PHYS 1443 – Section 001
Lecture #4

Monday, January 31, 2011

Dr. Jaehoon Yu

- 1D Motion under constant acceleration
- Free Fall
- Coordinate System
- Vectors and Scalars
- Motion in Two Dimensions
  - Motion under constant acceleration
  - Projectile Motion
  - Maximum ranges and heights

Today's homework is homework #3, due 10pm, Tuesday, Feb. 8!!

Announcements

- 53 out of 60 subscribed to e-mail list
  - Very important communication tool. Please subscribe to it ASAP.
- Quiz results
  - Class Average: 7.3/12
    - Equivalent to 60.8/100
  - Top score: 11.6/12
  - Not bad
- The first term exam is to be on Monday, Feb. 7
  - Time: 1 – 2:20pm
  - Will cover CH1 – what we finish this Wednesday, Feb. 3 + Appendices A and B
  - Mixture of multiple choices and free response problems
- No Department colloquium this week!

Reminder: Special Problems for Extra Credit

- Derive the quadratic equation for $Bx^2-Cx+A=0$ → 5 points
- Derive the kinematic equation $v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$ from first principles and the known kinematic equations → 10 points
- You must **show your work in detail** to obtain full credit
- Due at the start of the class, Monday, Feb. 7

Displacement, Velocity, Speed & Acceleration

Displacement
$$\boxed{\Delta x \equiv x_f - x_i}$$

Average velocity
$$\boxed{v_x \equiv \frac{x_f - x_i}{t_f - t_i} = \frac{\Delta x}{\Delta t}}$$

Average speed
$$\boxed{v \equiv \frac{\text{Total Distance Traveled}}{\text{Total Time Spent}}}$$

Instantaneous velocity
$$\boxed{v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}}$$

Instantaneous speed
$$\boxed{\left|v_x\right| = \left|\lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}\right| = \left|\frac{dx}{dt}\right|}$$

Average acceleration
$$a_x \equiv \frac{v_{xf} - v_{xi}}{t_f - t_i} = \frac{\Delta v_x}{\Delta t}$$

Instantaneous acceleration
$$a_x \equiv \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$

One Dimensional Motion

- Let's focus on the simplest case: acceleration is a constant ($a=a_0$)
- Using the definitions of average acceleration and velocity, we can derive equations of motion (description of motion, velocity and position as a function of time)

$$\bar{a}_x = a_x = \frac{v_{xf} - v_{xi}}{t_f - t_i}$$ (If $t_f=t$ and $t_i=0$) $$a_x = \frac{v_{xf} - v_{xi}}{t} \Rightarrow v_{xf} = v_{xi} + a_x t$$

For constant acceleration, average velocity is a simple numeric average

$$\bar{v}_x = \frac{v_{xi} + v_{xf}}{2} = \frac{2v_{xi} + a_x t}{2} = v_{xi} + \frac{1}{2} a_x t$$

$$\bar{v}_x = \frac{x_f - x_i}{t_f - t_i}$$ (If $t_f=t$ and $t_i=0$) $$\bar{v}_x = \frac{x_f - x_i}{t} \Rightarrow x_f = x_i + \bar{v}_x t$$

Resulting Equation of Motion becomes

$$x_f = x_i + \bar{v}_x t = x_i + v_{xi} t + \frac{1}{2} a_x t^2$$

Kinematic Equations of Motion on a Straight Line Under Constant Acceleration

$$\boxed{v_{xf}(t) = v_{xi} + a_x t}$$

Velocity as a function of time

$$\boxed{x_f - x_i = \frac{1}{2}\bar{v}_x t = \frac{1}{2}(v_{xf} + v_{xi})t}$$

Displacement as a function of velocities and time

$$\boxed{x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2}$$

Displacement as a function of time, velocity, and acceleration

$$\boxed{v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)}$$

Velocity as a function of Displacement and acceleration

You may use different forms of Kinematic equations, depending on the information given to you in specific physical problems!!

How do we solve a problem using a kinematic formula under constant acceleration?

- Identify what information is given in the problem.
  - Initial and final velocity?
  - Acceleration?
  - Distance, initial position or final position?
  - Time?
- Convert the units of all quantities to SI units to be consistent.
- Identify what the problem wants
- Identify which kinematic formula is **most appropriate and easiest** to solve for what the problem wants.
  - Frequently multiple formulae can give you the answer for the quantity you are looking for. ➔ Do not just use any formula but use the one that can be easiest to solve.
- Solve the equations for the quantity or quantities wanted.

