PHYS 1443 – Section 001
Lecture #3

Wednesday, January 26, 2011

Dr. Jaehoon Yu

- One Dimensional Motion
  - Displacement
  - Average Speed and Velocity
  - Instantaneous Speed and Velocity
  - Acceleration
- 1D Motion under constant acceleration
- Free Fall

Announcements

- Some homework tips
  - 57 out of 59 registered!
    - Very important! Please register ASAP
  - When inputting answers to the Quest homework system
    - Unless the problem explicitly asks for significant figures, input as many digits as you can
    - The Quest is dumb. So it does not know about anything other than numbers
- 49 out of 59 subscribed to e-mail list
  - 3 point extra credit if done by today, Jan. 26
- The first term exam is to be on Monday, Feb. 7
  - Will cover CH1 – what we finish on Wednesday, Feb. 3 + Appendices A and B
  - Mixture of multiple choices and free response problems

Special Problems for Extra Credit

- Derive the quadratic equation for Bx^2-Cx+A=0 ➔ 5 points
- Derive the kinematic equation $v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$ from first principles and the known kinematic equations ➔ 10 points
- You must **show your work in detail** to obtain full credit
- Due at the start of the class, Monday, Feb. 7

Displacement, Velocity and Speed

One dimensional displacement is defined as:

$$\Delta x \equiv x_f - x_i$$

A vector quantity

Displacement is the difference between initial and final potions of the motion and is a vector quantity. How is this different than distance?

Unit? **m**

The average velocity is defined as: $$v_x \equiv \frac{x_f - x_i}{t_f - t_i} = \frac{\Delta x}{\Delta t} \equiv \frac{\text{Displacement}}{\text{Elapsed Time}}$$

