PHYS 1441 – Section 002
Lecture #19

Wednesday, Apr. 15, 2009

Dr. Jaehoon Yu

- Relationship Between Linear and Angular Quantities
- Rolling Motion of a Rigid Body
- Rotational Dynamics
  - Torque
  - Equilibrium
  - Moment of Inertia
  - Torque and Angular Acceleration
- Rotational Kinetic Energy

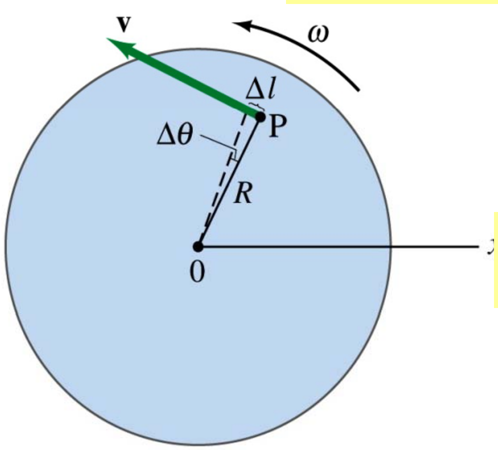
Announcements

- 2nd term exam
  - 1 – 2:20pm, Wednesday, Apr. 22, in SH103
  - Non-comprehensive exam
  - Covers: Ch. 6.1 – Ch 8.6
  - A help session in class Monday, Apr. 20 by Humphrey
  - One better of the two term exams will be used for final grading
- No colloquium today

Relationship Between Angular and Linear Quantities

What do we know about a rigid object that rotates about a fixed axis of rotation?

Every particle (or masslet) in the object moves in a circle centered at the same axis of rotation.



When a point rotates, it has both the linear and angular components in its motion.

What is the linear component of the motion you see?

Linear velocity along the tangential direction.

How do we related this linear component of the motion with angular component?

The direction of ω follows a right-hand rule.

The arc-length is $l = r\theta$ So the tangential speed v is $v = \frac{\Delta l}{\Delta t} = \frac{\Delta(r\theta)}{\Delta t} = r\frac{\Delta\theta}{\Delta t} = r\omega$

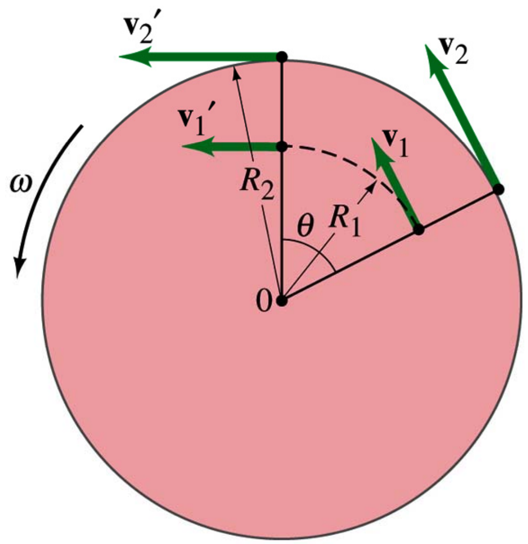
What does this relationship tell you about the tangential speed of the points in the object and their angular speed?:

Although every particle in the object has the same angular speed, its tangential speed differs and is proportional to its distance from the axis of rotation.

The farther away the particle is from the center of rotation, the higher the tangential speed.

Is the lion faster than the horse?

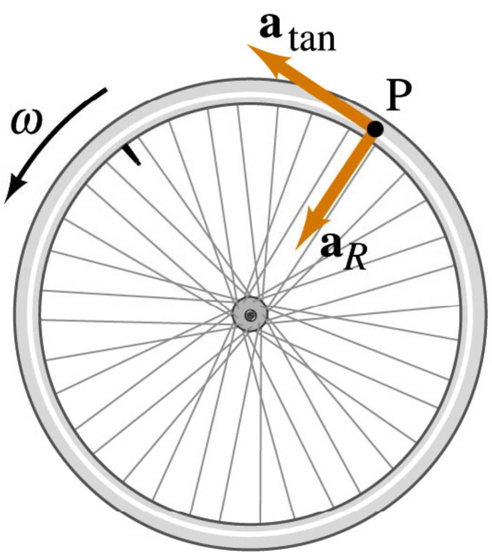
A rotating carousel has one child sitting on a horse near the outer edge and another child on a lion halfway out from the center. (a) Which child has the greater linear speed? (b) Which child has the greater angular speed?