Example 2.11

Suppose you want to design an air-bag system that can protect the driver in a head-on collision at a speed 100km/hr (~60miles/hr). Estimate how fast the air-bag must inflate to effectively protect the driver. Assume the car crumples upon impact over a distance of about 1m. How does the use of a seat belt help the driver?

How long does it take for the car to come to a full stop? ⇨ As long as it takes for it to crumple.

The initial speed of the car is $v_{xi} = 100km/h = \dfrac{100000m}{3600s} = 28m/s$

We also know that $v_{xf} = 0m/s$ and $x_f - x_i = 1m$

Using the kinematic formula $v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$

The acceleration is $a_x = \dfrac{v_{xf}^2 - v_{xi}^2}{2(x_f - x_i)} = \dfrac{0 - (28m/s)^2}{2\times 1m} = -390m/s^2$

Thus the time for air-bag to deploy is $t = \dfrac{v_{xf} - v_{xi}}{a} = \dfrac{0 - 28m/s}{-390m/s^2} = 0.07s$

Falling Motion

- Falling motion is a motion under the influence of the gravitational pull (gravity) only; Which direction is a freely falling object moving?

  Yes, down to the center of the earth!!

  – A motion under constant acceleration

  – All kinematic formula we learned can be used to solve for falling motions.
- Gravitational acceleration is inversely proportional to the distance between the object and the center of the earth
- The magnitude of the gravitational acceleration is $g=9.80 m/s^2$ on the surface of the earth, most of the time.
- The direction of gravitational acceleration is **ALWAYS** toward the center of the earth, which we normally call (-y); where up and down direction are indicated as the variable "y"
- Thus the correct denotation of gravitational acceleration on the surface of the earth is $g=-9.80 m/s^2$ when +y points upward

Example for Using 1D Kinematic Equations on a Falling object (sim to Ex. 2.16)

Stone was thrown straight upward at t=0 with +20.0m/s initial velocity on the roof of a 50.0m high building,

What is the acceleration in this motion? $g=-9.80m/s^2$

(a) Find the time the stone reaches at the maximum height.

What is so special about the maximum height? V=0

$$v_f = v_{yi} + a_y t = +20.0 - 9.80t = 0.00m/s$$

Solve for t

$$t = \frac{20.0}{9.80} = 2.04s$$

(b) Find the maximum height.

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2 = 50.0 + 20 \times 2.04 + \frac{1}{2} \times (-9.80) \times (2.04)^2$$

$$= 50.0 + 20.4 = 70.4(m)$$

Example of a Falling Object cnt'd

(c) Find the time the stone reaches back to its original height.

$$t = 2.04 \times 2 = 4.08 s$$

(d) Find the velocity of the stone when it reaches its original height.

$$v_{yf} = v_{yi} + a_y t = 20.0 + (-9.80) \times 4.08 = -20.0 (m/s)$$

(e) Find the velocity and position of the stone at t=5.00s.

