PHYS 1441 – Section 002
Lecture #18

Monday, Apr. 13, 2009

Dr. Jaehoon Yu

- Center of Mass
- Fundamentals of Rotational Motion
- Equations of Rotational Kinematics
- Relationship Between Linear and Angular Quantities
- Rolling Motion

Today's homework is HW #10, due 9pm, Tuesday, Apr. 21!!

Announcements

- 2nd term exam
  - 1 – 2:20pm, Wednesday, Apr. 22, in SH103
  - Non-comprehensive exam
  - Covers: Ch. 6.1 – what we complete this Wednesday, Apr. 15 (somewhere in Ch. 9)
  - A help session in class Monday, Apr. 20 by Humphrey
  - One better of the two term exams will be used for final grading
- Reading assignments
  - Ch. 7 – 9 and 7 – 10

Reminder: Extra-Credit Special Project

- Derive the formula for the final velocity of two objects which underwent an elastic collision as a function of known quantities m_1, m_2, v_{01} and v_{02} in page 14 of the Apr. 8 lecture note in a far greater detail than the note.
  - 20 points extra credit
- Show mathematically what happens to the final velocities if $m_1=m_2$ and describe in words the resulting motion.
  - 5 point extra credit
- Due: Start of the class this Wednesday, Apr. 15

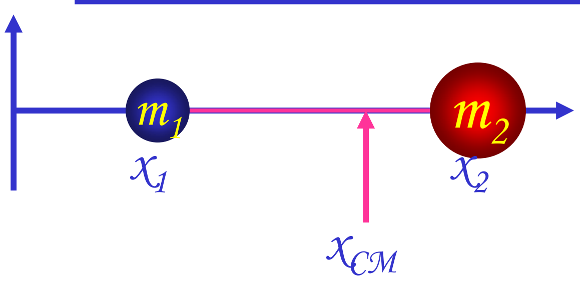
Center of Mass

We've been solving physical problems treating objects as sizeless points with masses, but in realistic situations objects have shapes with masses distributed throughout the body.

Center of mass of a system is the average position of the system's mass and represents the motion of the system as if all the mass is on that point.

What does above statement tell you concerning the forces being exerted on the system?

The total external force exerted on the system of total mass M causes the center of mass to move at an acceleration given by $\vec{a} = \sum \vec{F} / M$ *as if the entire mass of the system is on the center of mass.*



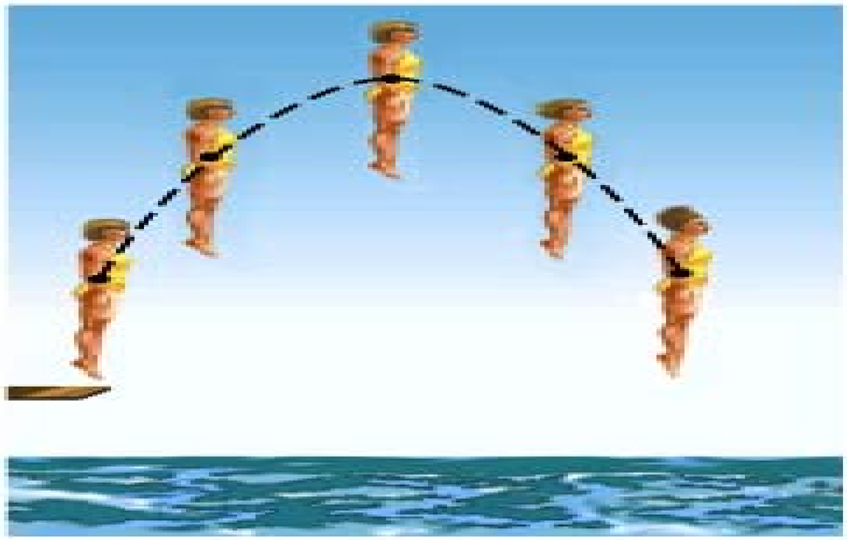
Consider a massless rod with two balls attached at either end.

The position of the center of mass of this system is the mass averaged position of the system

$$x_{CM} \equiv \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

CM is closer to the heavier object

Motion of a Diver and the Center of Mass



(a)

Diver performs a simple dive.
The motion of the center of mass follows a parabola since it is a projectile motion.

