PHYS 1441 – Section 002
Lecture #21

Monday, May 4, 2009
Dr. Jaehoon Yu

- Conditions for Equilibrium & Mechanical Equilibrium
- A Few Examples of Mechanical Equilibrium
- Elastic Property of Solids
- Fluid and Pressure
- Pascal's Principle
- Absolute and Gauge Pressure

Today's homework is None!!

Announcements

- Third Term Exam Results
  - Class Average: 60.3
  - Top score: 99/100
- The final exam
  - Date and time: 11am – 12:30pm, Monday, May 11
  - Comprehensive exam
  - Covers: Ch 1.1 – CH10.5 + Appendix A1 – A8
  - There will be a help session Wednesday, May 6, during the class
    - This session will be run by Edwin Baldelomar
    - Please be prepared to bring your own questions for him to work out with you in the session!!

Conditions for Equilibrium

What do you think the term "An object is at its equilibrium" means?

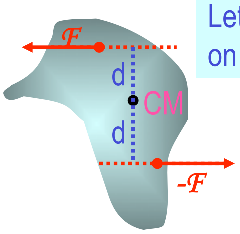
The object is either at rest (<u>Static Equilibrium</u>) or its center of mass is moving at a constant velocity (<u>Dynamic Equilibrium</u>).

When do you think an object is at its equilibrium?

Translational Equilibrium: Equilibrium in linear motion $\sum \vec{F} = 0$

Is this it?

The above condition is sufficient for a point-like object to be at its translational equilibrium. However for an object with size this is not sufficient. One more condition is needed. What is it?



Let's consider two forces equal in magnitude but in opposite direction acting on a rigid object as shown in the figure. What do you think will happen?

The object will rotate about the CM. The net torque acting on the object about any axis must be 0. $\sum \vec{\tau} = 0$

For an object to be at its *static equilibrium*, the object should not have linear or angular speed. $v_{CM} = 0 \quad \omega = 0$

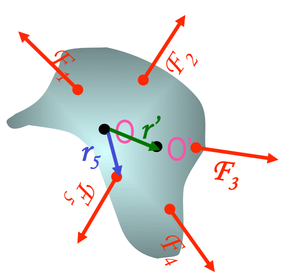
More on Conditions for Equilibrium

To simplify the problem, we will only deal with forces acting on x-y plane, giving torque only along z-axis. What do you think the conditions for equilibrium be in this case?

The six possible equations from the two vector equations turns to three equations.

$$\sum \vec{F} = 0 \Rightarrow \sum F_x = 0, \quad \sum F_y = 0 \quad \textbf{AND} \quad \sum \vec{\tau} = 0 \Rightarrow \sum \tau_z = 0$$

What happens if there are many forces exerting on an object?



If an object is at its translational static equilibrium, and if the net torque acting on the object is 0 about one axis, the net torque must be 0 about any arbitrary axis.

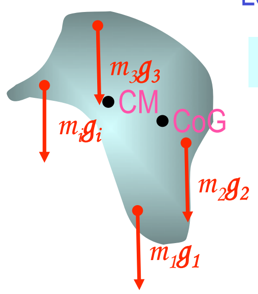
Why is this true?

Because the object is *<u>not moving</u>*, no matter what the rotational axis is, there should not be any motion. It is simply a matter of mathematical manipulation.

Center of Gravity Revisited

When is the center of gravity of a rigid body the same as the center of mass?

Under the uniform gravitational field throughout the body of the object.



Let's consider an arbitrary shaped object

The center of mass of this object is at

$$x_{CM} = \frac{\sum m_i x_i}{\sum m_i} = \frac{\sum m_i x_i}{M}$$

$$y_{CM} = \frac{\sum m_i y_i}{\sum m_i} = \frac{\sum m_i y_i}{M}$$

Let's now examine the case that the gravitational acceleration on each point is g_i

Since the CoG is the point as if all the gravitational force is exerted on, the torque due to this force becomes

$$(m_1 g_1 + m_2 g_2 + \cdots) x_{CoG} = m_1 g_1 x_1 + m_2 g_2 x_2 + \cdots$$

Generalized expression for different g throughout the body

If g is uniform throughout the body

$$(m_1 + m_2 + \cdots) g x_{CoG} = (m_1 x_1 + m_2 x_2 + \cdots) g$$

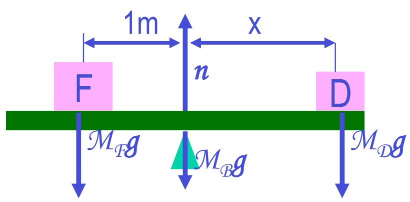
$$x_{CoG} = \frac{\sum m_i x_i}{\sum m_i} = x_{CM}$$

How do we solve static equilibrium problems?