Unit? **m/s**

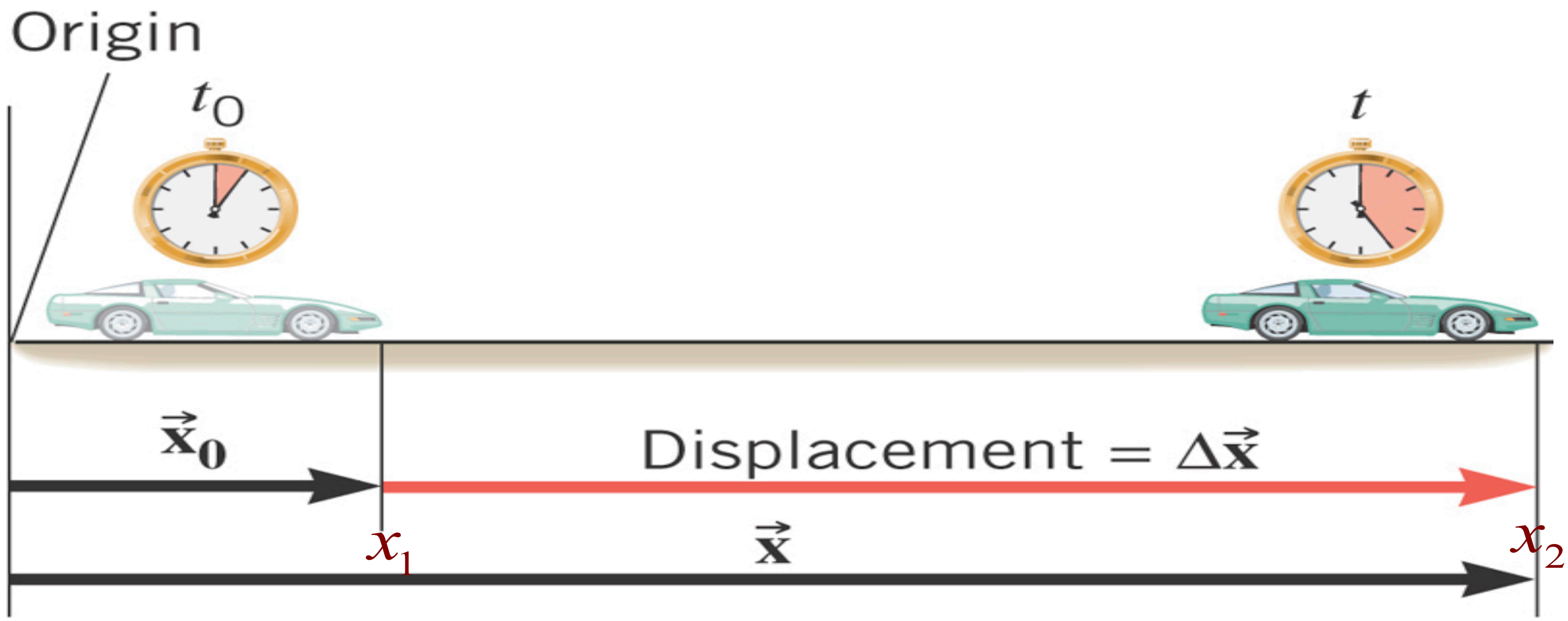
A vector quantity

Displacement per unit time in the period throughout the motion

The average speed is defined as: $$v \equiv \frac{\text{Total Distance Traveled}}{\text{Total Elapsed Time}}$$

Unit? **m/s**

A scalar quantity



What is the displacement? $\Delta x = x_2 - x_1$

How much is the elapsed time? $\Delta t = t - t_0$

Displacement, Velocity and Speed

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Unit? m

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Unit? m/s

Displacement per unit time in the period throughout the motion

The average speed is defined as: $$v \equiv \frac{\text{Total Distance Traveled}}{\text{Total Elapsed Time}}$$

Unit? m/s

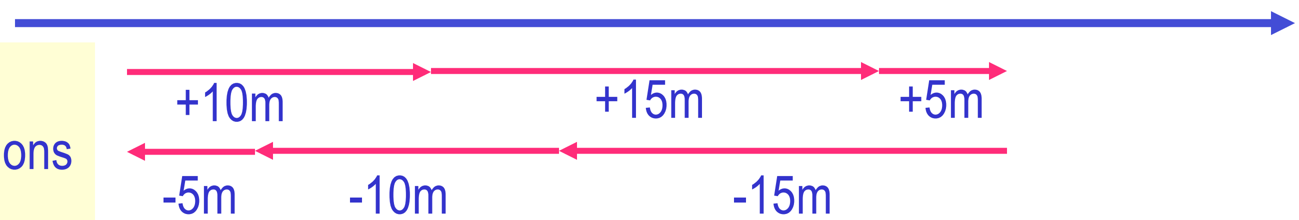
Can someone tell me what the difference between speed and velocity is?

Difference between Speed and Velocity

- Let's take a simple one dimensional translation that has many steps:

Let's call this line as X-axis

Let's have a couple of motions in a total time interval of 20 sec.



Total Displacement: $\Delta x \equiv x_f - x_i = x_i - x_i = 0(m)$

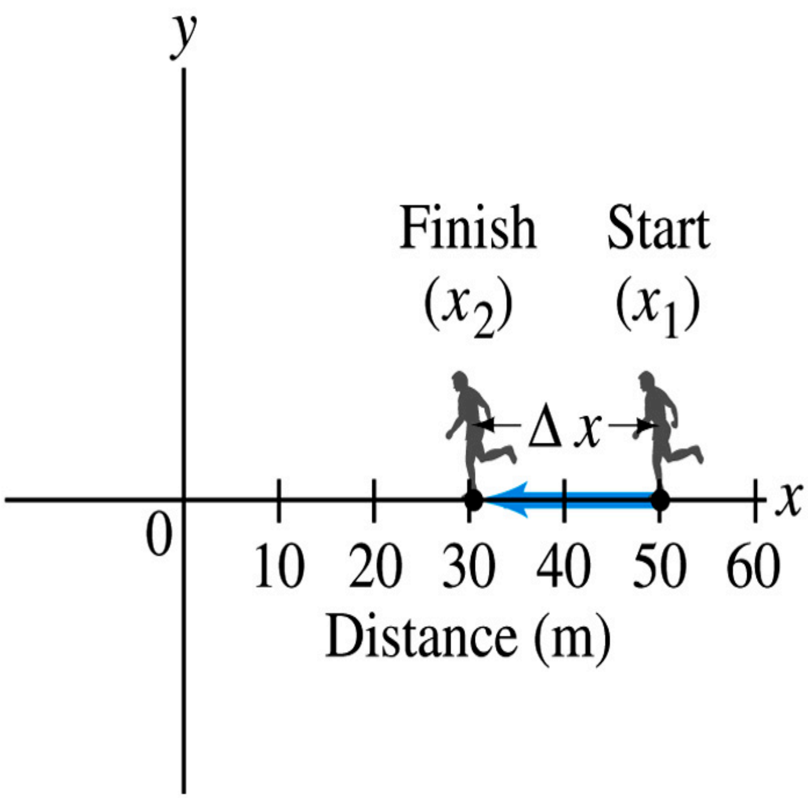
Average Velocity: $v_x \equiv \frac{x_f - x_i}{t_f - t_i} = \frac{\Delta x}{\Delta t} = \frac{0}{20} = 0(m/s)$

