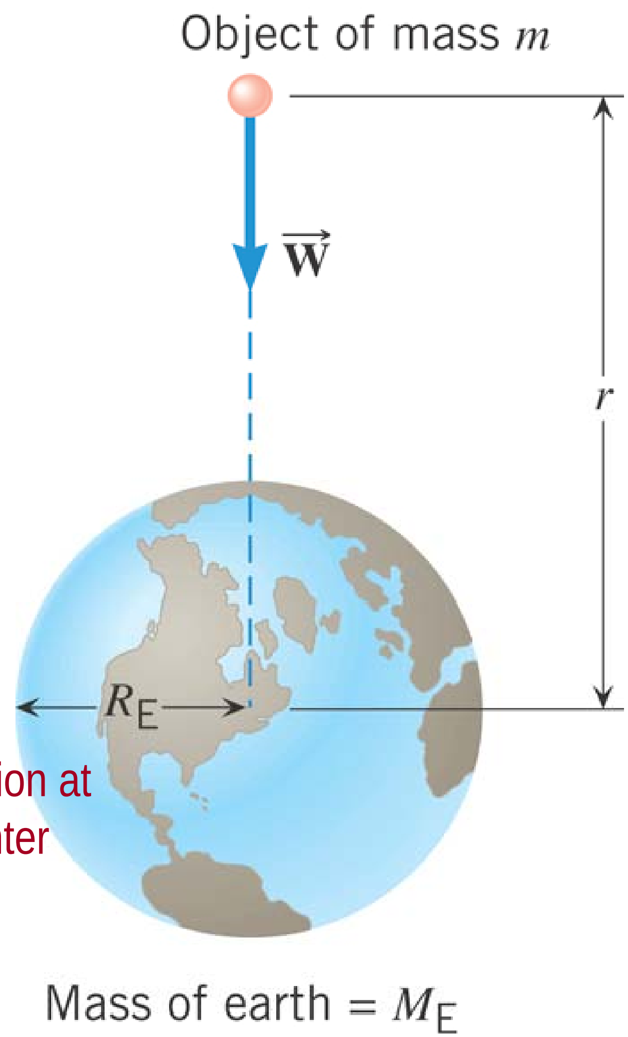
$$W = G\frac{M_E m}{r^2}$$

$$W = mg$$

$$\cancel{m}g = G\frac{M_E \cancel{m}}{r^2}$$

$$g = G\frac{M_E}{r^2}$$

Gravitational acceleration at distance r from the center of the earth!



What is the SI unit of g? $\boxed{m/s^2}$

Magnitude of the gravitational acceleration on the surface of the Earth

Gravitational force on the surface of the earth:

$$F_G = G\frac{M_E m}{r^2} = G\frac{M_E \cancel{m}}{R_E^2} = \cancel{m}g$$

$$g = G\frac{M_E}{R_E^2}$$

$G = 6.67\times10^{-11}\ \mathrm{N\cdot m^2/kg^2}$

$M_E = 5.98\times10^{24}\ \mathrm{kg};\quad R_E = 6.38\times10^{6}\ m$

$$= \left(6.67\times10^{-11}\ \mathrm{N\cdot m^2/kg^2}\right)\frac{\left(5.98\times10^{24}\ \mathrm{kg}\right)}{\left(6.38\times10^{6}\ \mathrm{m}\right)^2}$$

$$= 9.80\ \mathrm{m/s^2}$$

Example for Universal Gravitation

Using the fact that $g=9.80 m/s^2$ on the Earth's surface, find the average density of the Earth.