1. Select the object to which the equations for equilibrium are to be applied.
2. Identify all the forces and draw a free-body diagram with them indicated on it with their directions and locations properly indicated
3. Choose a convenient set of x and y axes and write down force equation for each x and y component with correct signs.
4. Apply the equations that specify the balance of forces at equilibrium. Set the net force in the x and y directions equal to 0.
5. Select the most optimal rotational axis for torque calculations ➔ Selecting the axis such that the torque of one of the unknown forces become 0 makes the problem easier to solve.
6. Write down the torque equation with proper signs.
7. Solve the force and torque equations for the desired unknown quantities.

Example for Mechanical Equilibrium

A uniform 40.0 N board supports the father and the daughter each weighing 800 N and 350 N, respectively, and is not moving. If the support (or fulcrum) is under the center of gravity of the board, and the father is 1.00 m from CoG, what is the magnitude of the normal force n exerted on the board by the support?



Since there is no linear motion, this system is in its translational equilibrium

$$\sum F_x = 0$$

$$\sum F_y = n - M_B g - M_F g - M_D g = 0$$

Therefore the magnitude of the normal force $n = 40.0 + 800 + 350 = 1190N$

Determine where the child should sit to balance the system.

The net torque about the fulcrum by the three forces are

$$\tau = M_B g \cdot 0 + n \cdot 0 + M_F g \cdot 1.00 - M_D g \cdot x = 0$$

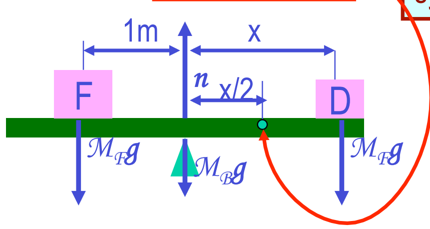
Therefore to balance the system the daughter must sit

$$x = \frac{M_F g}{M_D g} \cdot 1.00m = \frac{800}{350} \cdot 1.00m = 2.29m$$

Example for Mech. Equilibrium Cont'd

Determine the position of the child to balance the system for different position of axis of rotation.

Rotational axis



The net torque about the axis of rotation by all the forces are

$$\tau = M_B g \cdot x/2 + M_F g \cdot (1.00 + x/2) - n \cdot x/2 - M_D g \cdot x/2 = 0$$

Since the normal force is $n = M_B g + M_F g + M_D g$

The net torque can be rewritten

$$\tau = M_B g \cdot x/2 + M_F g \cdot (1.00 + x/2)$$
$$- (M_B g + M_F g + M_D g) \cdot x/2 - M_D g \cdot x/2$$
$$= M_F g \cdot 1.00 - M_D g \cdot x = 0$$

Therefore

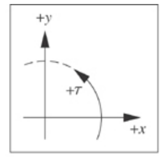
$$x = \frac{M_F g}{M_D g} \cdot 1.00m = \frac{800}{350} \cdot 1.00m = 2.29m$$

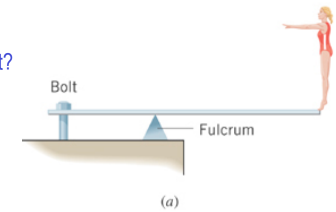
What do we learn?

No matter where the rotation axis is, net effect of the torque is identical.

Ex. A Diving Board

A woman whose weight is 530 N is poised at the right end of a diving board with length 3.90 m. The board has negligible weight and is supported by a fulcrum 1.40 m away from the left end. Find the forces that the bolt and the fulcrum exert on the board.





(a)

First the torque eq. $\sum \tau = F_2 \ell_2 - W \ell_W = 0$

How large is the torque by the bolt? None Why? Because the lever arm is 0.