(a) Linear speed is the distance traveled divided by the time interval. So the child sitting at the outer edge travels more distance within the given time than the child sitting closer to the center. Thus, the horse is faster than the lion.

(b) Angular speed is the angle traveled divided by the time interval. The angle both the children travel in the given time interval is the same. Thus, both the horse and the lion have the same angular speed.

How about the acceleration?



How many different linear acceleration components do you see in a circular motion and what are they? Two

Tangential, a_t, and the radial acceleration, a_r.

Since the tangential speed v is $$v_T = r\omega$$

The magnitude of tangential acceleration a_t is $$a_t = \frac{v_{Tf} - v_{T0}}{\Delta t} = \frac{r\omega_f - r\omega_0}{\Delta t} = r\frac{\omega_f - \omega_0}{\Delta t} = r\alpha$$

What does this relationship tell you?

Although every particle in the object has the same angular acceleration, its tangential acceleration differs proportional to its distance from the axis of rotation.

The radial or centripetal acceleration a_r is $$a_r = \frac{v^2}{r} = \frac{(r\omega)^2}{r} = r\omega^2$$

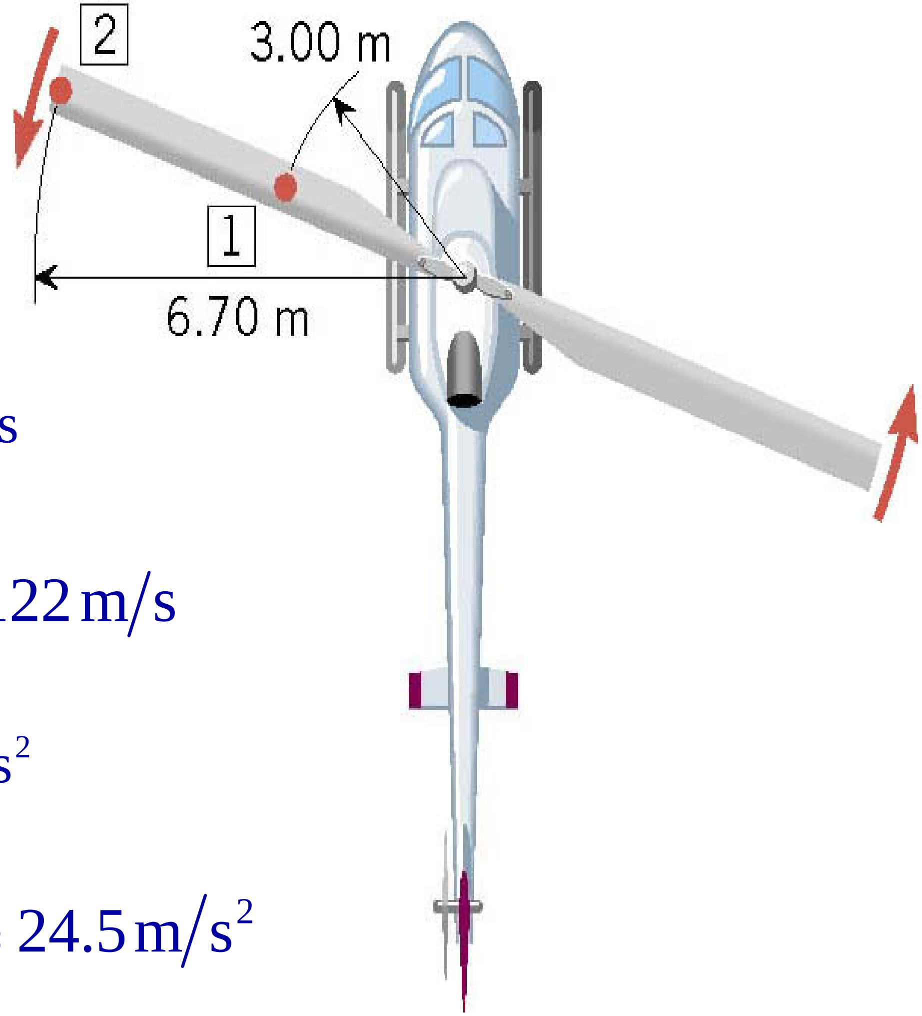
What does this tell you?

