PHYS 1443 – Section 001
Lecture #9

Monday, February 28, 2011

Dr. Jaehoon Yu

- Uniform Circular Motion
- Motion Under Resistive Forces
- Newton's Law of Universal Gravitation
- Free Fall Acceleration

Today's homework is homework #6, due 10pm, Friday, Mar. 11!!

Announcements

- Mid-term comprehensive exam
  - 19% of the total
  - 1 – 2:20pm, Monday, Mar. 7, SH103
  - Covers: CH.1.1 through what we complete this Wednesday, Mar. 2 (Ch6?) plus Appendices A and B
  - Mixture of multiple choices and free response problems
    - Please be sure to explain as much as possible
    - Just the answers in free response problems are not accepted.

Special Project

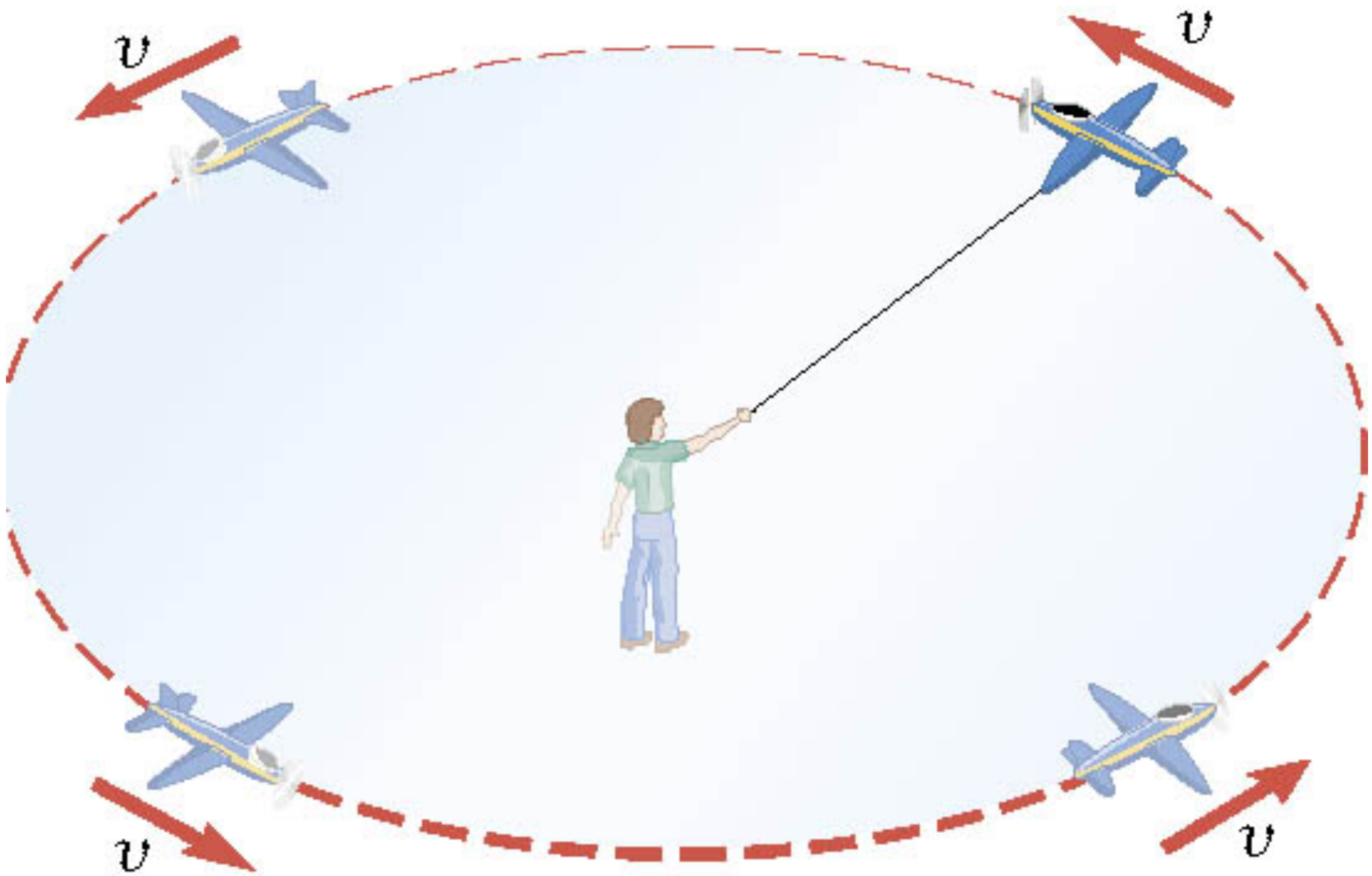
- Derive the formula for the gravitational acceleration (g_{in}) at the radius $R_{in} (< R_E)$ from the center, inside of the Earth. (10 points)
- Compute the fractional magnitude of the gravitational acceleration 1km and 500km inside the surface of the Earth with respect to that on the surface. (6 points, 3 points each)
- Due at the beginning of the class Wednesday, Mar. 9

