PHYS 1441 – Section 002
Lecture #14

Monay, Mar. 30, 2009

Dr. Jaehoon Yu

- Work-Kinetic Energy Theorem
- Work with friction
- Gravitational Potential Energy
- Elastic Potential Energy
- Conservation of Energy
- Power

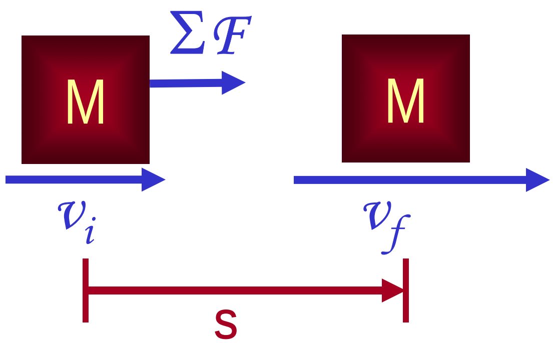
Today's homework is homework #8, due 9pm, Tuesday, Apr. 7!!

Announcements

- Trouble with Homework #6
  - There were so many people having trouble with HW6.
  - Will give average or the HW6 score whichever is higher for your HW6 grade
- Term exam results
  - Average score: 58.3/99
    - Equivalent to 58.9/100 → very good!!!
    - Previous exam: 46.7/106 →44.1/100
  - Top score: 99/99
- Evaluation criteria
  - Homework: 25%
  - Midterm and Final Comprehensive Exams: 19% each
  - One better of the two term Exams: 12%
  - Lab score: 15%
  - Pop-quizzes: 10%
  - Extra credits: 10%
- Mid-term grade discussion this Wednesday
  - Do not miss this

Kinetic Energy and Work-Kinetic Energy Theorem

- Some problems are hard to solve using Newton's second law
  - If forces exerting on an object during the motion are complicated
  - Relate the work done on the object by the net force to the change of the speed of the object



Suppose net force ΣF was exerted on an object for displacement s to increase its speed from v_i to v_f.

The work on the object by the net force ΣF is

$$W = \left(\sum \vec{F}\right) \cdot \vec{s} = \left(ma\cos 0\right)s = \left(ma\right)s$$

Using the kinematic equation of motion $2as = v_f^2 - v_0^2$ ⇒ $as = \dfrac{v_f^2 - v_0^2}{2}$

Kinetic Energy: $KE \equiv \dfrac{1}{2}mv^2$

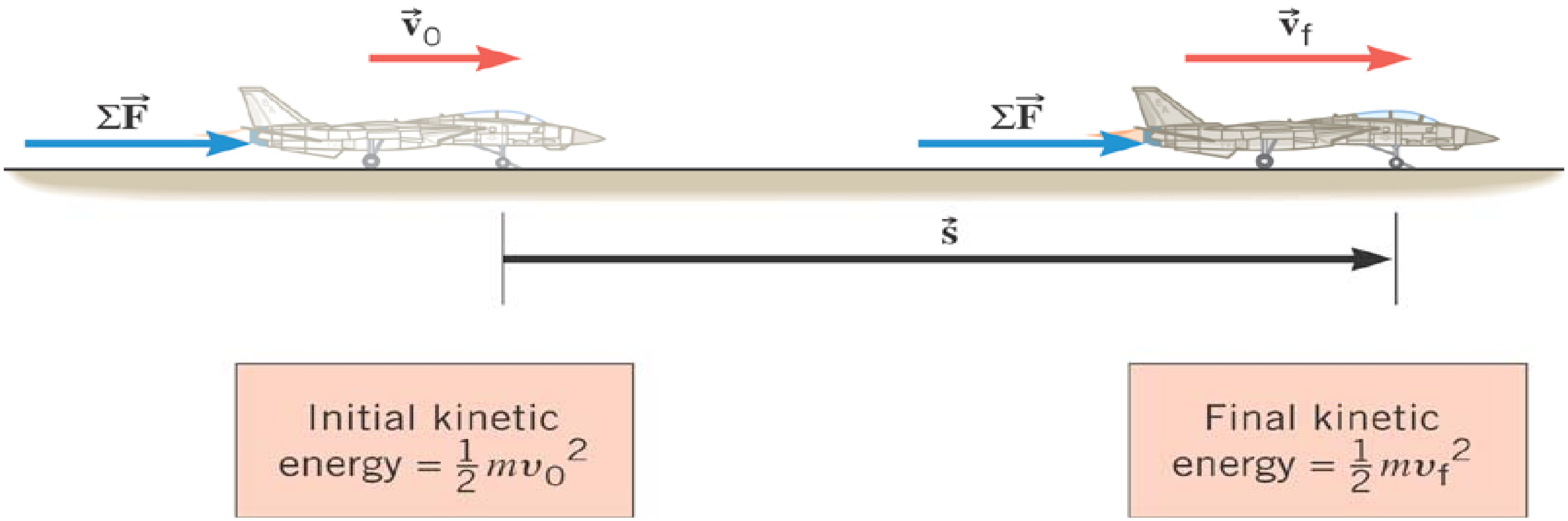
Work $W = \left(ma\right)s = \dfrac{1}{2}m\left(v_f^2 - v_0^2\right) = \dfrac{1}{2}mv_f^2 - \dfrac{1}{2}mv_0^2$