Velocity

$$v_{yf} = v_{yi} + a_y t = 20.0 + (-9.80) \times 5.00 = -29.0 (m/s)$$

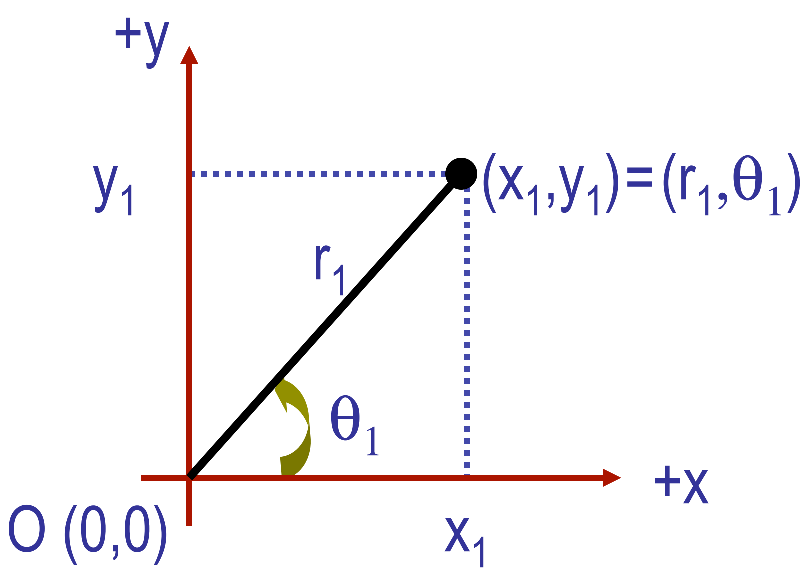
Position

$$y_f = y_i + v_{yi} t + \frac{1}{2} a_y t^2$$

$$= 50.0 + 20.0 \times 5.00 + \frac{1}{2} \times (-9.80) \times (5.00)^2 = +27.5 (m)$$

2D Coordinate Systems

- They make it easy and consistent to express locations or positions
- Two commonly used systems, depending on convenience, are
  - Cartesian (Rectangular) Coordinate System
    - Coordinates are expressed in (x,y)
  - Polar Coordinate System
    - Coordinates are expressed in distance from the origin ® and the angle measured from the x-axis, θ (r,θ)
- Vectors become a lot easier to express and compute



How are Cartesian and Polar coordinates related?

$$x_1 = r_1 \cos\theta_1$$

$$y_1 = r_1 \sin\theta_1$$

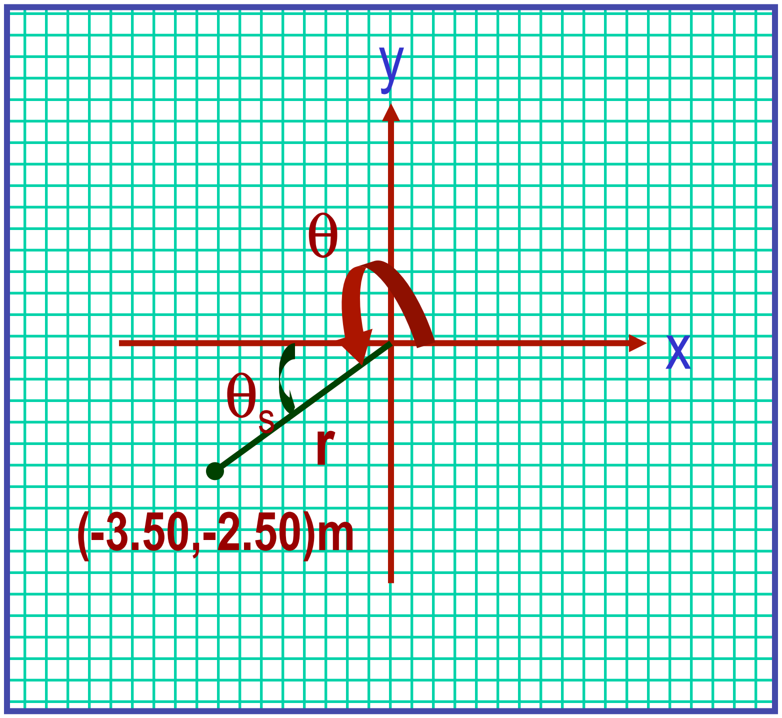
$$r_1 = \sqrt{\left(x_1^2 + y_1^2\right)}$$

$$\tan\theta_1 = \frac{y_1}{x_1}$$

$$\theta_1 = \tan^{-1}\left(\frac{y_1}{x_1}\right)$$

Example

Cartesian Coordinate of a point in the xy plane are (x,y)= (-3.50,-2.50)m. Find the equivalent polar coordinates of this point.



$$r = \sqrt{\left(x^2 + y^2\right)}$$

$$= \sqrt{\left((-3.50)^2 + (-2.50)^2\right)}$$

$$= \sqrt{18.5} = 4.30(m)$$

$$\theta = 180 + \theta_s$$

$$\tan\theta_s = \frac{-2.50}{-3.50} = \frac{5}{7}$$

$$\theta_s = \tan^{-1}\left(\frac{5}{7}\right) = 35.5^\circ$$

$$\therefore \theta = 180 + \theta_s = 180^\circ + 35.5^\circ = 216^\circ$$

Vector and Scalar

Vector quantities have both magnitudes (sizes) and directions

Force, gravitational acceleration, momentum

Normally denoted in **BOLD** letters, $\mathbf{F}$, or a letter with arrow on top $\overline{F}$