Special Project

- Two protons are separated by 1m.
  - Compute the gravitational force (F_G) between the two protons (3 points)
  - Compute the electric force (F_E) between the two protons (3 points)
  - Compute the ratio of FG/FE (3 points) and explain what this tells you (1 point)
- Due: Beginning of the class, Wednesday, Mar. 23

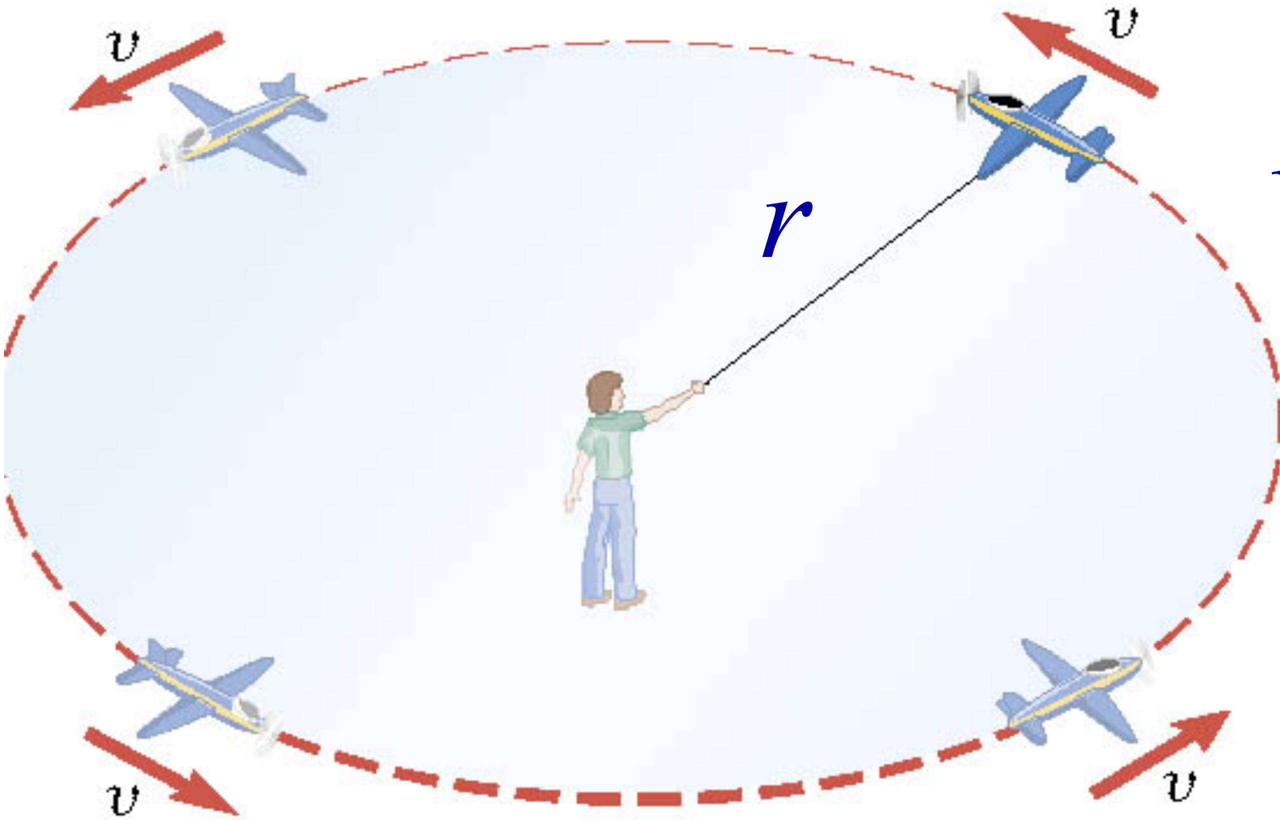
Definition of the Uniform Circular Motion