Work $W = \dfrac{1}{2}mv_f^2 - \dfrac{1}{2}mv_i^2 = KE_f - KE_i = \Delta KE$

Work done by the net force causes change in the object's kinetic energy.

Work-Kinetic Energy Theorem

Work-Kinetic Energy Theorem

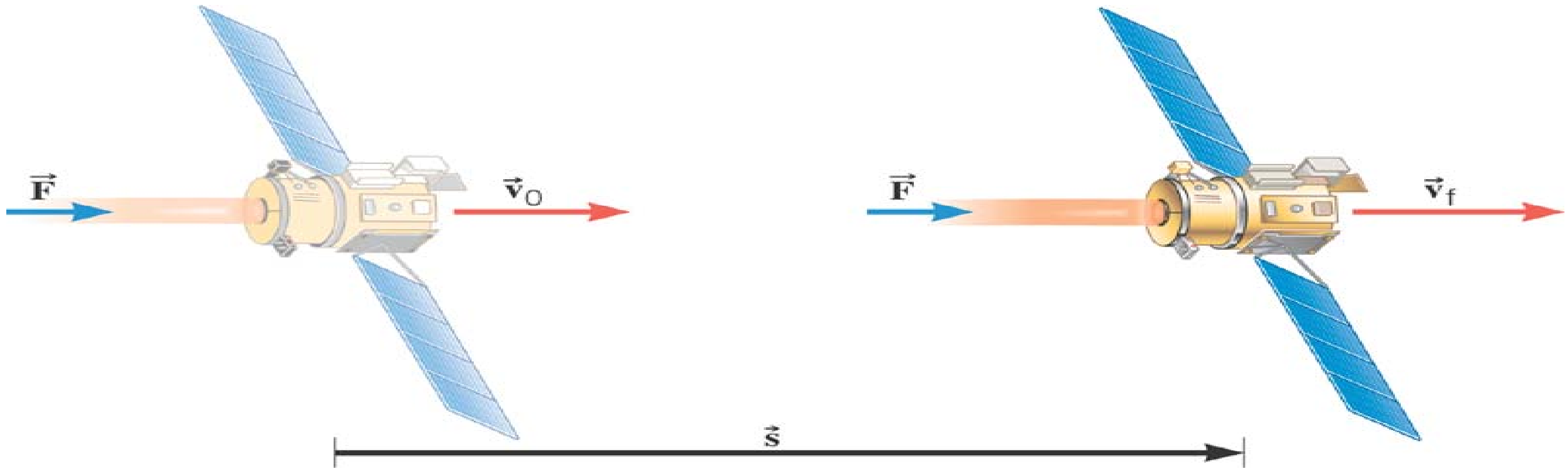


When a net external force by the jet engine does work on and object, the kinetic energy of the object changes according to

$$W = \mathrm{KE_f} - \mathrm{KE_o} = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_o^2$$

Ex. Deep Space 1

The mass of the space probe is 474-kg and its initial velocity is 275 m/s. If the 56.0-mN force acts on the probe parallel through a displacement of 2.42×10^{9}m, what is its final speed?