Total Distance Traveled: $D = 10 + 15 + 5 + 15 + 10 + 5 = 60(m)$

Average Speed: $v \equiv \frac{\text{Total Distance Traveled}}{\text{Total Time Interval}} = \frac{60}{20} = 3(m/s)$

Example 2.1

The position of a runner as a function of time is plotted as moving along the x axis of a coordinate system. During a 3.00-s time interval, the runner's position changes from x_1=50.0m to x_2=30.5 m, as shown in the figure. What was the runner's average velocity? What was the average speed?



- Displacement:

$$\Delta x \equiv x_f - x_i = x_2 - x_1 = 30.5 - 50.0 = -19.5(m)$$

- Average Velocity:

$$v_x \equiv \frac{x_f - x_i}{t_f - t_i} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t} = \frac{-19.5}{3.00} = -6.50(m/s)$$

- Average Speed:

$$v \equiv \frac{\text{Total Distance Traveled}}{\text{Total Time Interval}}$$

$$= \frac{50.0 - 30.5}{3.00} = \frac{+19.5}{3.00} = +6.50(m/s)$$

Instantaneous Velocity and Speed

- Can average quantities tell you the detailed story of the whole motion? **NO!!**
- Instantaneous velocity is defined as:

$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

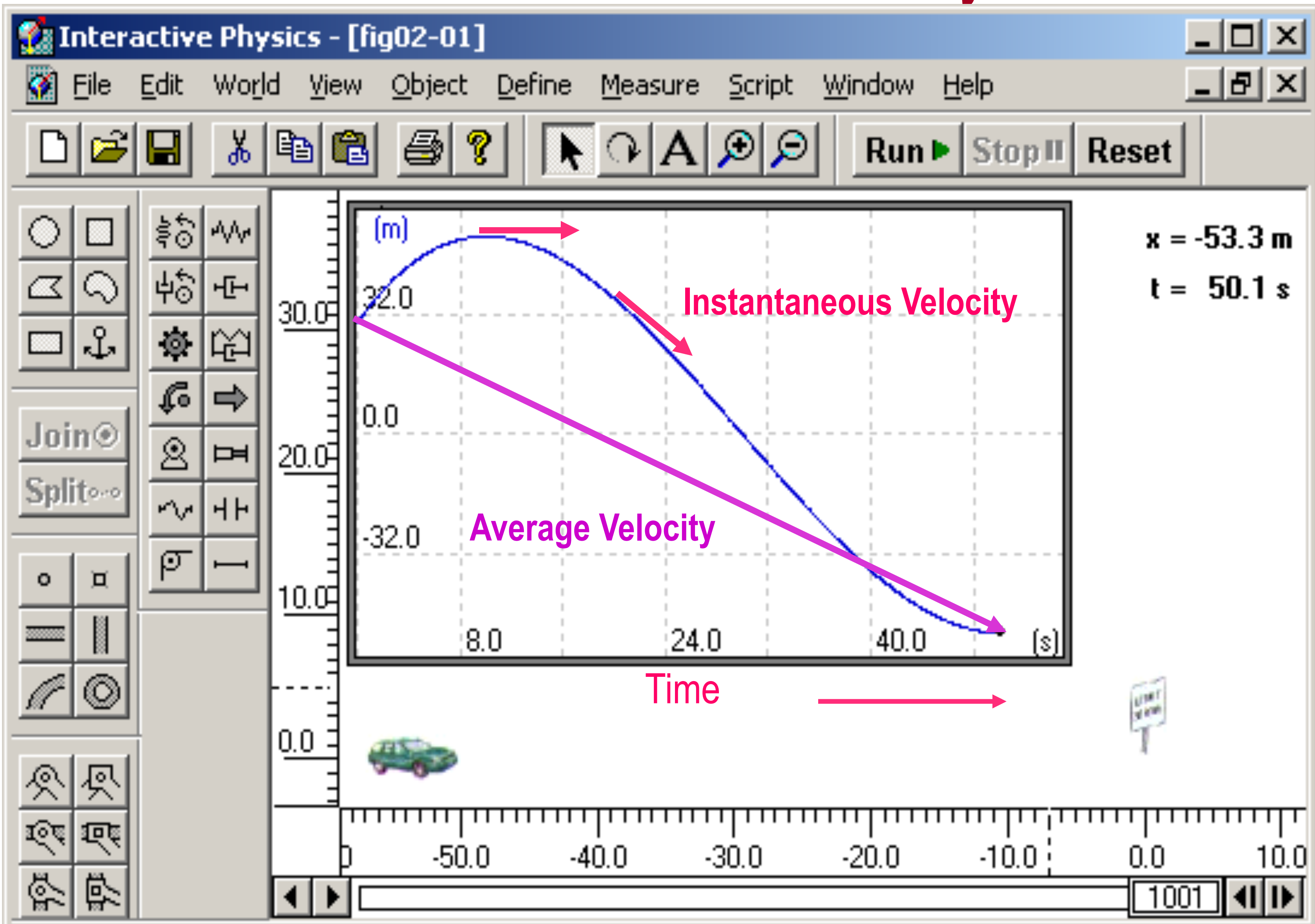
  - What does this mean?
    - Displacement in an infinitesimal time interval
    - Average velocity over a very, very short amount of time