The father away the particle is from the rotation axis, the more radial acceleration it receives. In other words, it receives more centripetal force.

Total linear acceleration is $$a = \sqrt{a_t^2 + a_r^2} = \sqrt{(r\alpha)^2 + (r\omega^2)^2} = r\sqrt{\alpha^2 + \omega^4}$$

Ex. A Helicopter Blade

A helicopter blade has an angular speed of 6.50 rev/s and an angular acceleration of 1.30 rev/s^2. For point 1 on the blade, find the magnitude of (a) the tangential speed and (b) the tangential acceleration.



$$\omega = \left(6.50\ \frac{\text{rev}}{\text{s}}\right)\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right) = 40.8\ \text{rad/s}$$

$$v_T = r\varpi = (3.00\ \text{m})(40.8\,\text{rad/s}) = 122\,\text{m/s}$$

$$\alpha = \left(1.30\ \frac{\text{rev}}{\text{s}^2}\right)\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right) = 8.17\ \text{rad/s}^2$$

$$a_T = r\alpha = (3.00\ \text{m})(8.17\,\text{rad/s}^2) = 24.5\,\text{m/s}^2$$

Rolling Motion of a Rigid Body

What is a rolling motion?

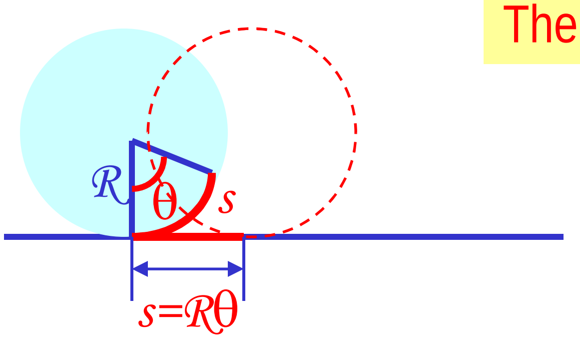
A more generalized case of a motion where the rotational axis moves together with an object

A rotational motion about a moving axis

To simplify the discussion, let's make a few assumptions

1. Limit our discussion on very symmetric objects, such as cylinders, spheres, etc
2. The object rolls on a flat surface

Let's consider a cylinder rolling on a flat surface, without slipping.



Under what condition does this "Pure Rolling" happen?

The total linear distance the CM of the cylinder moved is $s = R\theta$

Thus the linear speed of the CM is

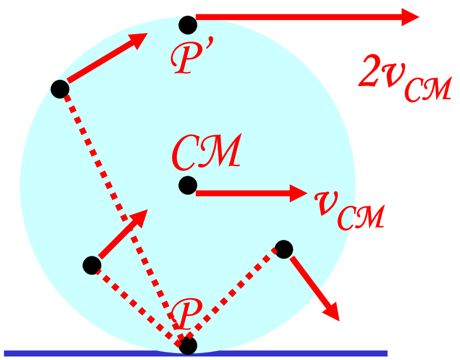
$$\bar{v}_{CM} = \frac{\Delta s}{\Delta t} = R\frac{\Delta\theta}{\Delta t} = R\omega$$

The condition for a "Pure Rolling motion"

More Rolling Motion of a Rigid Body

The magnitude of the linear acceleration of the CM is

$$a_{CM} = \frac{\Delta v_{CM}}{\Delta t} = R\frac{\Delta\omega}{\Delta t} = R\alpha$$