$$\left[\left(\sum \mathrm{F}\right)\cos\theta\right]s = \tfrac{1}{2}mv_{\mathrm{f}}^{2} - \tfrac{1}{2}mv_{o}^{2}$$

Solve for v_f

$$v_f = \sqrt{v_o^2 + 2\left(\sum F\cos\theta\right)s/m}$$

$$v_f = \sqrt{(275)^2 + 2\left(5.60\times10^{-2}\right)\cos 0^\circ\left(2.42\times10^{9}\right)/474} = 805\,\mathrm{m/s}$$

Ex. Satellite Motion and Work By the Gravity

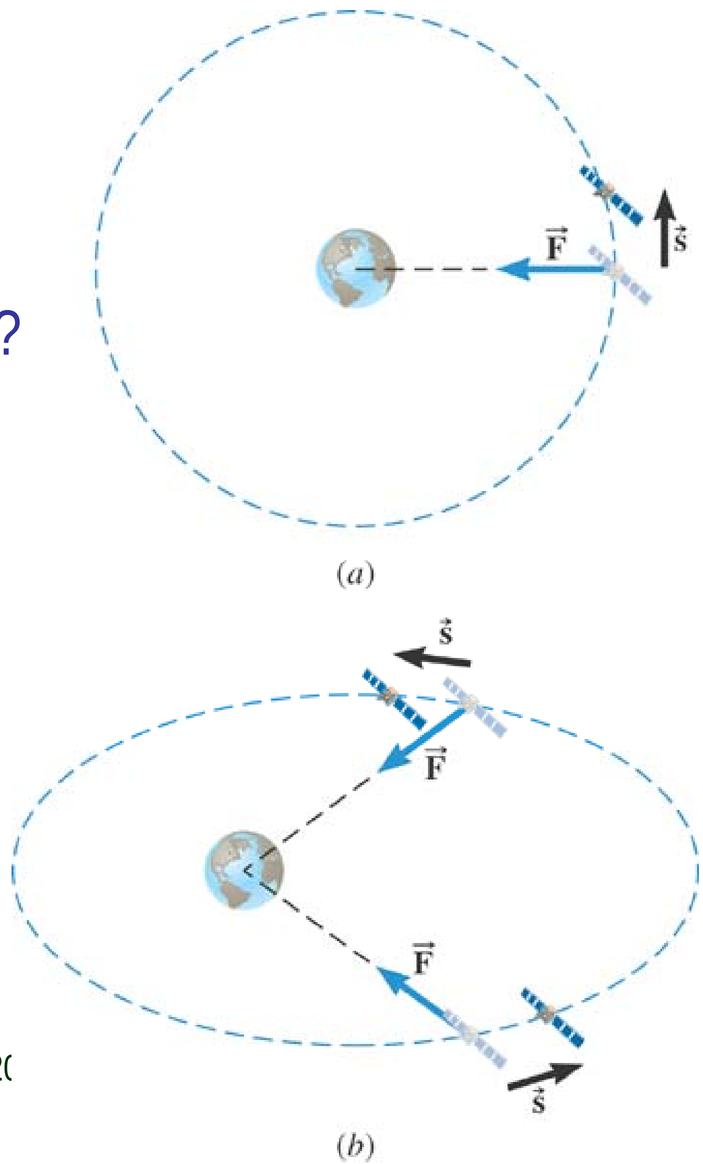
A satellite is moving about the earth in a circular orbit and an elliptical orbit. For these two orbits, determine whether the kinetic energy of the satellite changes during the motion.

For a circular orbit No change! Why not?

Gravitational force is the only external force but it is perpendicular to the displacement. So no work.

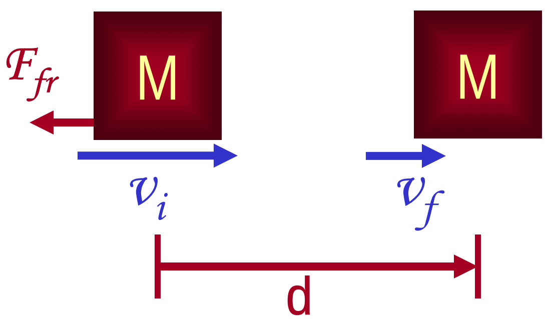
For an elliptical orbit Changes! Why?