Uniform circular motion is the motion of an object traveling at a constant speed on a circular path.



Speed of a uniform circular motion?

Let T be the period of this motion, the time it takes for the object to travel once around the complete circle whose radius is r is



$$v = \frac{\text{distance}}{\text{time}}$$

$$= \frac{2\pi r}{T}$$

Ex. : A Tire-Balancing Machine

The wheel of a car has a radius of 0.29m and is being rotated at 830 revolutions per minute on a tire-balancing machine. Determine the speed at which the outer edge of the wheel is moving.

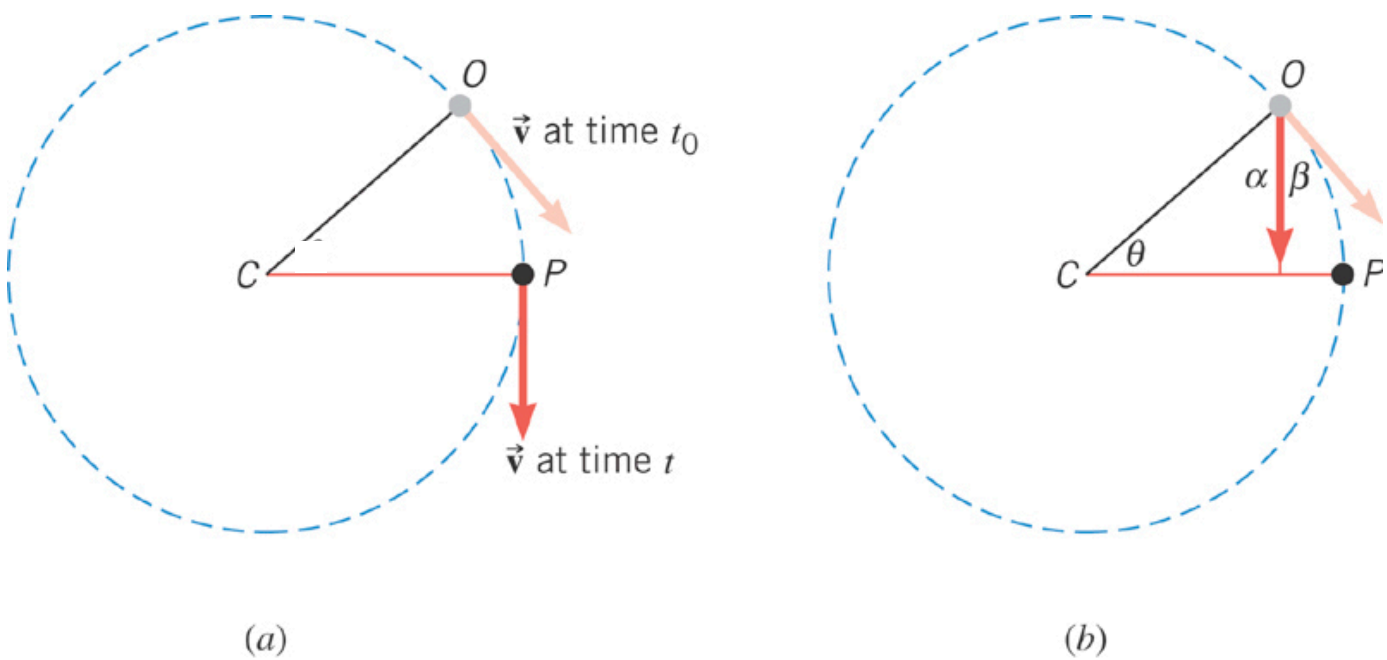
$$\frac{1}{830\,\text{revolutions/min}} = 1.2\times10^{-3}\,\text{min/revolution}$$