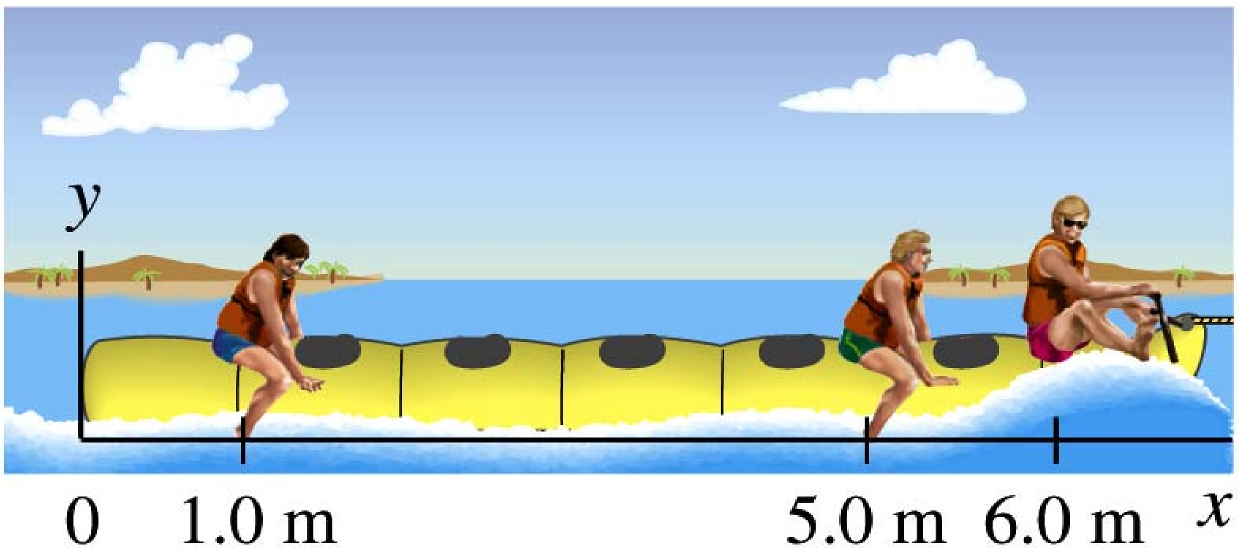
(b)

Diver performs a complicated dive.
The motion of the center of mass still follows the same parabola since it still is a projectile motion.

The motion of the center of mass of the diver is always the same.

Ex. 7 – 12 Center of Mass

Thee people of roughly equivalent mass M on a lightweight (air-filled) banana boat sit along the x axis at positions x_1=1.0m, x_2=5.0m, and x_3=6.0m. Find the position of CM.

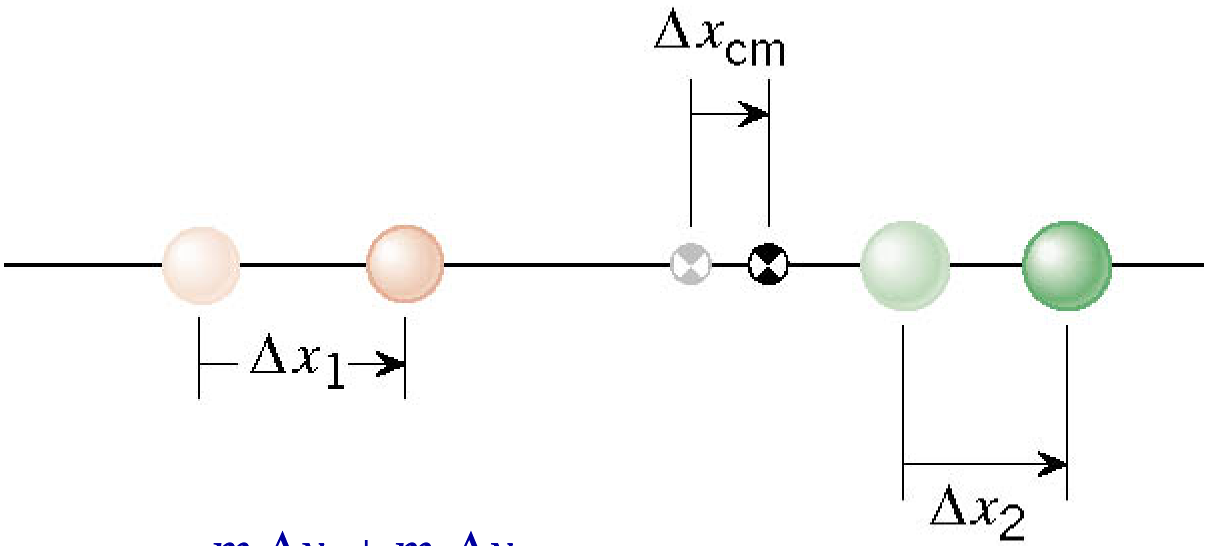


Using the formula for CM

$$x_{CM} = \frac{\sum_i m_i x_i}{\sum_i m_i}$$

$$= \frac{M \cdot 1.0 + M \cdot 5.0 + M \cdot 6.0}{M + M + M} = \frac{12.0M}{3M} = 4.0(m)$$