Gravitational force is the only external force but its angle with respect to the displacement varies. So it performs work.



Work and Energy Involving Kinetic Friction

- What do you think the work looks like if there is friction?
  - Static friction does not matter! Why? **It isn't there when the object is moving.**
  - Then which friction matters? **Kinetic Friction**



Friction force F_{fr} works on the object to slow down

The work on the object by the friction F_{fr} is

$$W_{fr} = F_{fr} d\cos(180) = -F_{fr} d \quad \Delta KE = -F_{fr} d$$

The negative sign means that the friction took away the energy!!

The final kinetic energy of an object, taking into account its initial kinetic energy, friction force and other source of work, is

$$KE_f = KE_i + \sum W - F_{fr} d$$

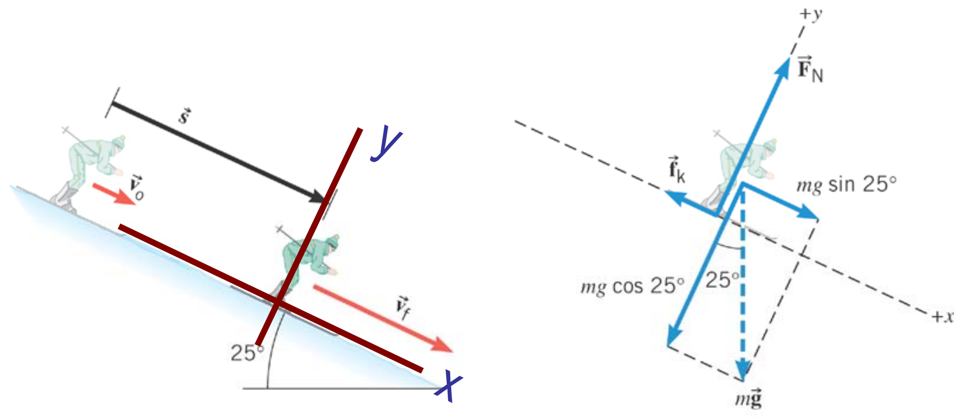
t=0, KE_i

Friction, Engine work

t=T, KE_f

Ex. Downhill Skiing

A 58kg skier is coasting down a 25^o slope. A kinetic frictional force of magnitude f_k=70N opposes her motion. Neat the top of the slope, the skier's speed is v_0=3.6m/s. Ignoring air resistance, determine the speed v_f at the point that is displaced 57m downhill.



(a)
(b) Free-body diagram for the skier

What are the forces in this motion?

Gravitational force: F_g Normal force: F_N Kinetic frictional force: f_k

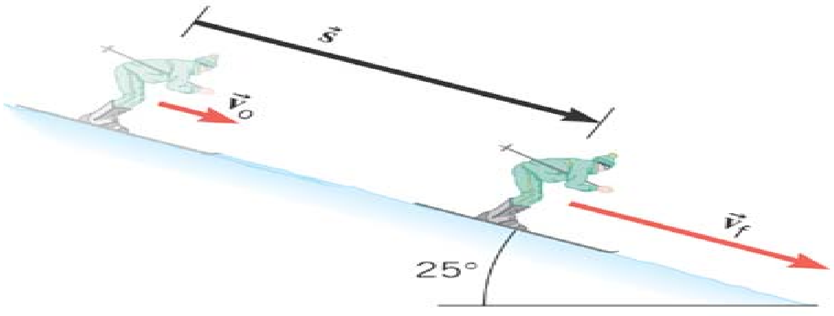
What are the X and Y component of the net force in this motion?