So the force by the fulcrum is $F_2 = \dfrac{W \ell_W}{\ell_2}$

$$F_2 = \frac{(530\ \mathrm{N})(3.90\ \mathrm{m})}{1.40\ \mathrm{m}} = 1480\ \mathrm{N}$$

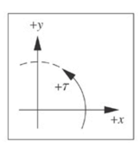
Now the force eq. $\sum F_y = -F_1 + F_2 - W = 0$

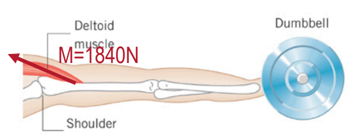
$$-F_1 + 1480\ \mathrm{N} - 530\ \mathrm{N} = 0$$

So the force by the bolt is $F_1 = 950\ \mathrm{N}$

Ex. Bodybuilding

The arm is horizontal and weighs 31.0 N. The deltoid muscle can supply 1840 N of force. What is the weight of the heaviest dumbell he can hold?





(a)

First the torque eq.

$$\sum \tau = -W_a \ell_a - W_d \ell_d + M \ell_M = 0$$

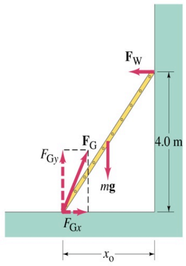
the lever arm by the deltoid muscle is $\ell_M = (0.150\ \mathrm{m}) \sin 13.0^\circ$

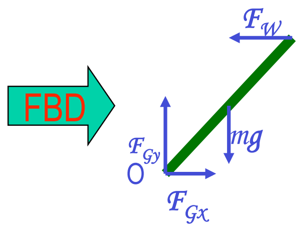
$$W_d = \frac{-W_a \ell_a + M \ell_M}{\ell_d}$$

$$= \frac{-(31.0\ \mathrm{N})(0.280\ \mathrm{m}) + (1840\ \mathrm{N})(0.150\ \mathrm{m}) \sin 13.0^\circ}{0.620\ \mathrm{m}} = 86.1\ \mathrm{N}$$

Example 9 – 7

A 5.0 m long ladder leans against a wall at a point 4.0m above the ground. The ladder is uniform and has mass 12.0kg. Assuming the wall is frictionless (but ground is not), determine the forces exerted on the ladder by the ground and the wall.





First the translational equilibrium, using components

$$\sum F_x = F_{Gx} - F_W = 0$$

$$\sum F_y = -mg + F_{Gy} = 0$$

Thus, the y component of the force by the ground is

$$F_{Gy} = mg = 12.0 \times 9.8N = 118N$$

The length x_0 is, from Pythagorian theorem

$$x_0 = \sqrt{5.0^2 - 4.0^2} = 3.0m$$

Example 9 – 7 cont'd

From the rotational equilibrium $\sum \tau_O = -mg\, x_0/2 + F_W 4.0 = 0$

Thus the force exerted on the ladder by the wall is

$$F_W = \frac{mg\, x_0/2}{4.0} = \frac{118 \cdot 1.5}{4.0} = 44N$$

The x component of the force by the ground is

$$\sum F_x = F_{Gx} - F_W = 0$$

Solve for F_{Gx}

$$F_{Gx} = F_W = 44N$$

Thus the force exerted on the ladder by the ground is

$$F_G = \sqrt{F_{Gx}^2 + F_{Gy}^2} = \sqrt{44^2 + 118^2} \approx 130N$$

The angle between the ground force to the floor

$$\theta = \tan^{-1}\left(\frac{F_{Gy}}{F_{Gx}}\right) = \tan^{-1}\left(\frac{118}{44}\right) = 70^\circ$$

Elastic Properties of Solids

We have been assuming that the objects do not change their shapes when external forces are exerting on it. It this realistic?

No. In reality, the objects get deformed as external forces act on it, though the internal forces resist the deformation as it takes place.

Deformation of solids can be understood in terms of Stress and Strain

Stress: A quantity proportional to the force causing the deformation.
Strain: Measure of the degree of deformation

It is empirically known that for small stresses, strain is proportional to stress

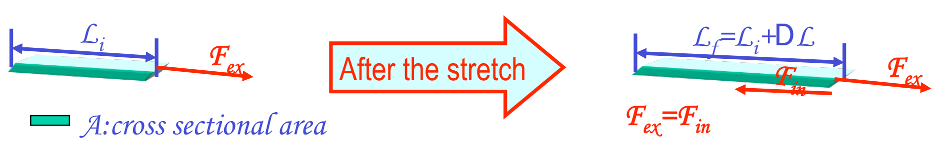
The constants of proportionality are called Elastic Modulus $\text{Elastic Modulus} \equiv \dfrac{\text{stress}}{\text{strain}}$

Three types of Elastic Modulus

1. **Young's modulus**: Measure of the elasticity in a length
2. **Shear modulus**: Measure of the elasticity in an area
3. **Bulk modulus**: Measure of the elasticity in a volume

Young's Modulus

Let's consider a long bar with cross sectional area A and initial length L_i.