Velocity of Center of Mass



$$\Delta x_{cm} = \frac{m_1 \Delta x_1 + m_2 \Delta x_2}{m_1 + m_2}$$

$$\Rightarrow \quad v_{cm} = \frac{\Delta x_{cm}}{\Delta t} = \frac{m_1 \Delta x_1 / \Delta t + m_2 \Delta x_2 / \Delta t}{m_1 + m_2} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

In an isolated system, the total linear momentum does not change, therefore the velocity of the center of mass does not change.

Another Look at the Ice Skater Problem

Starting from rest, two skaters push off against each other on ice where friction is negligible. One is a 54-kg woman and one is a 88-kg man. The woman moves away with a speed of +2.5 m/s.

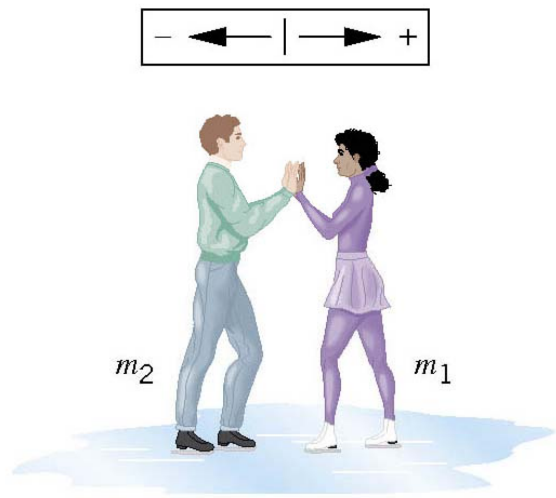
$$v_{10} = 0\, m/s \qquad v_{20} = 0\, m/s$$

$$v_{cm0} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = 0$$

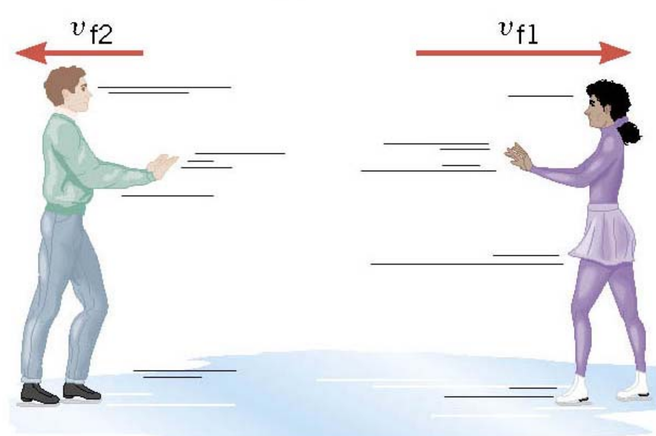
$$v_{1f} = +2.5\, m/s \qquad v_{2f} = -1.5\, m/s$$

$$v_{cmf} = \frac{m_1 v_{1f} + m_2 v_{2f}}{m_1 + m_2}$$

$$= \frac{54 \cdot (+2.5) + 88 \cdot (-1.5)}{54 + 88} = \frac{3}{142} = 0.02 \approx 0\, m/s$$