Y component $$\sum F_y = F_{gy} + F_N = -mg\cos 25^o + F_N = 0$$

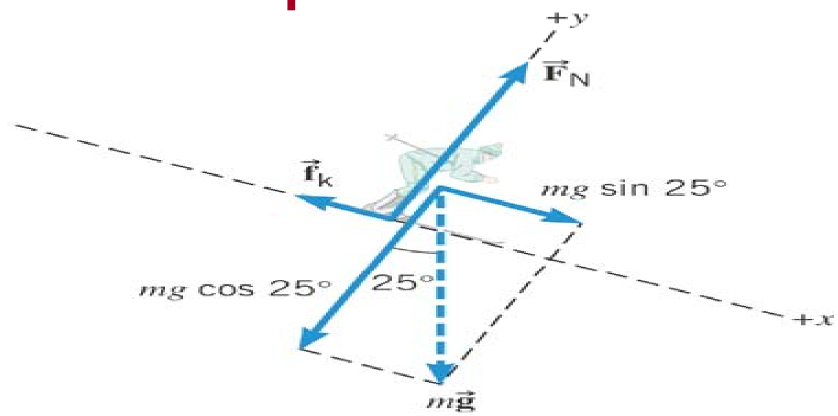
From this we obtain $$F_N = mg\cos 25^o = 58\cdot 9.8\cdot\cos 25^o = 515N$$

What is the coefficient of kinetic friction? $$f_k = \mu_k F_N \Rightarrow \mu_k = \frac{f_k}{F_N} = \frac{70}{515} = 0.14$$

Ex. Now with the X component



(a)



(b) Free-body diagram for the skier

X component $\sum F_x = F_{gx} - f_k = mg\sin 25^o - f_k = \left(58\cdot 9.8\cdot \sin 25^o - 70\right) = 170N = ma$

Total work by this force $W = \left(\sum F_x\right)\cdot s = \left(mg\sin 25^o - f_k\right)\cdot s = \left(58\cdot 9.8\cdot \sin 25^o - 70\right)\cdot 57 = 9700J$

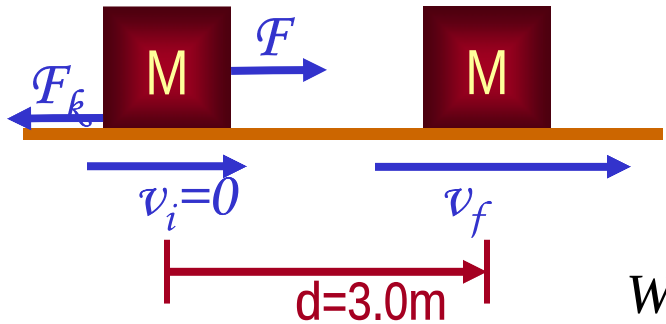
From work-kinetic energy theorem $W = KE_f - KE_i$ ⇒ $KE_f = \frac{1}{2}mv_f^2 = W + KE_i = W + \frac{1}{2}mv_0^2$

Solving for v_f $v_f^2 = \frac{2W + mv_0^2}{m}$ ⇒ $v_f = \sqrt{\frac{2W + mv_0^2}{m}} = \sqrt{\frac{2\cdot 9700 + 58\cdot (3.6)^2}{58}} = 19m/s$

What is her acceleration? $\sum F_x = ma$ ⇒ $a = \frac{\sum F_x}{m} = \frac{170}{58} = 2.93m/s^2$

Example of Work Under Friction

A 6.0kg block initially at rest is pulled to East along a horizontal surface with coefficient of kinetic friction μ_k=0.15 by a constant horizontal force of 12N. Find the speed of the block after it has moved 3.0m.



Work done by the force $\mathcal{F}$ is

$$W_F = \left|\vec{F}\right|\left|\vec{d}\right|\cos\theta = 12\times 3.0\cos 0 = 36\,(J)$$

Work done by friction $\mathcal{F}_k$ is

$$W_k = \vec{F}_k\bullet\vec{d} = \left|\vec{F}_k\right|\left|\vec{d}\right|\cos\theta = \left|\mu_k mg\right|\left|\vec{d}\right|\cos\theta$$

$$= 0.15\times 6.0\times 9.8\times 3.0\cos 180 = -26\,(J)$$

Thus the total work is

$$W = W_F + W_k = 36 - 26 = 10(J)$$

Using work-kinetic energy theorem and the fact that initial speed is 0, we obtain

$$W = W_F + W_k = \frac{1}{2}mv_f^2$$

Solving the equation for v_f, we obtain

$$v_f = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2\times 10}{6.0}} = 1.8m/s$$

Potential Energy

Energy associated with a system of objects → Stored energy which has the potential or the possibility to work or to convert to kinetic energy

What does this mean?