Since the gravitational acceleration is

$$F_g = G\frac{M_E m}{R_E^2} = mg$$

Solving for g

$$g = G\frac{M_E}{R_E^2} = 6.67\times 10^{-11}\frac{M_E}{R_E^2}$$

Solving for M_E

$$M_E = \frac{R_E^2 g}{G}$$

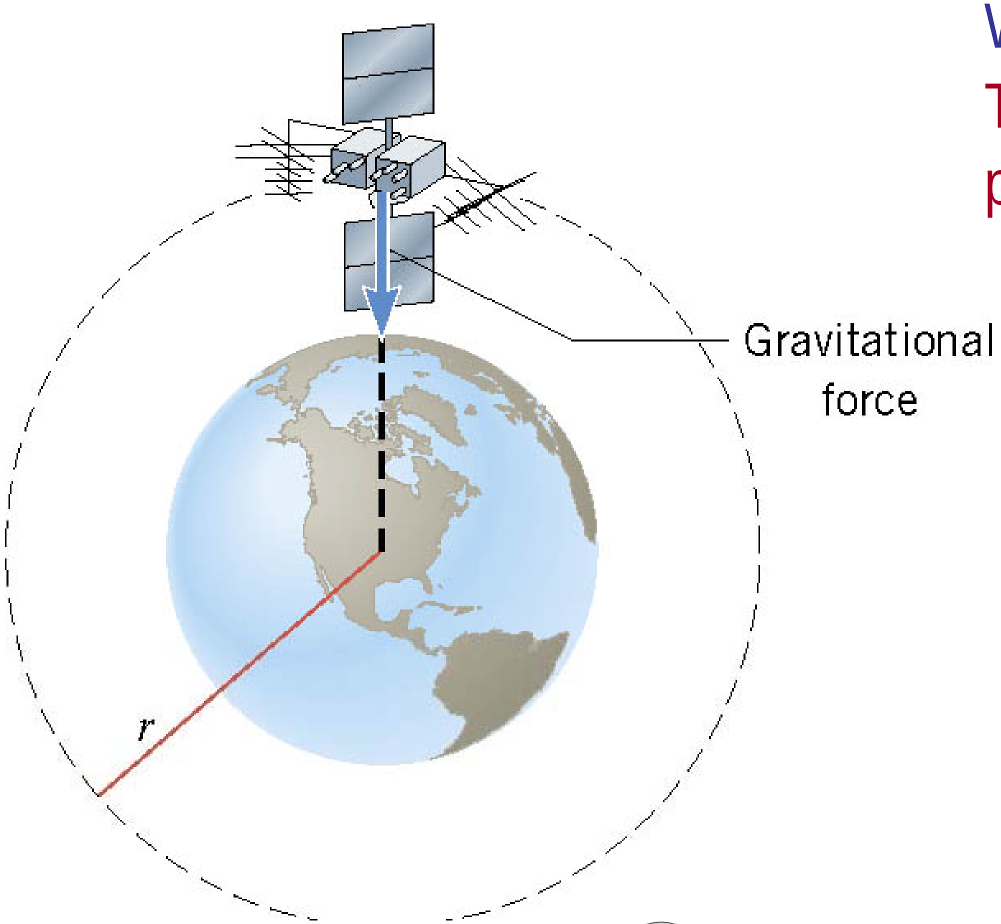
Therefore the density of the Earth is

$$\rho = \frac{M_E}{V_E} = \frac{\frac{R_E^2 g}{G}}{\frac{4\pi}{3}R_E^3} = \frac{3g}{4\pi G R_E}$$

$$= \frac{3\times 9.80}{4\pi\times 6.67\times 10^{-11}\times 6.37\times 10^6} = 5.50\times 10^3 kg/m^3$$

Satellite in Circular Orbits

There is only one speed that a satellite can have if the satellite is to remain in an orbit with a fixed radius.



What is the centripetal force?

The gravitational force of the earth pulling the satellite!

$$F_c = G\frac{mM_E}{r^2} = m\frac{v^2}{r}$$

$$v^2 = \frac{GM_E}{r}$$

$$\boxed{v = \sqrt{\frac{GM_E}{r}}}$$

Ex. Orbital Speed of the Hubble Space Telescope

Determine the speed of the Hubble Space Telescope orbiting at a height of 598 km above the earth's surface.

$$v = \sqrt{\frac{GM_E}{r}}$$

$$= \sqrt{\frac{\left(6.67\times10^{-11}\,\mathrm{N\cdot m^2/kg^2}\right)\left(5.98\times10^{24}\,\mathrm{kg}\right)}{6.38\times10^{6}\,\mathrm{m}+598\times10^{3}\,\mathrm{m}}}$$