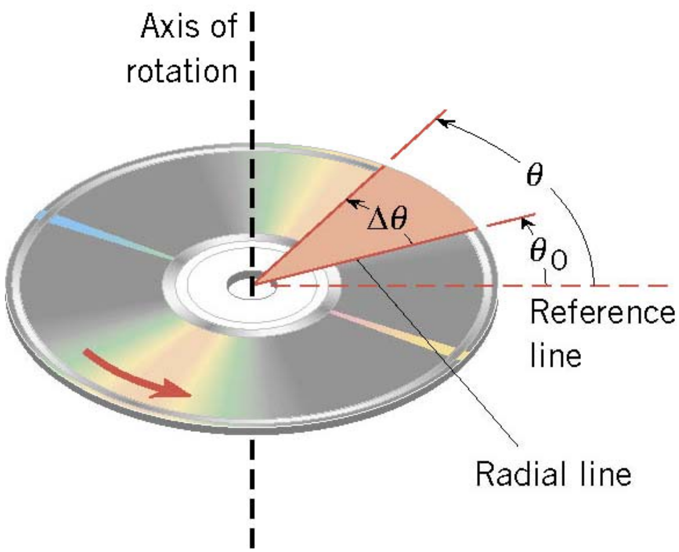
(*a*) Before



(*b*) After

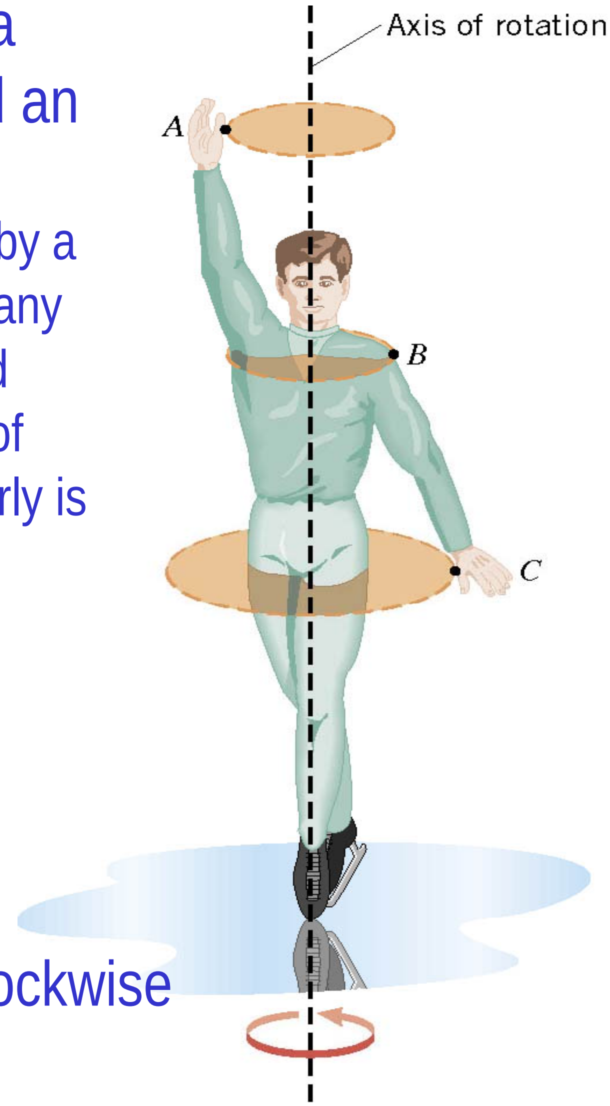
Rotational Motion and Angular Displacement

In the simplest kind of rotation, points on a rigid object move on circular paths around an *axis of rotation.*



The angle swept out by a line passing through any point on the body and intersecting the axis of rotation perpendicularly is called the *angular displacement.*

$$\Delta\theta = \theta - \theta_o$$



It's a vector!! So there must be directions…

How do we define directions?

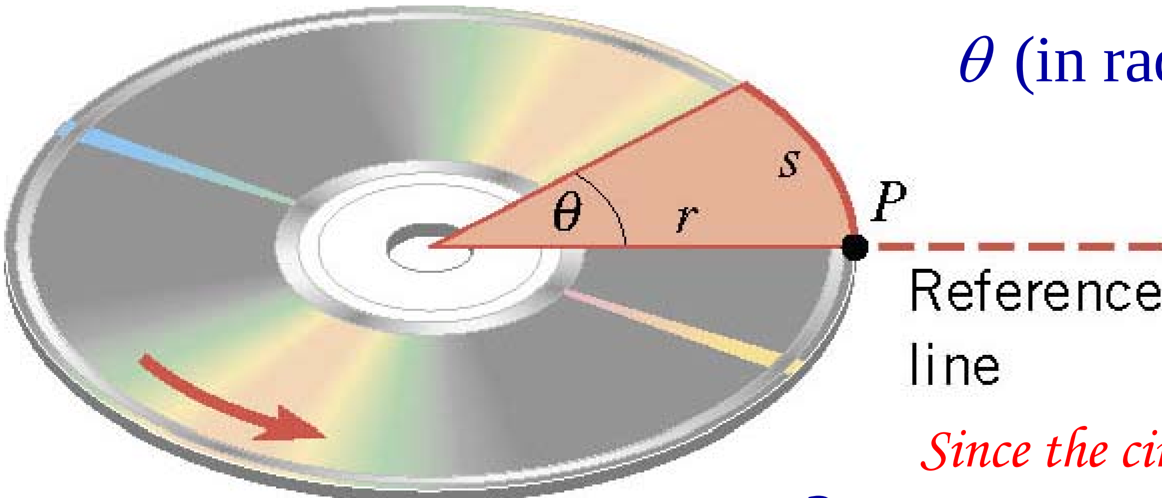
+:if counter-clockwise

-:if clockwise

The direction vector points gets determined based on the right-hand rule.