Tensile stress $\text{Tensile Stress} \equiv \dfrac{F_{ex}}{A}$ Tensile strain $\text{Tensile Strain} \equiv \dfrac{\Delta L}{L_i}$

Young's Modulus is defined as $$Y \equiv \frac{\text{Tensile Stress}}{\text{Tensile Strain}} = \frac{F_{ex}/A}{\Delta L/L_i}$$

Used to characterize a rod or wire stressed under tension or compression

What is the unit of Young's Modulus? Force per unit area

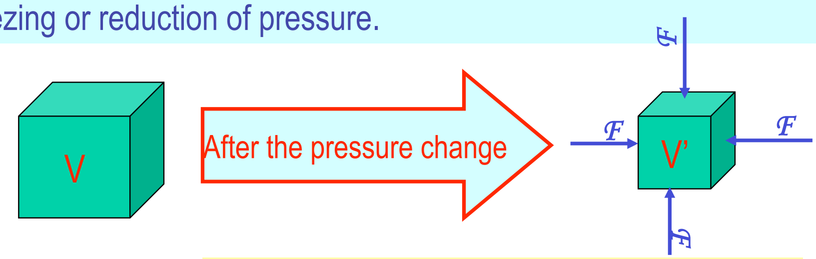
Experimental Observations

1. For a fixed external force, the change in length is proportional to the original length
2. The necessary force to produce the given strain is proportional to the cross sectional area

Elastic limit: Maximum stress that can be applied to the substance before it becomes permanently deformed

Bulk Modulus

Bulk Modulus characterizes the response of a substance to uniform squeezing or reduction of pressure.



Volume stress =pressure

$$\text{Pressure} \equiv \frac{\text{Normal Force}}{\text{Surface Area the force applies}} = \frac{F}{A}$$

If the pressure on an object changes by DP=DF/A, the object will undergo a volume change DV.

Bulk Modulus is defined as

$$\mathrm{B} \equiv \frac{\text{Volume Stress}}{\text{Volume Strain}} = -\frac{\Delta F / A}{\Delta V / V_i} = -\frac{\Delta P}{\Delta V / V_i}$$

Because the change of volume is reverse to change of pressure.

Compressibility is the reciprocal of Bulk Modulus

Example for Solid's Elastic Property

A solid brass sphere is initially under normal atmospheric pressure of 1.0x10^{5}N/m^2. The sphere is lowered into the ocean to a depth at which the pressures is 2.0x10^{7}N/m^2. The volume of the sphere in air is 0.5m^3. By how much its volume change once the sphere is submerged?

Since bulk modulus is $\mathrm{B} = -\frac{\Delta P}{\Delta V / V_i}$

The amount of volume change is $\Delta V = -\frac{\Delta P V_i}{\mathrm{B}}$

From table 12.1, bulk modulus of brass is 6.1x10^{10} N/m^2

The pressure change DP is $\Delta P = P_f - P_i = 2.0\times10^7 - 1.0\times10^5 \approx 2.0\times10^7$

Therefore the resulting volume change DV is $\Delta V = V_f - V_i = -\frac{2.0\times10^7 \times 0.5}{6.1\times10^{10}} = -1.6\times10^{-4}\, m^3$

The volume has decreased.

Density and Specific Gravity

Density, r (rho), of an object is defined as mass per unit volume

$$\rho \equiv \frac{M}{V}$$

Unit? kg/m^3

Dimension? $[ML^{-3}]$

Specific Gravity of a substance is defined as the ratio of the density of the substance to that of water at 4.0 °C (r_{H2O}=1.00g/cm^3).

$$SG \equiv \frac{\rho_{\text{substance}}}{\rho_{H_2O}}$$

Unit? None

Dimension? None

What do you think would happen of a substance in the water dependent on SG?

$SG > 1$ Sink in the water

$SG < 1$ Float on the surface

Fluid and Pressure

What are the three states of matter? Solid, Liquid and Gas

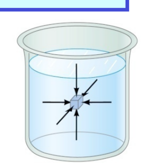
How do you distinguish them? Using the time it takes for a particular substance to change its shape in reaction to external forces.

What is a fluid? A collection of molecules that are **randomly arranged** and **loosely bound** by forces between them or by an external container.