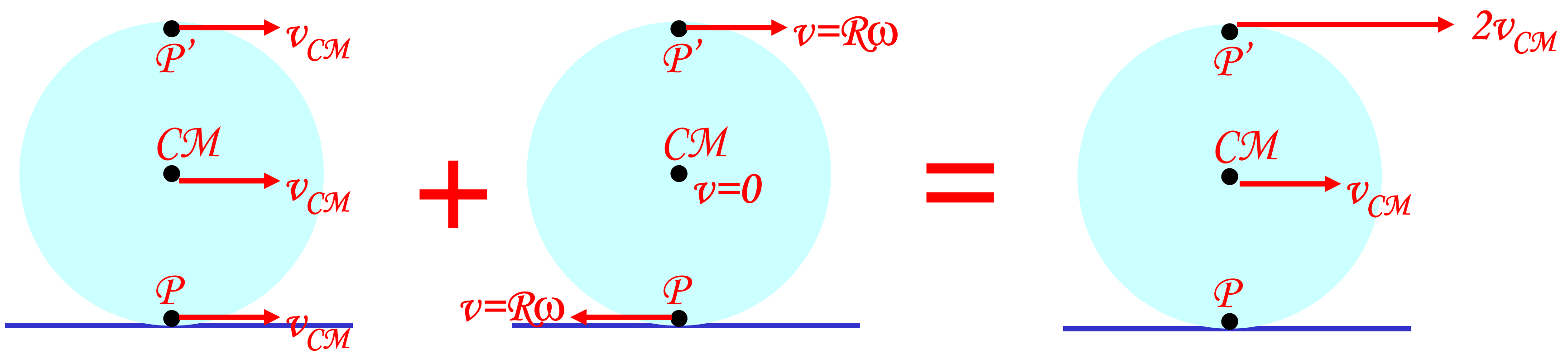
As we learned in rotational motion, all points in a rigid body moves at the same angular speed but at different linear speeds.

CM is moving at the same speed at all times.

At any given time, the point that comes to P has 0 linear speed while the point at P' has twice the speed of CM

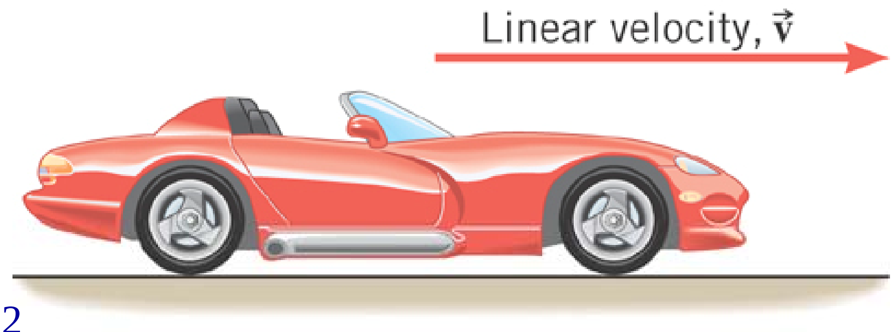
Why??

A rolling motion can be interpreted as the sum of Translation and Rotation



Ex. An Accelerating Car

Starting from rest, the car accelerates for 20.0 s with a constant linear acceleration of 0.800 m/s^2. The radius of the tires is 0.330 m. What is the angle through which each wheel has rotated?



(a)

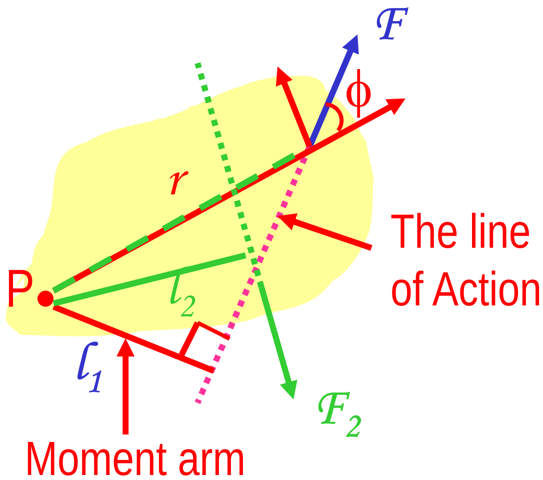
$$\alpha = \frac{a}{r} = \frac{0.800\,\mathrm{m/s^2}}{0.330\ \mathrm{m}} = 2.42\,\mathrm{rad/s^2}$$

θ	α	ω	ω_o	t
?	-2.42 rad/s^2		0 rad/s	20.0 s

$$\theta = \omega_o t + \tfrac{1}{2}\alpha t^2$$
$$= \tfrac{1}{2}\left(-2.42\,\mathrm{rad/s^2}\right)^2 \left(20.0\ \mathrm{s}\right)^2$$
$$= -484\ \mathrm{rad}$$

Torque

Torque is the tendency of a force to rotate an object about an axis. Torque, τ, is a vector quantity.