These are just conventions!!

SI Unit of the Angular Displacement



$$\theta \text{ (in radians)} = \frac{\text{Arc length}}{\text{Radius}} = \frac{s}{r}$$

Dimension? None

For one full revolution:

Since the circumference of a circle is 2πr

$$\theta = \frac{2\pi r}{r} = 2\pi \text{ rad} \Rightarrow 2\pi \text{ rad} = 360^o$$

How many degrees is in one radian?

1 radian is $1 \text{ rad} = \frac{360^o}{2\pi} \cdot 1rad = \frac{180^o}{\pi} \cdot 1rad \cong \frac{180^\circ}{3.14} \cdot 1rad \cong 57.3^\circ$

How radians is one degree?

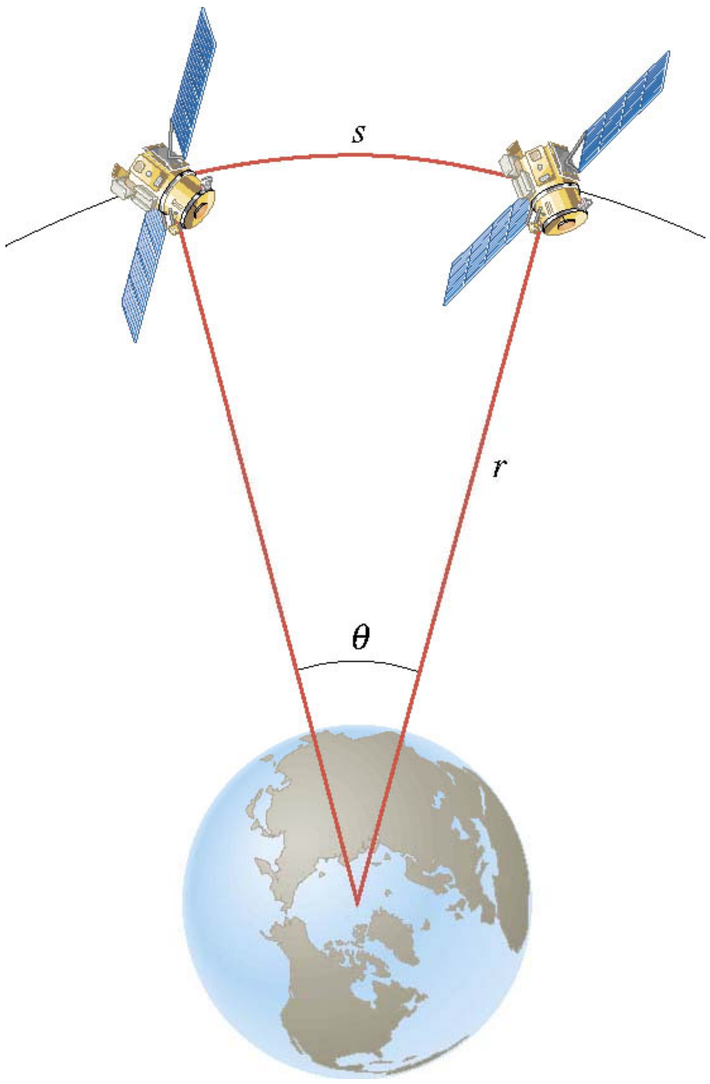
And one degrees is $1^o = \frac{2\pi}{360^o} \cdot 1^o = \frac{\pi}{180^o} \cdot 1^o \cong \frac{3.14}{180^\circ} \cdot 1^\circ \cong 0.0175rad$

How many radians are in 10.5 revolutions? $10.5rev = 10.5rev \cdot 2\pi \frac{rad}{rev} = 21\pi\left(rad\right)$

Very important: In solving angular problems, all units, degrees or revolutions, must be converted to radians.

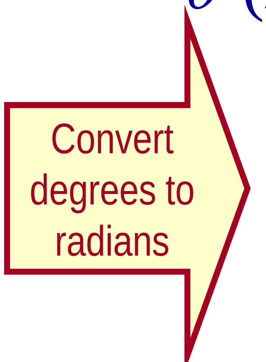
Ex. Adjacent Synchronous Satellites

Synchronous satellites are put into an orbit whose radius is 4.23×10^7m. If the angular separation of the two satellites is 2.00 degrees, find the arc length that separates them.