•Instantaneous speed is the size (magnitude) of the velocity vector:

$$|v_x| = \left| \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} \right|$$

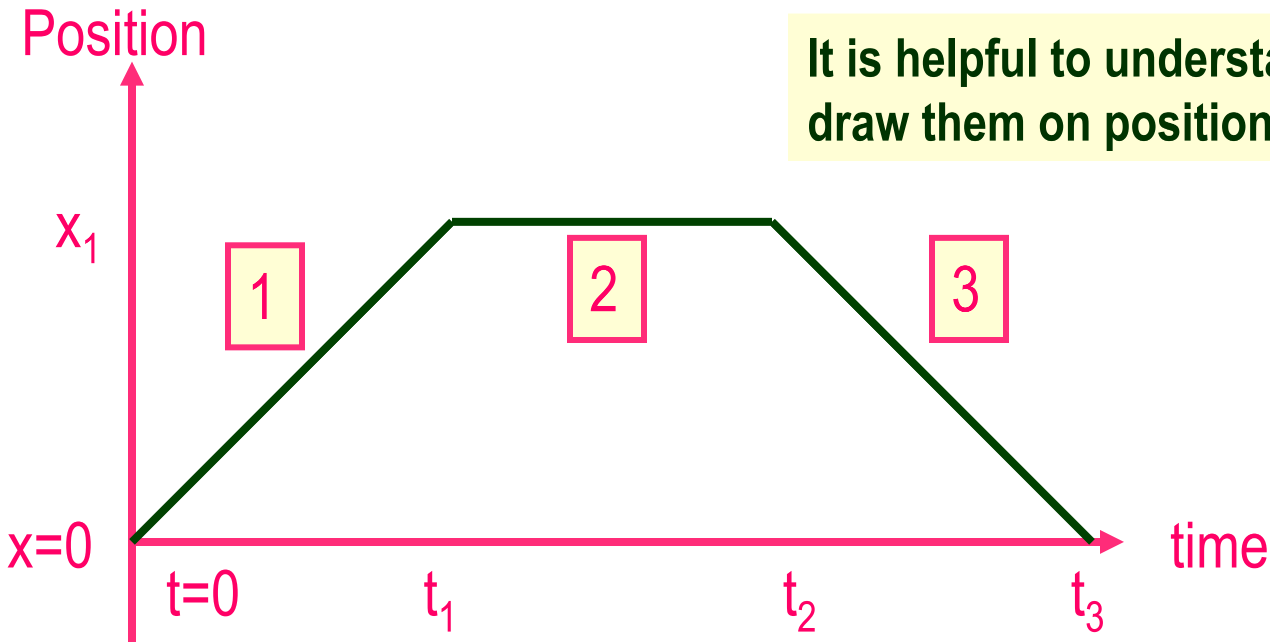
*Magnitude of Vectors are Expressed in absolute values