Their sizes or magnitudes are denoted with normal letters, F, or absolute values: $\left|\vec{F}\right|$ or $\left|F\right|$

Scalar quantities have magnitudes only

Can be completely specified with a value and its unit

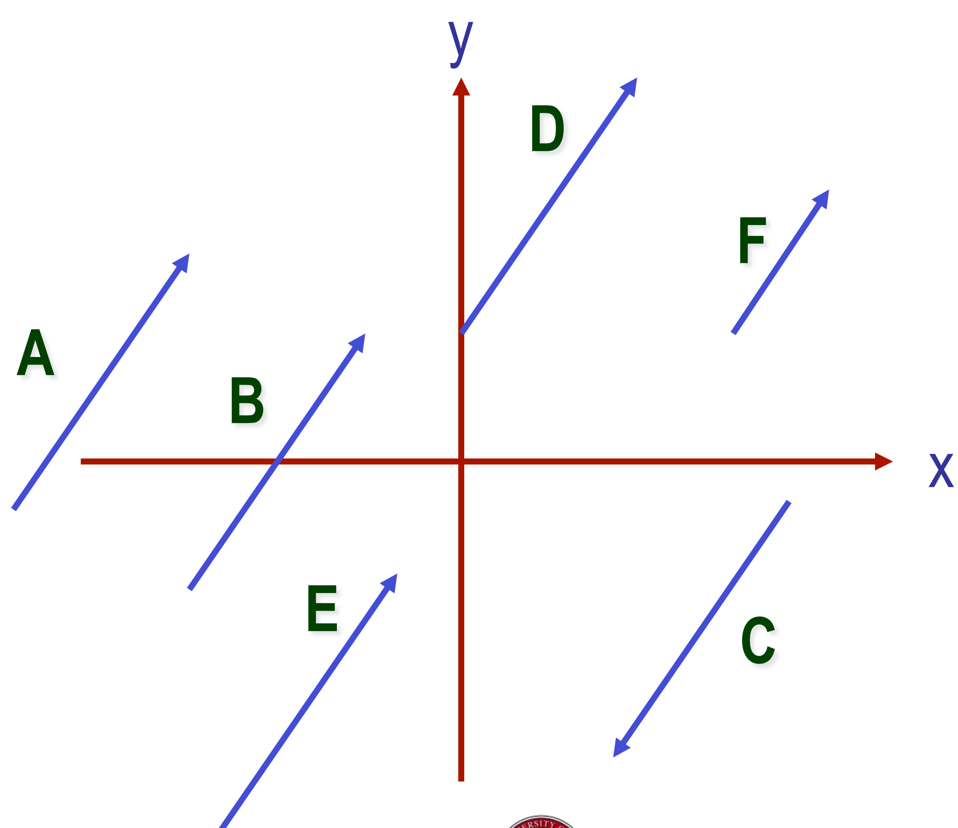
Energy, heat, mass, time

Normally denoted in normal letters, E

Both have units!!!

Properties of Vectors

- Two vectors are the same if their sizes and the directions are the same, no matter where they are on a coordinate system!!



Which ones are the same vectors?

A=B=E=D

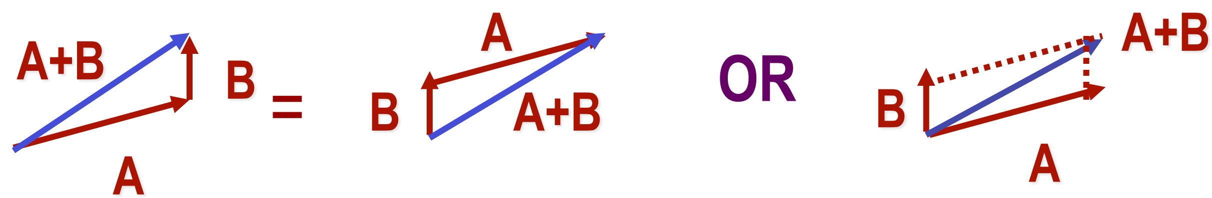
Why aren't the others?