What do we need to find out? The Arc length!!!

$$\theta \text{ (in radians)} = \frac{\text{Arc length}}{\text{Radius}} = \frac{s}{r}$$



$$2.00\,\text{deg}\left(\frac{2\pi \text{ rad}}{360\,\text{deg}}\right) = 0.0349 \text{ rad}$$

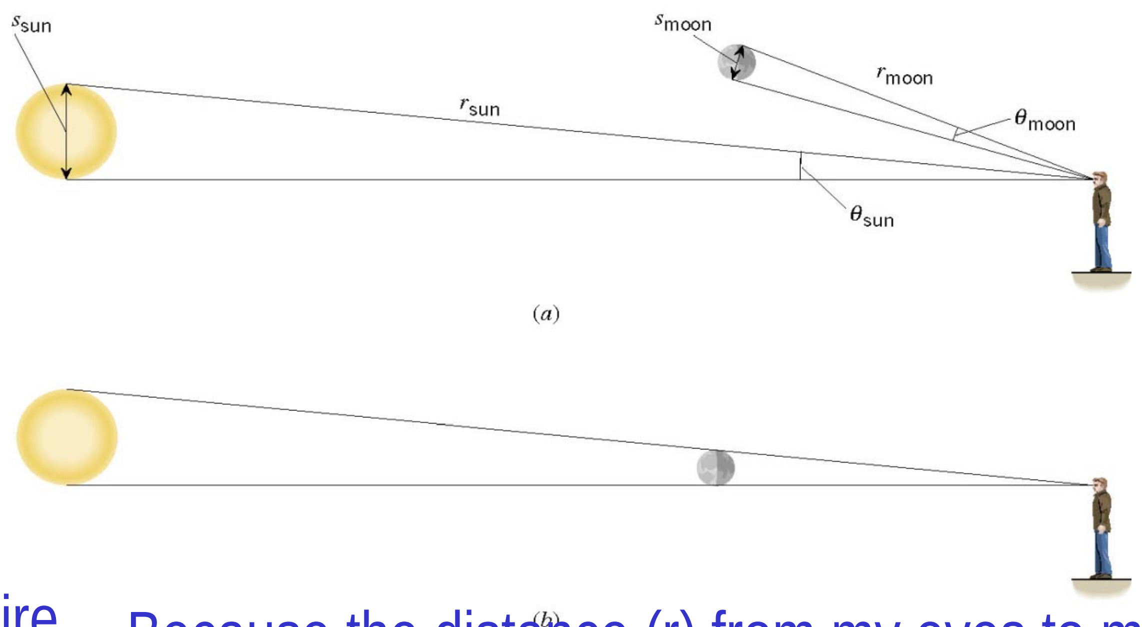
$$s = r\theta = \left(4.23\times10^7\,\text{m}\right)\left(0.0349 \text{ rad}\right)$$

$$= 1.48\times10^6\,\text{m} \ \ (920 \text{ miles})$$

Ex. A Total Eclipse of the Sun

The diameter of the sun is about 400 times greater than that of the moon. By coincidence, the sun is also about 400 times farther from the earth than is the moon. For an observer on the earth, compare the angle subtended by the moon to the angle subtended by the sun and explain why this result leads to a total solar eclipse.

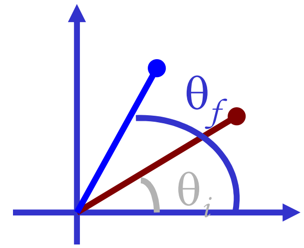
$$\theta \text{ (in radians)} = \frac{\text{Arc length}}{\text{Radius}} = \frac{s}{r}$$



I can even cover the entire sun with my thumb!! Why?

Because the distance (r) from my eyes to my thumb is far shorter than that to the sun.