Instantaneous Velocity



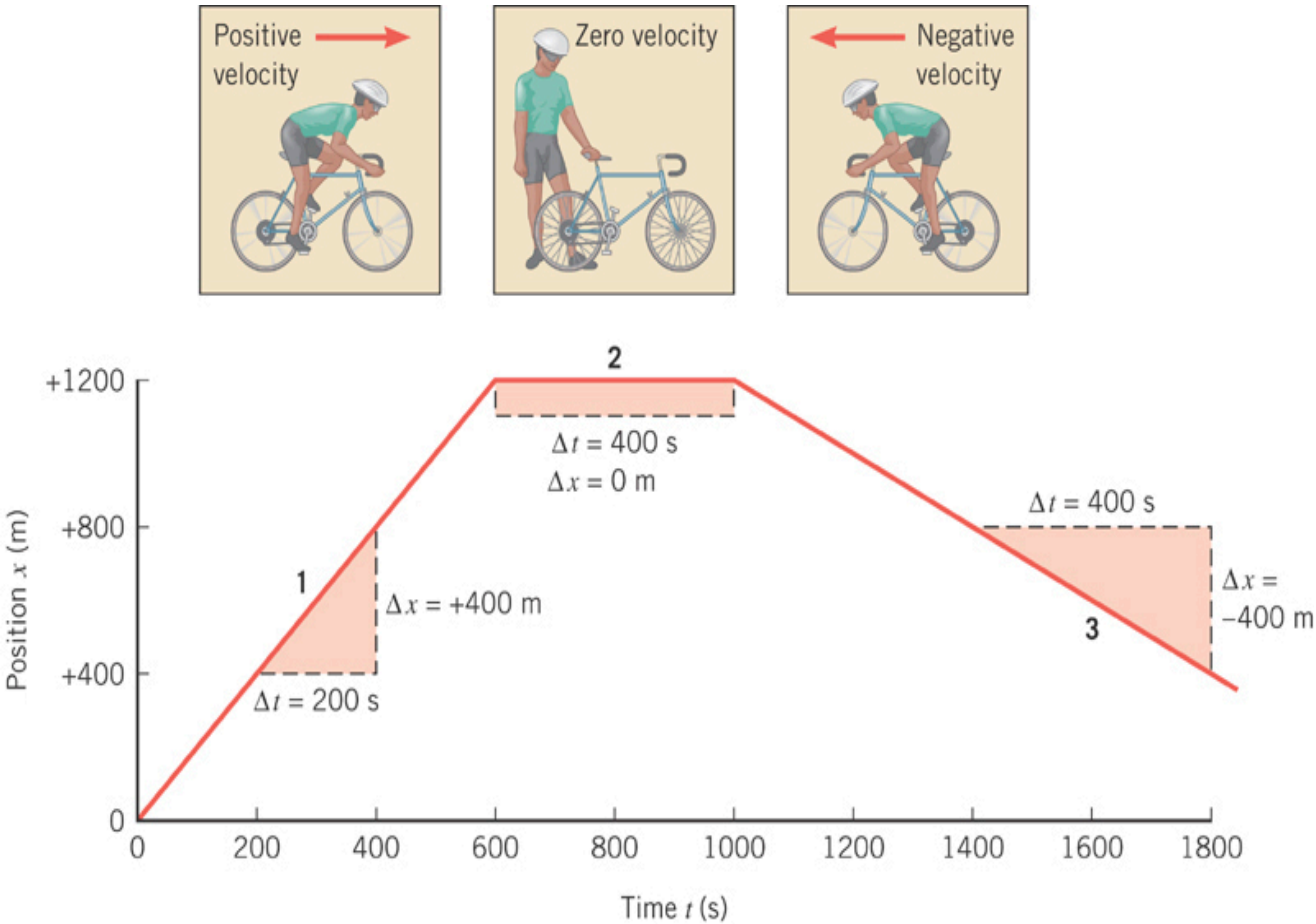
Position vs Time Plot

It is helpful to understand motions to draw them on position vs time plots.



1. Running at a constant velocity (go from x=0 to $x=x_1$ in t_1, Displacement is + x_1 in t_1 time interval)
2. Velocity is 0 (go from x_1 to x_1 no matter how much time changes)
3. Running at a constant velocity but in the reverse direction as 1. (go from x_1 to x=0 in t_3-t_2 time interval, Displacement is - x_1 in t_3-t_2 time interval)

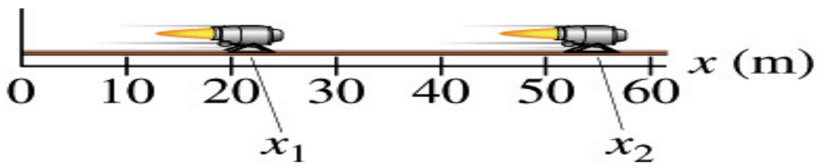
Does this motion physically make sense?