In order to describe potential energy, U, a system must be defined.

The concept of potential energy can only be used under the special class of forces called the conservative force which results in the principle of ***conservation of mechanical energy.***

$$E_M \equiv KE_i + PE_i = KE_f + PE_f$$

What are other forms of energies in the universe?

Mechanical Energy *Chemical Energy* *Biological Energy*

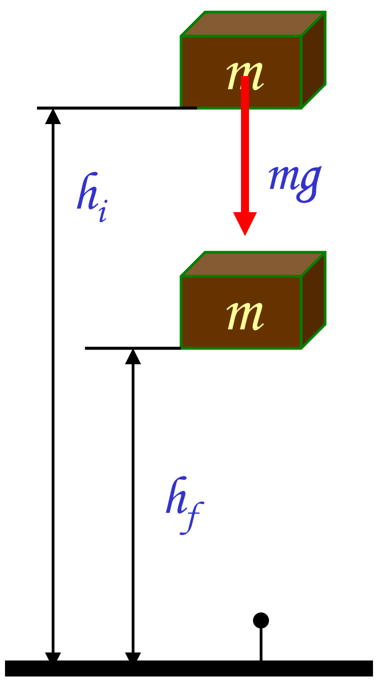
Electromagnetic Energy *Nuclear Energy*

These different types of energies are stored in the universe in many different forms!!!

If one takes into account ALL forms of energy, the total energy in the entire universe is conserved. It just transforms from one form to another.

Gravitational Potential Energy

The potential energy is given to an object by the gravitational field in the system of Earth by virtue of the object's height from an arbitrary zero level



When an object is falling, the gravitational force, Mg, performs the work on the object, increasing the object's kinetic energy. So the potential energy of an object at a height y, which is the potential to do work, is expressed as

$$PE = \vec{F}_g \cdot \vec{y} = \left|\vec{F}_g\right|\left|\vec{y}\right|\cos\theta = \left|\vec{F}_g\right|\left|\vec{y}\right| = mgh \qquad PE \equiv mgh$$

The work done on the object by the gravitational force as the brick drops from y_i to y_f is:

$$W_g = PE_i - PE_f$$
$$= mgh_i - mgh_f = -\Delta PE$$

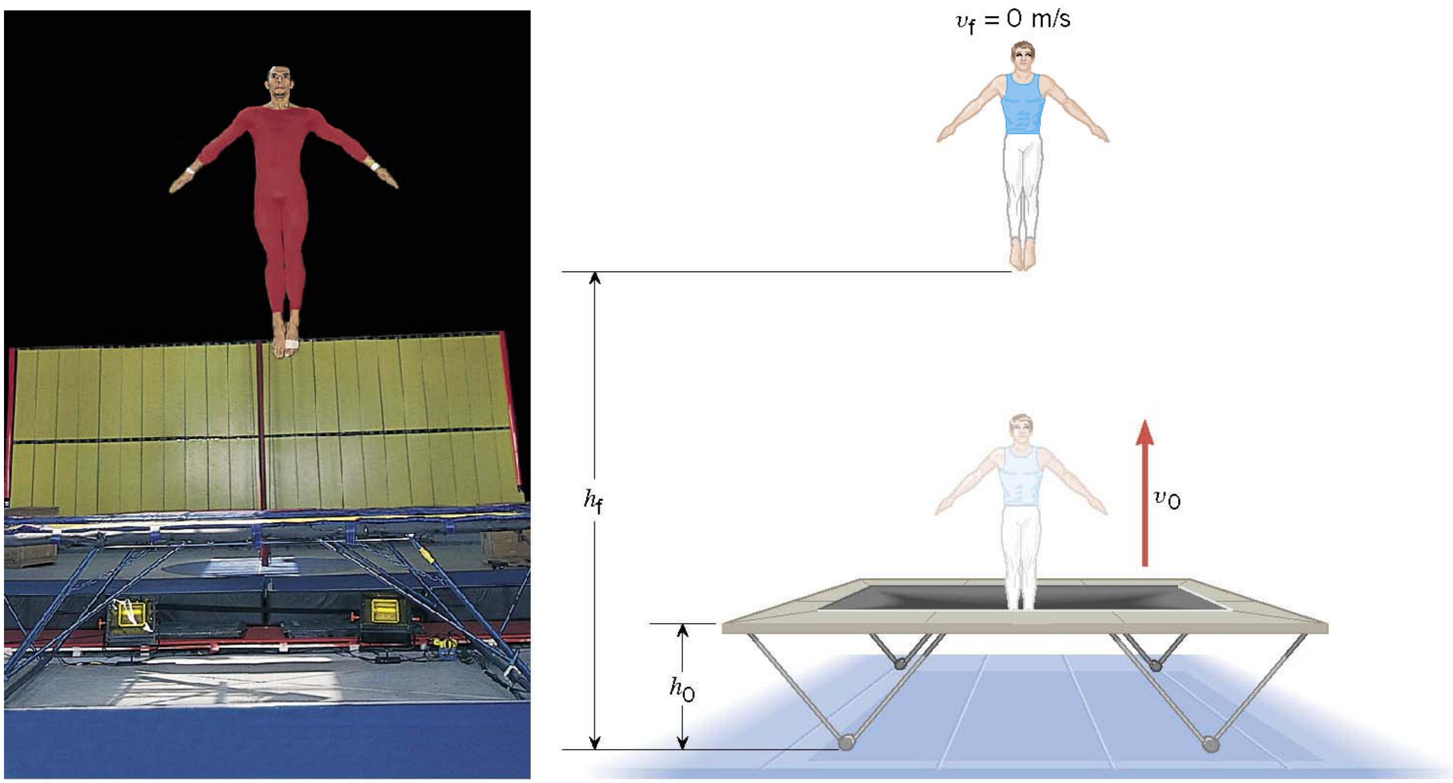
What does this mean?

Work by the gravitational force as the brick drops from y_i to y_f is the negative change of the system's potential energy

➔ Potential energy was spent in order for the gravitational force to increase the brick's kinetic energy.

Ex. A Gymnast on a Trampoline

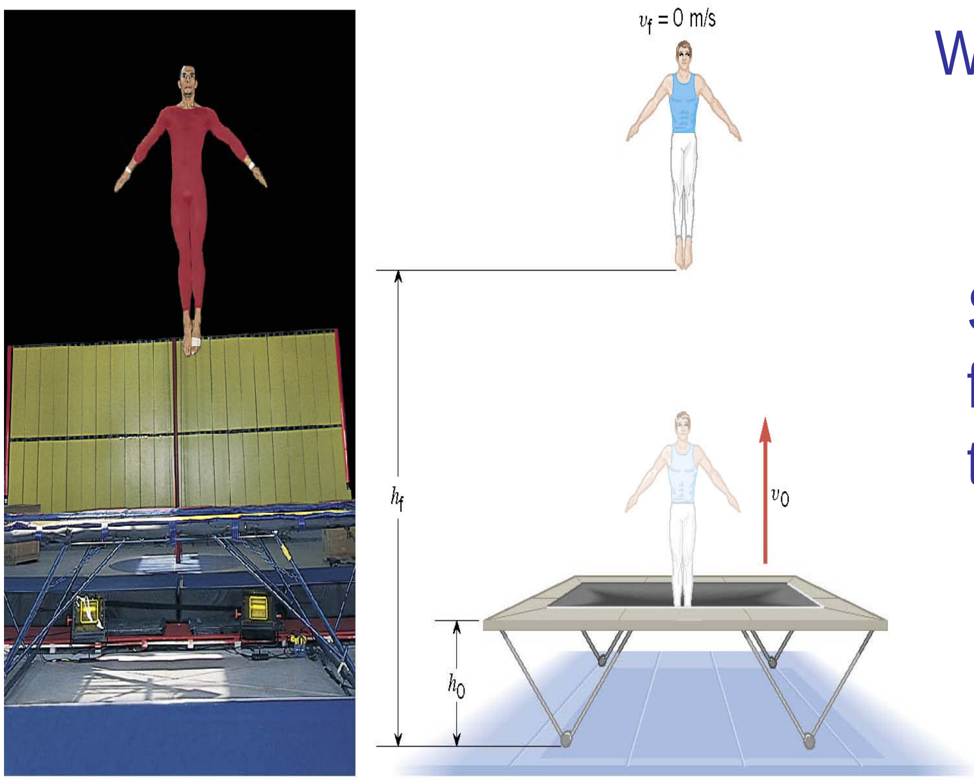
The gymnast leaves the trampoline at an initial height of 1.20 m and reaches a maximum height of 4.80 m before falling back down. What was the initial speed of the gymnast?