C: The same magnitude but opposite direction: **C=-A:**A negative vector

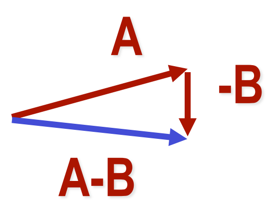
F: The same direction but different magnitude

Vector Operations

- Addition:
  - Triangular Method: One can add vectors by connecting the head of one vector to the tail of the other (head-to-tail)
  - Parallelogram method: Connect the tails of the two vectors and extend
  - Addition is commutative: Changing order of operation does not affect the results **A+B=B+A**, **A+B+C+D+E=E+C+A+B+D**



- Subtraction:
  - The same as adding a negative vector: **A** - **B** = **A** + (-**B**)



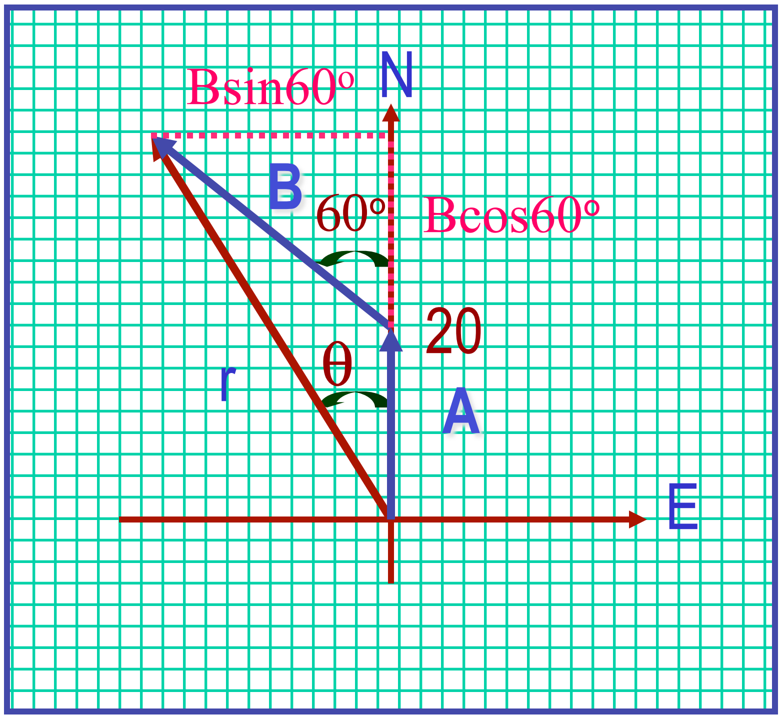
Since subtraction is the equivalent to adding a negative vector, subtraction is also commutative!!!

- Multiplication by a scalar is increasing the magnitude **A**, **B**=2**A**

$$|\mathcal{B}| = 2|\mathcal{A}|$$

Example for Vector Addition

A car travels 20.0km due north followed by 35.0km in a direction 60.0º west of north. Find the magnitude and direction of resultant displacement.



$$r = \sqrt{\left(A + B\cos\theta\right)^2 + \left(B\sin\theta\right)^2}$$

$$= \sqrt{A^2 + B^2\left(\cos^2\theta + \sin^2\theta\right) + 2AB\cos\theta}$$

$$= \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$$= \sqrt{(20.0)^2 + (35.0)^2 + 2\times 20.0\times 35.0\cos 60}$$

$$= \sqrt{2325} = 48.2(km)$$

$$\theta = \tan^{-1}\frac{\left|\vec{B}\right|\sin 60}{\left|\vec{A}\right| + \left|\vec{B}\right|\cos 60}$$