Consider an object pivoting about the point P by the force F being exerted at a distance r from P.

The line that extends out of the tail of the force vector is called the line of action.

The perpendicular distance from the pivoting point P to the line of action is called the moment arm.

Magnitude of torque is defined as the product of the force exerted on the object to rotate it and the moment arm.

$$\left|\vec{\tau}\right| \equiv (\text{Magnitude of the Force}) \times (\text{Lever Arm}) = (F)(r\sin\phi) = Fl$$

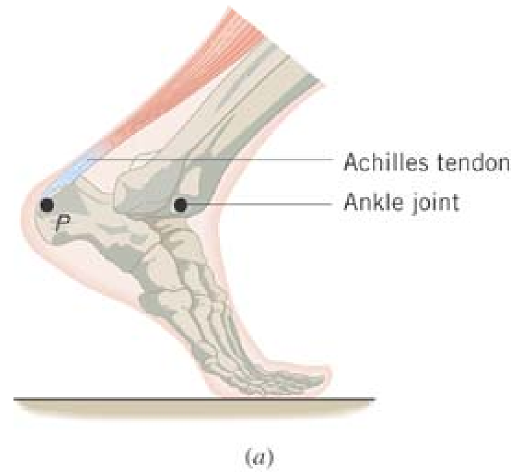
When there are more than one force being exerted on certain points of the object, one can sum up the torque generated by each force vectorially. The convention for sign of the torque is positive if rotation is in counter-clockwise and negative if clockwise.

$$\sum \tau = \tau_1 + \tau_2 = F_1 l_1 - F_2 l_2$$

Unit? $N \cdot m$

Ex. The Achilles Tendon

The tendon exerts a force of magnitude 790 N on the point P. Determine the torque (magnitude and direction) of this force about the ankle joint which is located $3.6\text{x}10^{-2}$m away from point P.



790 N

First, let's find the lever arm length

$$\cos 55^{\circ} = \frac{l}{3.6\times10^{-2}\,\text{m}}$$

$$l = 3.6\times10^{-2}\cos 55^{\circ} =$$

$$= 3.6\times10^{-2}\sin\left(90^{\circ}-55^{\circ}\right) = 2.1\times10^{-2}\left(m\right)$$

So the torque is

$$\tau = Fl$$

$$= \left(720\text{ N}\right)\left(3.6\times10^{-2}\,\text{m}\right)\cos 55^{\circ}$$

$$= \left(720\text{ N}\right)\left(3.6\times10^{-2}\,\text{m}\right)\sin 35^{\circ} = 15\text{ N}\cdot\text{m}$$

Since the rotation is in clock-wise $\tau = -15 N\cdot m$

Moment of Inertia

Rotational Inertia: Measure of resistance of an object to changes in its rotational motion. Equivalent to mass in linear motion.

For a group of objects $I \equiv \sum_i m_i r_i^2$

For a rigid body $I \equiv \int r^2 dm$

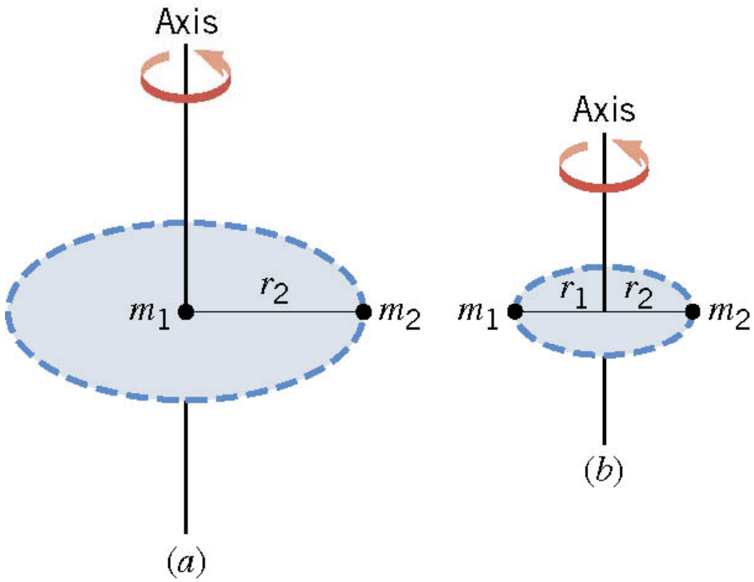
What are the dimension and unit of Moment of Inertia? $\left[ML^2\right]$ $kg \cdot m^2$

Determining Moment of Inertia is extremely important for computing equilibrium of a rigid body, such as a building.