(*a*) (*b*)

Ex. Continued

From the work-kinetic energy theorem $W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_o^2$



(a) (b)

Work done by the gravitational force

$$W_{gravity} = mg\left(h_o - h_f\right)$$

Since at the maximum height, the final speed is 0. Using work-KE theorem, we obtain

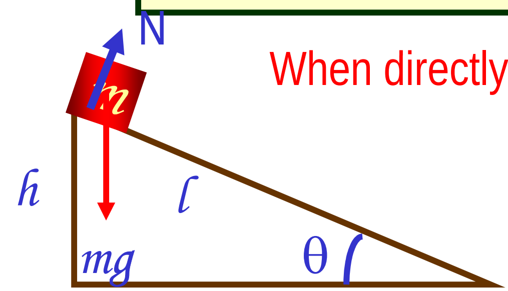
$$\not{m}g\left(h_o - h_f\right) = -\frac{1}{2}\not{m}v_o^2$$

$$v_o = \sqrt{-2g\left(h_o - h_f\right)}$$

$$\therefore v_o = \sqrt{-2\left(9.80\,\mathrm{m/s^2}\right)\left(1.20\text{ m} - 4.80\text{ m}\right)} = 8.40\,\mathrm{m/s}$$

Conservative and Non-conservative Forces

The work done on an object by the gravitational force does not depend on the object's path in the absence of a retardation force.



When directly falls, the work done on the object by the gravitation force is $W_g = mgh$

When sliding down the hill of length *l*, the work is

$$W_g = F_{g-incline} \times l = mg\sin\theta \times l$$
$$= mg(l\sin\theta) = mgh$$

How about if we lengthen the incline by a factor of 2, keeping the height the same??

Still the same amount of work☺

$$W_g = mgh$$

So the work done by the gravitational force on an object is independent of the path of the object's movements. It only depends on the difference of the object's initial and final position in the direction of the force.

Forces like gravitational and elastic forces are called the conservative force

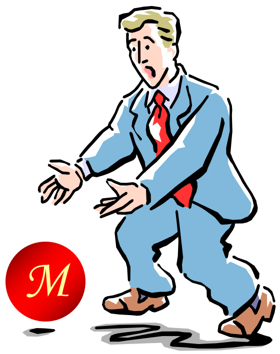
1. If the work performed by the force does not depend on the path.
2. If the work performed on a closed path is 0.

Total mechanical energy is conserved!!

$$E_M \equiv KE_i + PE_i = KE_f + PE_f$$

Example for Potential Energy

A bowler drops bowling ball of mass 7kg on his toe. Choosing the floor level as y=0, estimate the total work done on the ball by the gravitational force as the ball falls on the toe.



Let's assume the top of the toe is 0.03m from the floor and the hand was 0.5m above the floor.

$$U_i = mgy_i = 7\times 9.8\times 0.5 = 34.3J \quad U_f = mgy_f = 7\times 9.8\times 0.03 = 2.06J$$

$$W_g = -\Delta U = -\left(U_f - U_i\right) = 32.24J \cong 30J$$

b) Perform the same calculation using the top of the bowler's head as the origin.

What has to change?

First we must re-compute the positions of the ball in his hand and on his toe.

Assuming the bowler's height is 1.8m, the ball's original position is –1.3m, and the toe is at –1.77m.

$$U_i = mgy_i = 7\times 9.8\times\left(-1.3\right) = -89.2J \quad U_f = mgy_f = 7\times 9.8\times\left(-1.77\right) = -121.4J$$

$$W_g = -\Delta U = -\left(U_f - U_i\right) = 32.2J \cong 30J$$