Angular Displacement, Velocity, and Acceleration



Angular displacement is defined as

$$\Delta\theta = \theta_f - \theta_i$$

How about the average angular velocity, the rate of change of angular displacement?

$$\overline{\omega} \equiv \frac{\theta_f - \theta_i}{t_f - t_i} = \frac{\Delta\theta}{\Delta t}$$

Unit? rad/s Dimension? $[T^{-1}]$

By the same token, the average angular acceleration, rate of change of the angular velocity, is defined as…

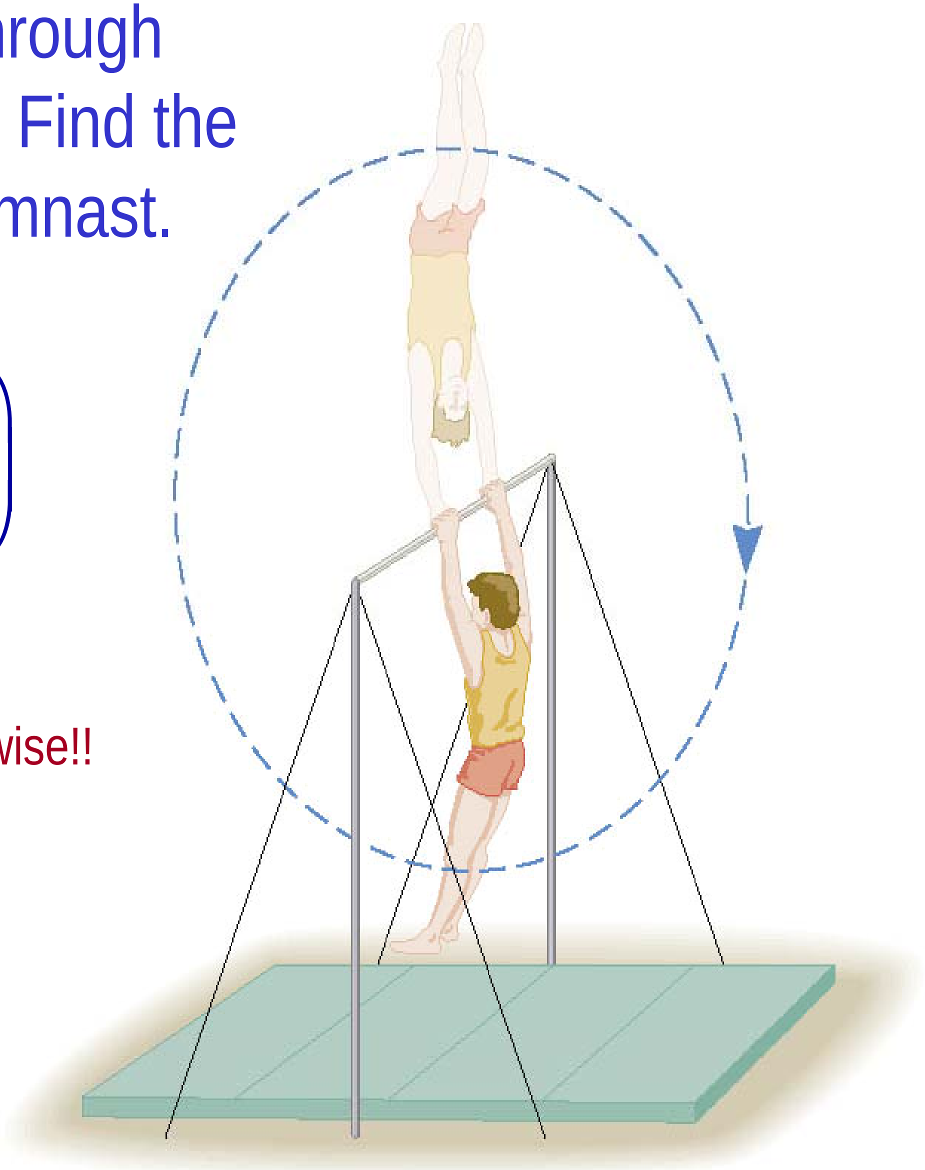
$$\overline{\alpha} \equiv \frac{\omega_f - \omega_i}{t_f - t_i} = \frac{\Delta\omega}{\Delta t}$$

Unit? rad/s^2 Dimension? $[T^{-2}]$

When rotating about a fixed axis, every particle on a rigid object rotates through the same angle and has the same angular speed and angular acceleration.

Ex. Gymnast on a High Bar

A gymnast on a high bar swings through two revolutions in a time of 1.90 s. Find the average angular velocity of the gymnast.