$$= 7.56\times10^{3}\,\mathrm{m/s} \quad \left(16900\,\mathrm{mi/h}\right)$$

Period of a Satellite in an Orbit

Speed of a satellite $$v = \sqrt{\frac{GM_E}{r}} = \frac{2\pi r}{T}$$

$$\frac{GM_E}{r} = \left(\frac{2\pi r}{T}\right)^2$$

Square either side and solve for T2

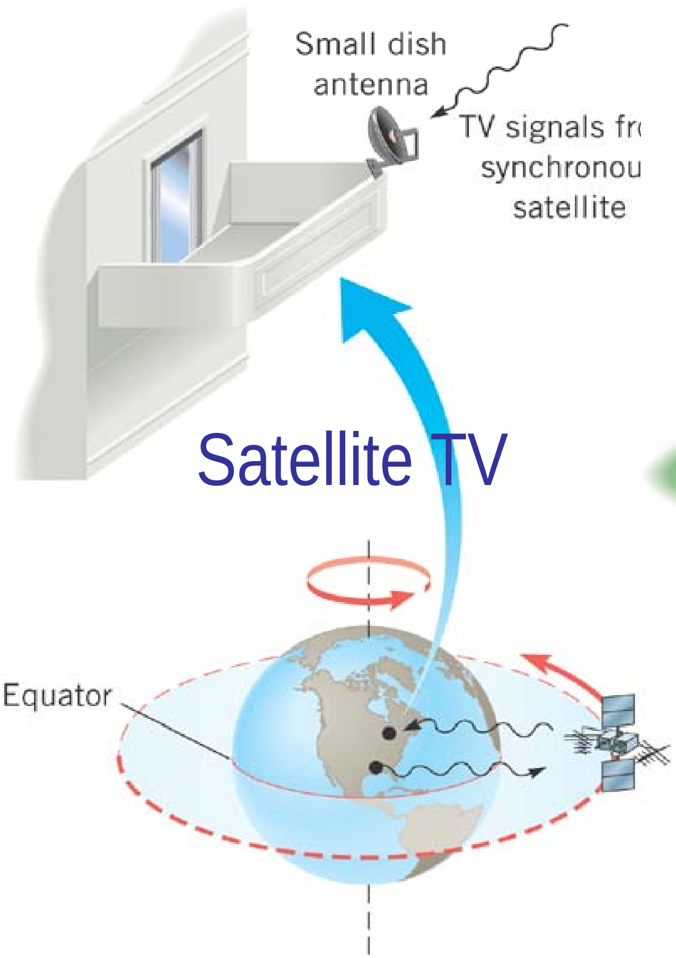
$$T^2 = \frac{(2\pi)^2 r^3}{GM_E}$$

Period of a satellite $$T = \frac{2\pi r^{3/2}}{\sqrt{GM_E}}$$

Kepler's 3rd Law

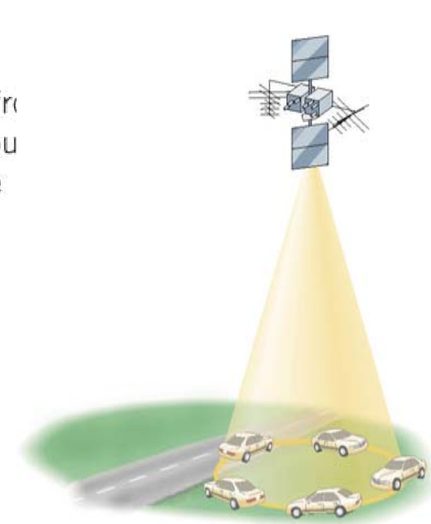
This is applicable to any satellite or even for planets and moons.

Geo-synchronous Satellites



Satellite TV

Global Positioning System (GPS)