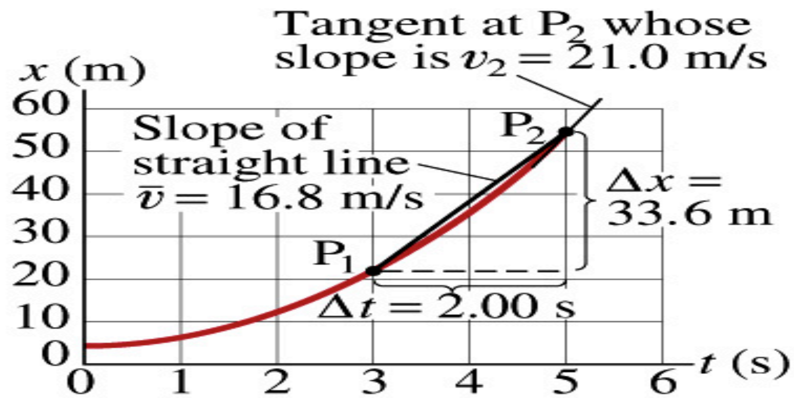


Positive velocity
Zero velocity
Negative velocity
2
+1200
Δt = 400 s
Δx = 0 m
+800
Δt = 400 s
1
Δx = +400 m
Δx = −400 m
3
+400
Δt = 200 s
0
Position x (m)
0
200
400
600
800
1000
1200
1400
1600
1800
Time t (s)

Example 2.3

A jet engine moves along a track. Its position as a function of time is given by the equation $x=At^2+B$ where A=2.10m/s^2 and B=2.80m.





(a) Determine the displacement of the engine during the interval from t_1=3.00s to t_2=5.00s.

$$x_1 = x_{t_1=3.00} = 2.10\times(3.00)^2 + 2.80 = 21.7m \qquad x_2 = x_{t_2=5.00} = 2.10\times(5.00)^2 + 2.80 = 55.3m$$

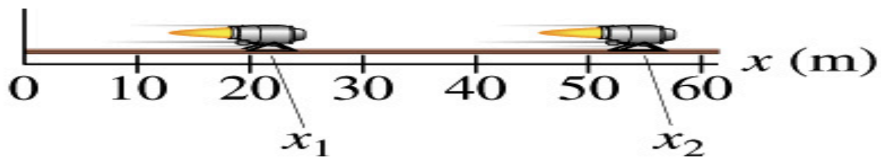
Displacement is, therefore:

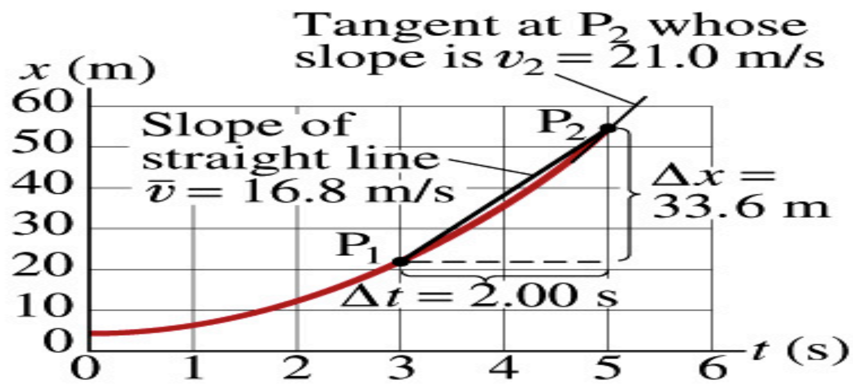
$$\Delta x = x_2 - x_1 = 55.3 - 21.7 = +33.6(m)$$

(b) Determine the average velocity during this time interval.

$$\bar{v}_x = \frac{\Delta x}{\Delta t} = \frac{33.6}{5.00-3.00} = \frac{33.6}{2.00} = 16.8(m/s)$$

Example 2.3 cont'd





(c) Determine the instantaneous velocity at $t=t_2$=5.00s.