What is the angular displacement?

$$\Delta\theta = -2.00 \text{ rev}\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)$$

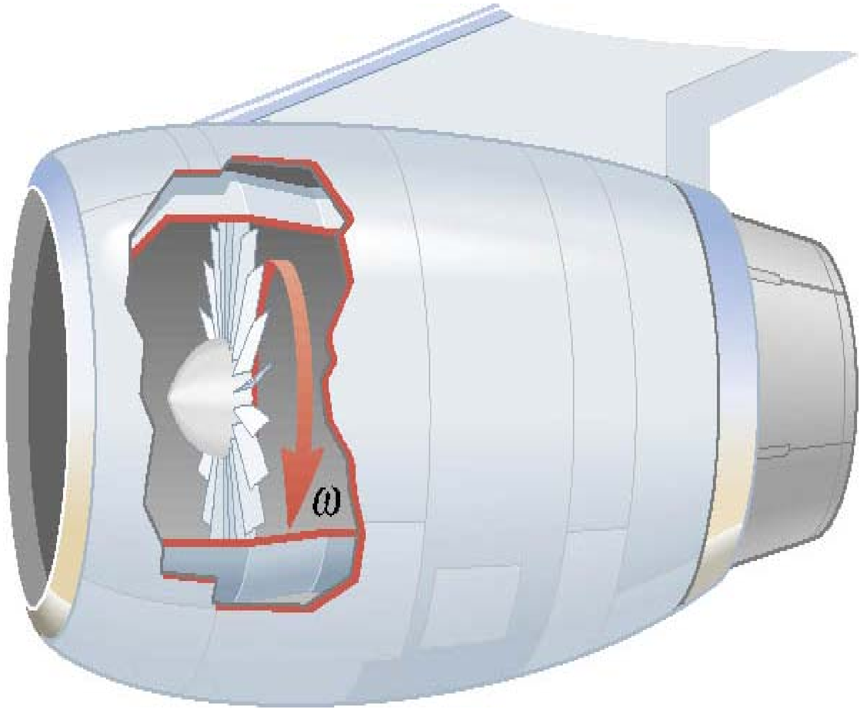
$$= -12.6 \text{ rad}$$

Why negative? Because he is rotating clockwise!!

$$\bar{\omega} = \frac{-12.6 \text{ rad}}{1.90 \text{ s}} = -6.63 \text{ rad/s}$$

Ex. A Jet Revving Its Engines

As seen from the front of the engine, the fan blades are rotating with an angular speed of -110 rad/s. As the plane takes off, the angular velocity of the blades reaches -330 rad/s in a time of 14 s. Find the angular acceleration, assuming it to be constant.



$$\bar{\alpha} = \frac{\omega_f - \omega_i}{t_f - t_i} = \frac{\Delta\omega}{\Delta t}$$

$$= \frac{(-330\,\text{rad/s}) - (-110\,\text{rad/s})}{14\text{ s}} = -16\,\text{rad/s}^2$$

Rotational Kinematics

The first type of motion we have learned in linear kinematics was under a constant acceleration. We will learn about the rotational motion under constant angular acceleration, because these are the simplest motions in both cases.

Just like the case in linear motion, one can obtain

Angular velocity under constant angular acceleration:

$$\omega_f = \omega_0 + \alpha t$$

Linear kinematics $v = v_o + at$

Angular displacement under constant angular acceleration:

$$\theta_f = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

Linear kinematics $x_f = x_0 + v_o t + \frac{1}{2} a t^2$

One can also obtain

$$\omega_f^2 = \omega_0^2 + 2\alpha\left(\theta_f - \theta_0\right)$$

Linear kinematics $v_f^2 = v_o^2 + 2a\left(x_f - x_i\right)$

Problem Solving Strategy

- Visualize the problem by drawing a picture
- Decide which directions are to be called positive (+) and negative (-).
- Write down the values that are given for any of the five kinematic variables and convert them to SI units.
- Verify that the information contains values for at least three of the five kinematic variables. Select the appropriate equation.
- When the motion is divided into segments, remember that the final angular velocity of one segment is the initial velocity for the next.
- Keep in mind that there may be two possible answers to a kinematics problem.

Ex. Blending with a Blender

The blades are whirling with an angular velocity of +375 rad/s when the "puree" button is pushed in. When the "blend" button is pushed, the blades accelerate and reach a greater angular velocity after the blades have rotated through an angular displacement of +44.0 rad. The angular acceleration has a constant value of +1740 rad/s^2. Find the final angular velocity of the blades.

θ	α	ω	ω_o	t
+44.0rad	+1740rad/s^2	?	+375rad/s	

Which kinematics eq? $\omega^2 = \omega_o^2 + 2\alpha\theta$

$$\omega = \pm\sqrt{\omega_o^2 + 2\alpha\theta}$$

$$= \pm\sqrt{(375\,\text{rad/s})^2 + 2(1740\,\text{rad/s}^2)(44.0\,\text{rad})} = \pm 542\,\text{rad/s}$$

Which sign? $\omega = +542\,\text{rad/s}$ *Why?* *Because it is accelerating in counter-clockwise!*