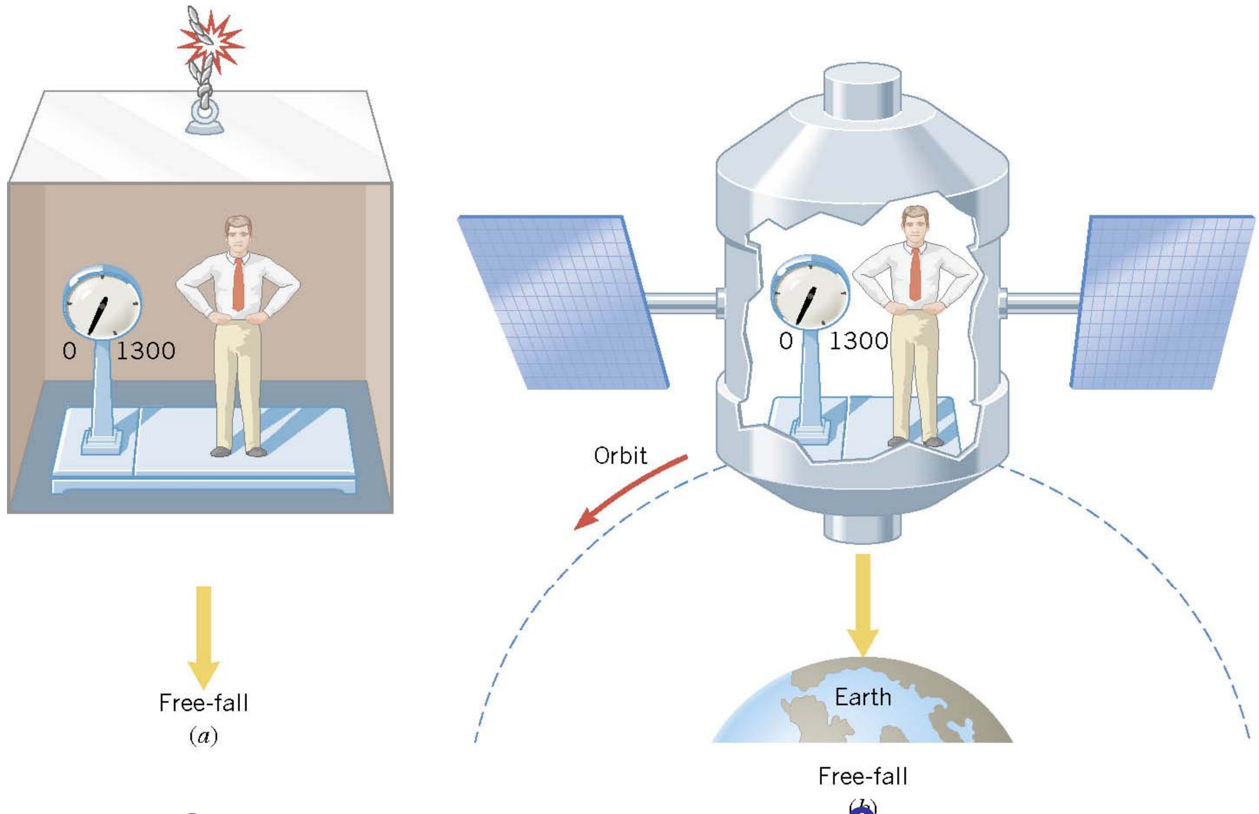


(a)

What period should these satellites have?

The same as the earth!! 24 hours

Ex. Apparent Weightlessness and Free Fall



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In each case, what is the weight recorded by the scale?

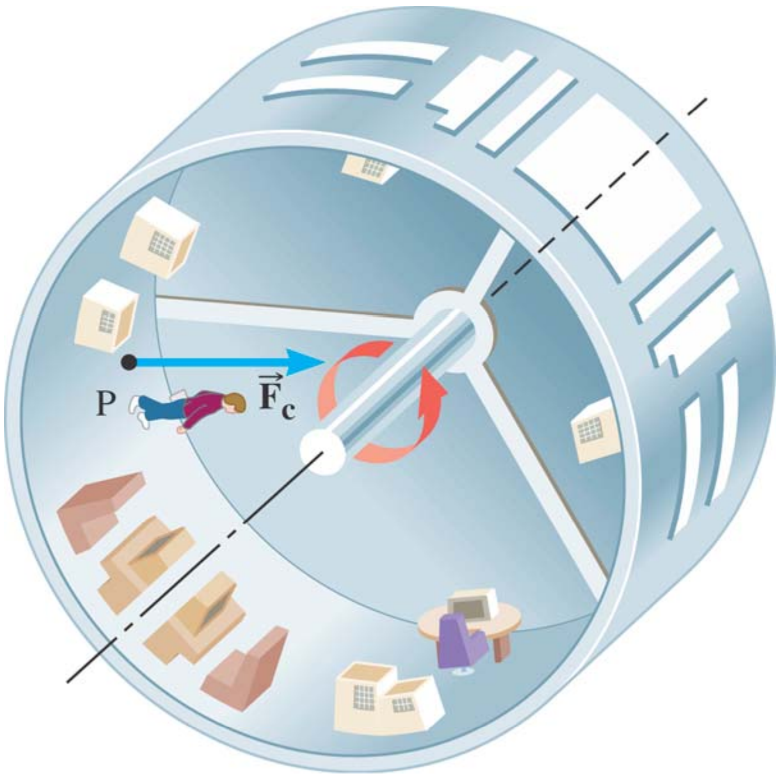
Ex. Artificial Gravity

At what speed must the surface of the space station move so that the astronaut experiences a push on his feet equal to his weight on earth? The radius is 1700 m.