Calculus formula for derivative $\frac{d}{dt}\left(Ct^n\right) = nCt^{n-1}$ and $\frac{d}{dt}\left(C\right) = 0$

The derivative of the engine's equation of motion is

$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} = \frac{d}{dt}\left(At^2 + B\right) = 2At$$

The instantaneous velocity at t=5.00s is

$$v_x\left(t = 5.00s\right) = 2A \times 5.00 = 2.10 \times 10.0 = 21.0\left(m/s\right)$$

Displacement, Velocity and Speed

Displacement

$$\Delta x \equiv x_f - x_i$$

Average velocity

$$v_x \equiv \frac{x_f - x_i}{t_f - t_i} = \frac{\Delta x}{\Delta t}$$

Average speed

$$v \equiv \frac{\text{Total Distance Traveled}}{\text{Total Time Spent}}$$

Instantaneous velocity

$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

Instantaneous speed

$$\left|v_x\right| = \left|\lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}\right| = \left|\frac{dx}{dt}\right|$$

Acceleration

Change of velocity in time (what kind of quantity is this?) Vector

- Definition of the average acceleration:

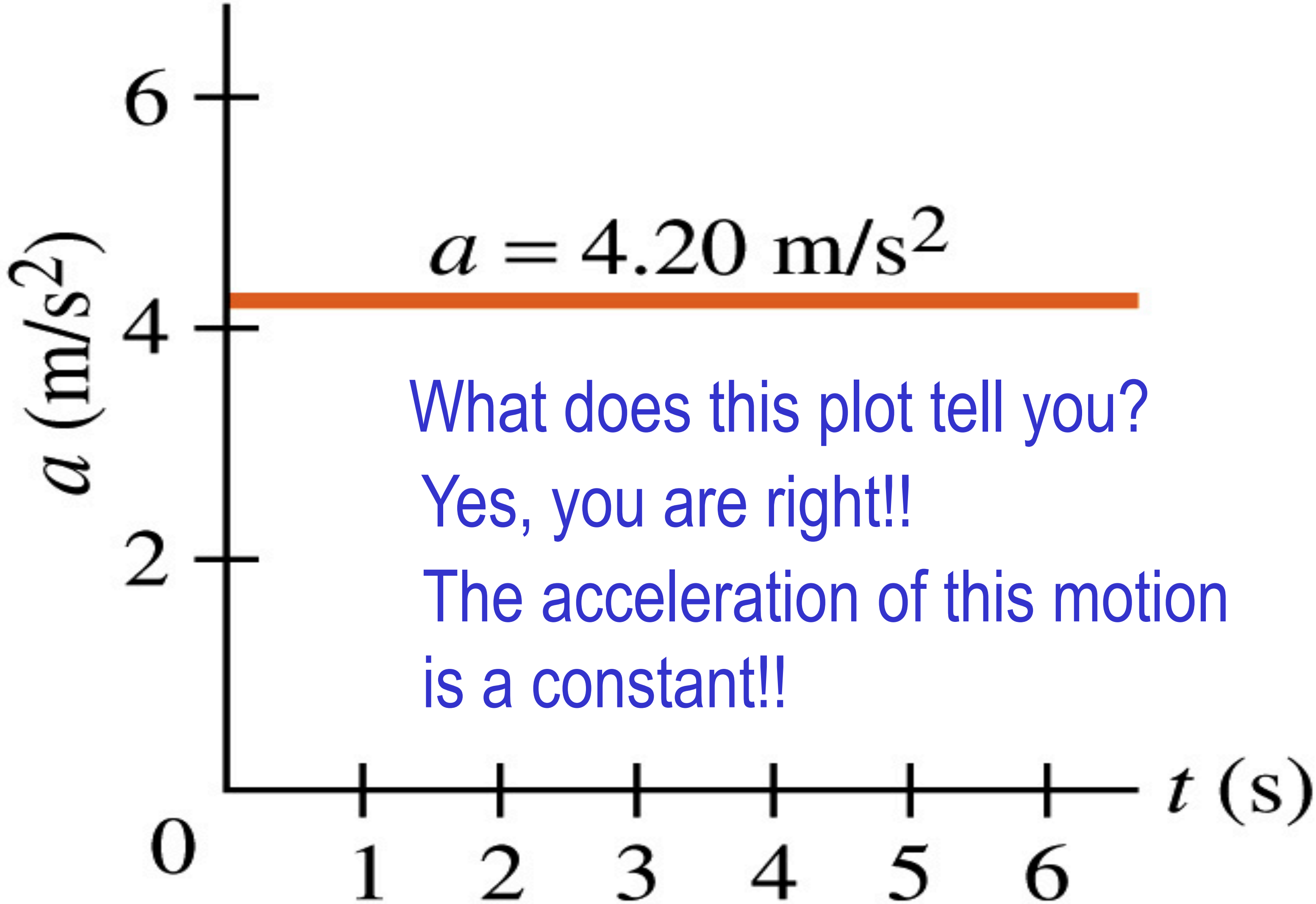
$$a_x \equiv \frac{v_{xf} - v_{xi}}{t_f - t_i} = \frac{\Delta v_x}{\Delta t}$$ analogous to $$v_x \equiv \frac{x_f - x_i}{t_f - t_i} = \frac{\Delta x}{\Delta t}$$