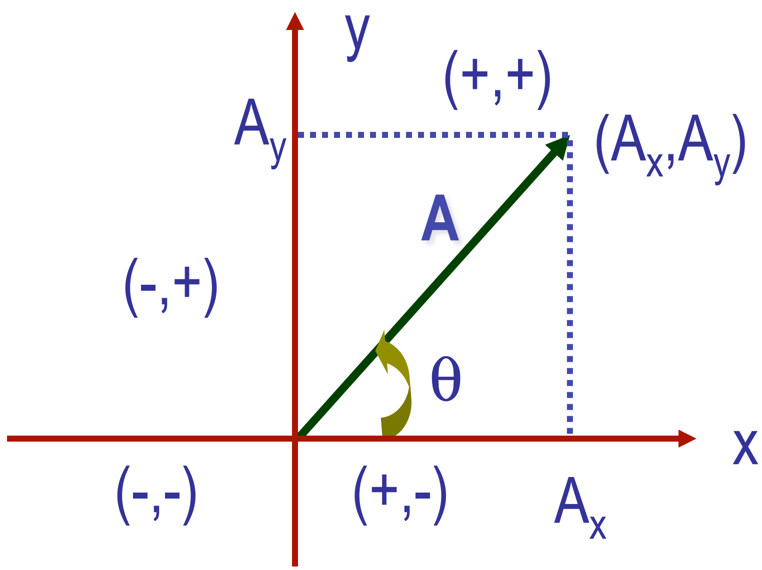
$$= \tan^{-1}\frac{35.0\sin 60}{20.0 + 35.0\cos 60}$$

$$= \tan^{-1}\frac{30.3}{37.5} = 38.9^\circ \text{ to W wrt N}$$

Do this using components!!

Components and Unit Vectors

Coordinate systems are useful in expressing vectors in their components



$$\left.\begin{array}{l} \boxed{A_x = \left|\vec{A}\right| \cos\theta} \\ \boxed{A_y = \left|\vec{A}\right| \sin\theta} \end{array}\right\} \text{Components}$$

$$\left.\boxed{\left|\vec{A}\right| = \sqrt{A_x^2 + A_y^2}}\right\} \text{Magnitude}$$

$$\left|\vec{A}\right| = \sqrt{\left(\left|\vec{A}\right|\cos\theta\right)^2 + \left(\left|\vec{A}\right|\sin\theta\right)^2}$$

$$= \sqrt{\left|\vec{A}\right|^2\left(\cos^2\theta + \sin^2\theta\right)} = \left|\vec{A}\right|$$

Unit Vectors

- Unit vectors are the ones that tells us the directions of the components
- **<u>Dimensionless</u>**
- **<u>Magnitudes these vectors are exactly 1</u>**
- Unit vectors are usually expressed in **i, j, k** or

$\vec{i}, \ \vec{j}, \ \vec{k}$

So a vector **A** can be expressed as

$$\vec{A} = A_x\vec{i} + A_y\vec{j} = \left|\vec{A}\right|\cos\theta\,\vec{i} + \left|\vec{A}\right|\sin\theta\,\vec{j}$$

Examples of Vector Operations

Find the resultant vector which is the sum of **A**=(2.0**i**+2.0**j**) and **B** =(2.0**i**-4.0**j**)

$$\vec{C} = \vec{A} + \vec{B} = \left(2.0\vec{i} + 2.0\vec{j}\right) + \left(2.0\vec{i} - 4.0\vec{j}\right)$$

$$= \left(2.0 + 2.0\right)\vec{i} + \left(2.0 - 4.0\right)\vec{j} = 4.0\vec{i} \quad -2.0\vec{j}\left(m\right)$$

$$\left|\vec{C}\right| = \sqrt{\left(4.0\right)^2 + \left(-2.0\right)^2}$$

$$= \sqrt{16 + 4.0} = \sqrt{20} = 4.5(m)$$

$$\theta = \tan^{-1}\frac{C_y}{C_x} = \tan^{-1}\frac{-2.0}{4.0} = -27^\circ$$

Find the resultant displacement of three consecutive displacements: **d_1**=(15**i**+30**j** +12**k**)cm, **d_2**=(23**i**+14**j** -5.0**k**)cm, and **d_3**=(-13**i**+15**j**)cm

$$\vec{D} = \vec{d}_1 + \vec{d}_2 + \vec{d}_3 = \left(15\vec{i} + 30\vec{j} + 12\vec{k}\right) + \left(23\vec{i} + 14\vec{j} - 5.0\vec{k}\right) + \left(-13\vec{i} + 15\vec{j}\right)$$

$$= \left(15 + 23 - 13\right)\vec{i} + \left(30 + 14 + 15\right)\vec{j} + \left(12 - 5.0\right)\vec{k} = 25\vec{i} + 59\vec{j} + 7.0\vec{k}(cm)$$

Magnitude

$$\left|\vec{D}\right| = \sqrt{\left(25\right)^2 + \left(59\right)^2 + \left(7.0\right)^2} = 65(cm)$$