We will first learn about mechanics of fluid at rest, *fluid statics.*

In what ways do you think fluid exerts stress on the object submerged in it?

Fluid cannot exert shearing or tensile stress. Thus, the only force the fluid exerts on an object immersed in it is the force perpendicular to the surface of the object.



This force by the fluid on an object usually is expressed in the form of the force per unit area at the given depth, the pressure, defined as

$$P \equiv \frac{F}{A}$$

Expression of pressure for an infinitesimal area dA by the force dF is

$$P = \frac{dF}{dA}$$

Note that pressure is a scalar quantity because it's the magnitude of the force on a surface area A.

What is the unit and the dimension of pressure? Unit: N/m^2 Dim.: $[M][L^{-1}][T^{-2}]$

Special SI unit for pressure is Pascal

$$1Pa \equiv 1N/m^2$$

Example for Pressure

The mattress of a water bed is 2.00m long by 2.00m wide and 30.0cm deep. a) Find the weight of the water in the mattress.

The volume density of water at the normal condition (0^{o}C and 1 atm) is 1000kg/m^3. So the total mass of the water in the mattress is

$$m=\rho_W V_M=1000\times 2.00\times 2.00\times 0.300=1.20\times 10^3\,kg$$

Therefore the weight of the water in the mattress is

$$W=mg=1.20\times 10^3\times 9.8=1.18\times 10^4\,N$$

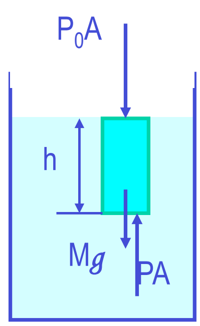
b) Find the pressure exerted by the water on the floor when the bed rests in its normal position, assuming the entire lower surface of the mattress makes contact with the floor.

Since the surface area of the mattress is 4.00 m^2, the pressure exerted on the floor is

$$P=\frac{F}{A}=\frac{mg}{A}=\frac{1.18\times 10^4}{4.00}=2.95\times 10^3$$

Variation of Pressure and Depth

Water pressure increases as a function of depth, and the air pressure decreases as a function of altitude. Why?



It seems that the pressure has a lot to do with the total mass of the fluid above the object that puts weight on the object.

Let's imagine the liquid contained in a cylinder with height *h* and the cross sectional area *A* immersed in a fluid of density *r* at rest, as shown in the figure, and the system is in its equilibrium.

If the liquid in the cylinder is the same substance as the fluid, the mass of the liquid in the cylinder is $M=\rho V=\rho A h$

Since the system is in its equilibrium $PA-P_0A-Mg=PA-P_0A-\rho Ahg=0$

Therefore, we obtain $P=P_0+\rho gh$

The pressure at the depth *h* below the surface of the fluid open to the atmosphere is greater than the atmospheric pressure by *rgh*.

Atmospheric pressure P_0 is

$$1.00atm=1.013\times 10^5\,Pa$$

Pascal's Principle and Hydraulics

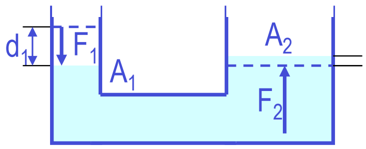
A change in the pressure applied to a fluid is transmitted undiminished to every point of the fluid and to the walls of the container.

$$P = P_0 + \rho g h$$

What happens if P_0 is changed?

The resultant pressure P at any given depth h increases as much as the change in P_0.

This is the principle behind hydraulic pressure. How?



Since the pressure change caused by the the force F_1 applied onto the area A_1 is transmitted to the F_2 on an area A_2.

$$P = \frac{F_1}{A_1} = \frac{F_2}{A_2}$$

Therefore, the resultant force F_2 is $F_2 = \frac{A_2}{A_1} F_1$

In other words, the force gets multiplied by the ratio of the areas A_2/A_1 and is transmitted to the force F_2 on the surface.

This seems to violate some kind of conservation law, doesn't it?

No, the actual displaced volume of the fluid is the same. And the work done by the forces are still the same.

$$F_2 = \frac{d_1}{d_2} F_1$$

Example for Pascal's Principle

In a car lift used in a service station, compressed air exerts a force on a small piston that has a circular cross section and a radius of 5.00cm. This pressure is transmitted by a liquid to a piston that has a radius of 15.0cm. What force must the compressed air exert to lift a car weighing 13,300N? What air pressure produces this force?