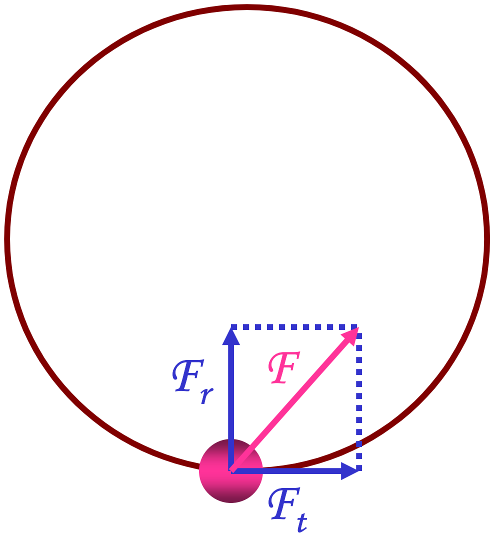
$$F_c = m\frac{v^2}{r} = mg$$

$$v = \sqrt{rg}$$

$$= \sqrt{(1700\ \mathrm{m})(9.80\,\mathrm{m/s^2})}$$



Forces in Non-uniform Circular Motion



The object has both tangential and radial accelerations.

What does this statement mean?

The object is moving under both tangential and radial forces.

$$\vec{F} = \vec{F}_r + \vec{F}_t$$

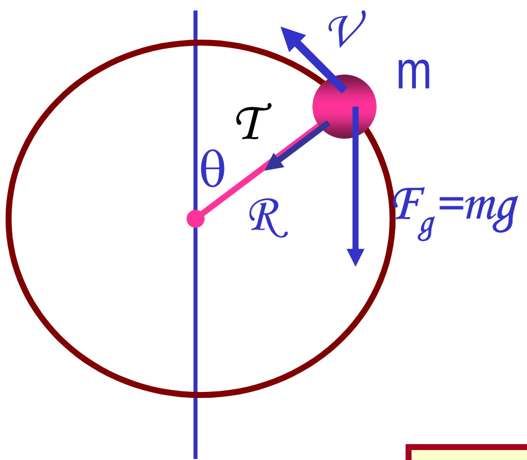
These forces cause not only the velocity but also the speed of the ball to change. The object undergoes a curved motion in the absence of constraints, such as a string.

What is the magnitude of the net acceleration?

$$a = \sqrt{a_r^2 + a_t^2}$$

Example for Non-Uniform Circular Motion

A ball of mass m is attached to the end of a cord of length R. The ball is moving in a vertical circle. Determine the tension of the cord at any instance in which the speed of the ball is v and the cord makes an angle θ with vertical.



What are the forces involved in this motion?

- The gravitational force F_g
- The radial force, T, providing the tension.

tangential comp.

$$\sum F_t = mg\sin\theta = ma_t$$

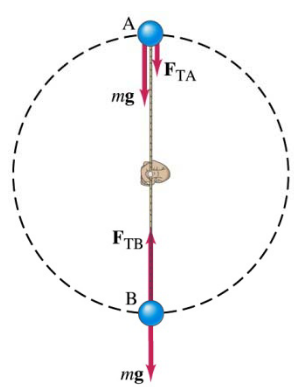
$$\boxed{a_t = g\sin\theta}$$

Radial comp.

$$\sum F_r = T + mg\cos\theta = ma_r = m\frac{v^2}{R}$$

$$\boxed{T = m\left(\frac{v^2}{R} - g\cos\theta\right)}$$

At what angles the tension becomes the maximum and the minimum. What are the tensions?



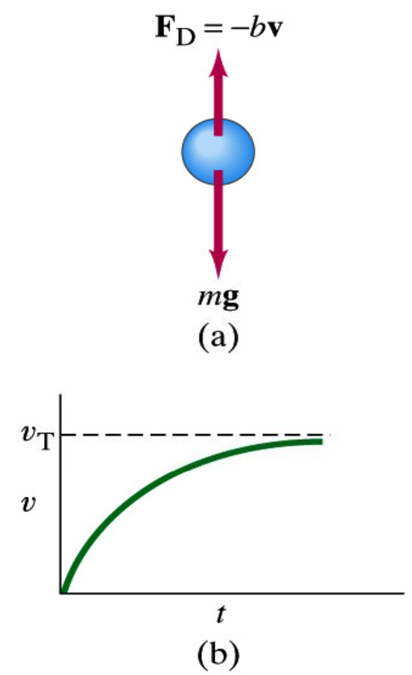
Motion in Resistive Forces

Medium can exert resistive forces on an object moving through it due to viscosity or other types frictional properties of the medium.

Some examples? $\boxed{\text{Air resistance, viscous force of liquid, etc}}$

These forces are exerted on moving objects in opposite direction of the movement.

These forces are proportional to such factors as speed. They almost always increase with increasing speed.



Two different cases of proportionality:

1. Forces linearly proportional to speed: Slowly moving or very small objects
2. Forces proportional to square of speed: Large objects w/ reasonable speed