- Definition of the instantaneous acceleration:

$$a_x \equiv \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$ analogous to $$v_x \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

- In calculus terms: The slope (derivative) of the velocity vector with respect to time or the change of slopes of position as a function of time

Acceleration vs Time Plot



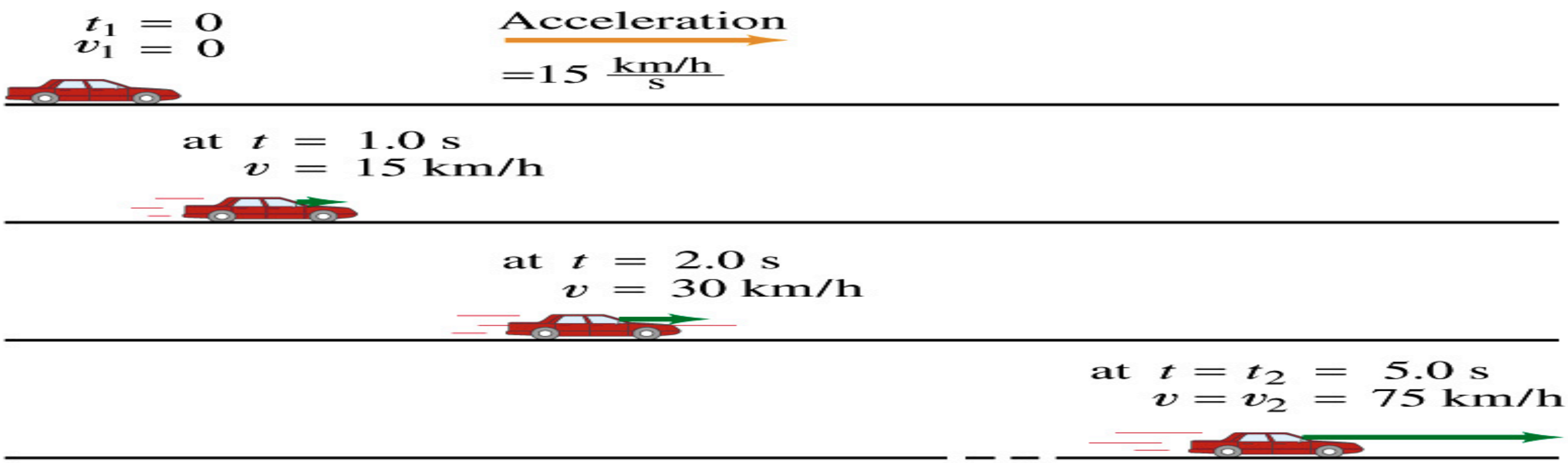
What does this plot tell you?

Yes, you are right!!

The acceleration of this motion is a constant!!

Example 2.4

A car accelerates along a straight road from rest to 75km/h in 5.0s.



What is the magnitude of its average acceleration?

$$v_{xi} = 0\ m/s$$

$$v_{xf} = \frac{75000m}{3600s} = 21\ m/s$$

$$\bar{a}_x = \frac{v_{xf} - v_{xi}}{t_f - t_i} = \frac{\Delta v_x}{\Delta t} = \frac{21-0}{5.0} = \frac{21}{5.0} = 4.2(m/s^2)$$

$$= \frac{4.2 \times (3600)^2}{1000} = 5.4 \times 10^4 (km/h^2)$$

Few Confusing Things on Acceleration

- When an object is moving in a constant velocity ($v=v_0$), there is no acceleration ($a=0$)
  - Is there any acceleration when an object is not moving?
- When an object is moving faster as time goes on, ($v=v(t)$), acceleration is positive ($a>0$).
  - Incorrect, since the object might be moving in negative direction initially
- When an object is moving slower as time goes on, ($v=v(t)$), acceleration is negative ($a<0$)
  - Incorrect, since the object might be moving in negative direction initially
- In all cases, velocity is positive, unless the direction of the movement changes.
  - Incorrect, since the object might be moving in negative direction initially
- Is there acceleration if an object moves in a constant speed but changes direction?

The answer is YES!!