Using the Pascal's principle, one can deduce the relationship between the forces, the force exerted by the compressed air is

$$F_1 = \frac{A_1}{A_2} F_2 = \frac{\pi (0.05)^2}{\pi (0.15)^2} \times 1.33 \times 10^4 = 1.48 \times 10^3 N$$

Therefore the necessary pressure of the compressed air is

$$P = \frac{F_1}{A_1} = \frac{1.48 \times 10^3}{\pi (0.05)^2} = 1.88 \times 10^5 Pa$$

Example for Pascal's Principle

Estimate the force exerted on your eardrum due to the water above when you are swimming at the bottom of the pool with a depth 5.0 m.

We first need to find out the pressure difference that is being exerted on the eardrum. Then estimate the area of the eardrum to find out the force exerted on the eardrum.

Since the outward pressure in the middle of the eardrum is the same as normal air pressure

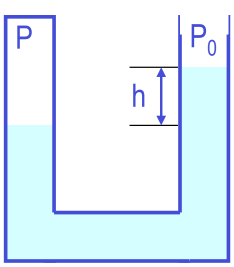
$$P - P_0 = \rho_W g h = 1000 \times 9.8 \times 5.0 = 4.9 \times 10^4 \, Pa$$

Estimating the surface area of the eardrum at $1.0 cm^2 = 1.0 \times 10^{-4} \, m^2$, we obtain

$$F = (P - P_0) A \approx 4.9 \times 10^4 \times 1.0 \times 10^{-4} \approx 4.9 N$$

Absolute and Relative Pressure

How can one measure pressure?



One can measure the pressure using an open-tube manometer, where one end is connected to the system with unknown pressure P and the other open to air with pressure P_0.

The measured pressure of the system is $P = P_0 + \rho g h$

This is called the **absolute pressure**, because it is the actual value of the system's pressure.

In many cases we measure the pressure difference with respect to the atmospheric pressure to avoid the effect of the changes in P_0 that depends on the environment. This is called **gauge or relative pressure**.

$$P_G = P - P_0 = \rho g h$$

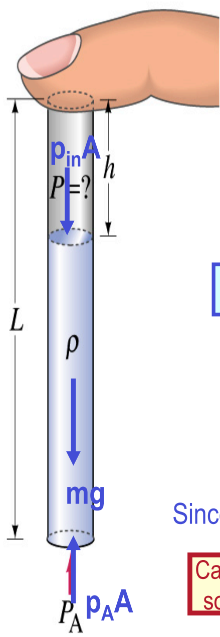
The common barometer which consists of a mercury column with one end closed at vacuum and the other open to the atmosphere was invented by Evangelista Torricelli.

Since the closed end is at vacuum, it does not exert any force. 1 atm of air pressure pushes mercury up 76cm. So 1 atm is

$$P_0 = \rho g h = (13.595 \times 10^3 kg/m^3)(9.80665 m/s^2)(0.7600 m)$$
$$= 1.013 \times 10^5 Pa = 1 atm$$

If one measures the tire pressure with a gauge at 220kPa the actual pressure is 101kPa+220kPa=303kPa.

Finger Holds Water in Straw



You insert a straw of length *L* into a tall glass of your favorite beverage. You place your finger over the top of the straw so that no air can get in or out, and then lift the straw from the liquid. You find that the straw strains the liquid such that the distance from the bottom of your finger to the top of the liquid is *h*. Does the air in the space between your finger and the top of the liquid in the straw have a pressure P that is (a) greater than, (b) equal to, or (c) less than, the atmospheric pressure P_A outside the straw? **Less**

What are the forces in this problem?

Gravitational force on the mass of the liquid $F_g = mg = \rho A(L-h)g$

Force exerted on the top surface of the liquid by inside air pressure $F_{in} = p_{in}A$

Force exerted on the bottom surface of the liquid by the outside air $F_{out} = -p_A A$

Since it is at equilibrium $F_{out} + F_g + F_{in} = 0 \Rightarrow -p_A \cancel{A} + \rho g(L-h)\cancel{A} + p_{in}\cancel{A} = 0$

Cancel A and solve for p_{in} $\Rightarrow$ $p_{in} = p_A - \rho g(L-h)$

So p_{in} is less than P_A by rg(L-h).

Congratulations!!!!

You all have done very well!!!

I certainly had a lot of fun with ya'll and am truly proud of you!

Good luck with your exam!!!

Have a safe summer!!