Dependent on the axis of rotation!!!

Ex. The Moment of Inertia Depends on Where the Axis Is.

Two particles each have mass and are fixed at the ends of a thin rigid rod. The length of the rod is *L*. Find the moment of inertia when this object rotates relative to an axis that is perpendicular to the rod at (a) one end and (b) the center.



(a) $I = \sum\left(mr^2\right) = m_1 r_1^2 + m_2 r_2^2$

$m_1 = m_2 = m \qquad r_1 = 0 \quad r_2 = L$

$$I = m(0)^2 + m(L)^2 = mL^2$$

(b) $I = \sum\left(mr^2\right) = m_1 r_1^2 + m_2 r_2^2$

$m_1 = m_2 = m \qquad r_1 = L/2 \quad r_2 = L/2$

$$I = m(L/2)^2 + m(L/2)^2 = \tfrac{1}{2} mL^2$$

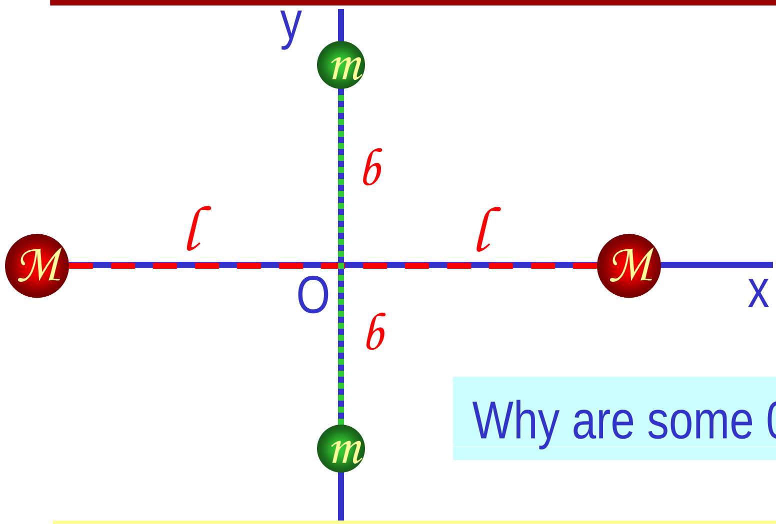
Which case is easier to spin?

Case (b)

Why? Because the moment of inertia is smaller

Example for Moment of Inertia

In a system of four small spheres as shown in the figure, assuming the radii are negligible and the rods connecting the particles are massless, compute the moment of inertia and the rotational kinetic energy when the system rotates about the y-axis at angular speed ω.



Since the rotation is about y axis, the moment of inertia about y axis, I_y, is

$$I = \sum_i m_i r_i^2 = Ml^2 + Ml^2 + m \cdot 0^2 + m \cdot 0^2 = 2Ml^2$$

This is because the rotation is done about y axis, and the radii of the spheres are negligible.

Why are some 0s?

Thus, the rotational kinetic energy is

$$K_R = \frac{1}{2} I \omega^2 = \frac{1}{2}\left(2Ml^2\right)\omega^2 = Ml^2\omega^2$$

Find the moment of inertia and rotational kinetic energy when the system rotates on the x-y plane about the z-axis that goes through the origin O.

$$I = \sum_i m_i r_i^2 = Ml^2 + Ml^2 + mb^2 + mb^2 = 2\left(Ml^2 + mb^2\right) \qquad K_R = \frac{1}{2} I \omega^2 = \frac{1}{2}\left(2Ml^2 + 2mb^2\right)\omega^2 = \left(Ml^2 + mb^2\right)\omega^2$$

Check out Figure 8 – 21 for moment of inertia for various shaped objects