$$T = 1.2\times10^{-3}\ \text{min} = 0.072\ \text{s}$$

$$v = \frac{2\pi r}{T} = \frac{2\pi(0.29\ \text{m})}{0.072\ \text{s}} = 25\,\text{m/s}$$

Centripetal Acceleration

In uniform circular motion, the speed is constant, but the direction of the velocity vector is *not constant.*



$$\alpha + \beta = 90^\circ$$

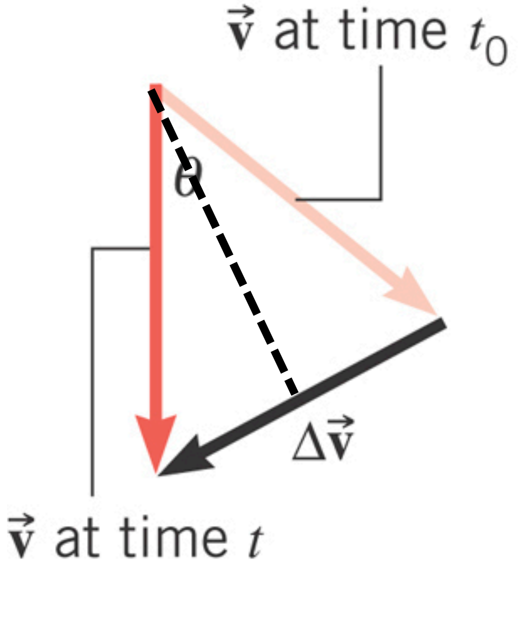
$$\alpha + \theta = 90^\circ$$

$$\beta - \theta = 0$$

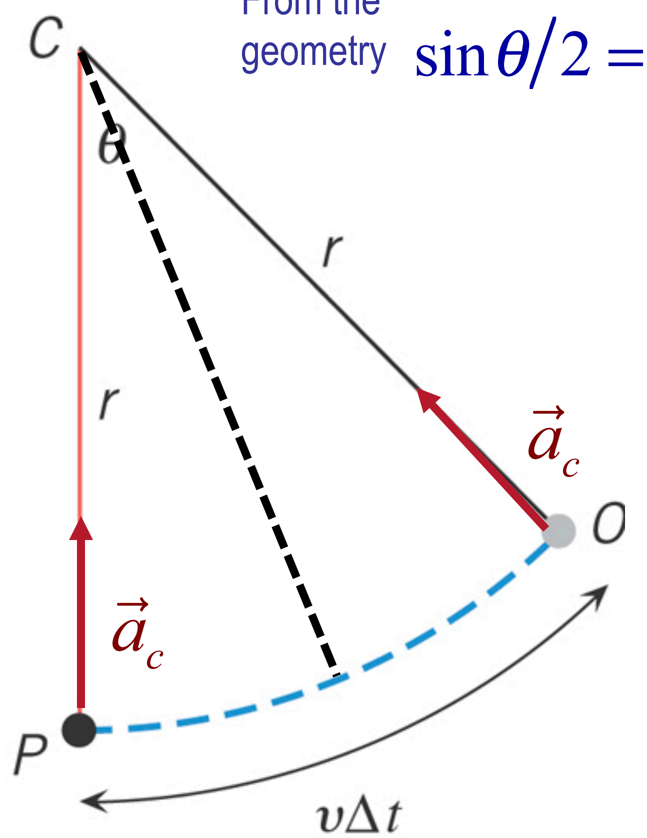
$$\beta = \theta$$

The change of direction of the velocity is the same as the change of the angle in the circular motion!

Centripetal Acceleration



(a)



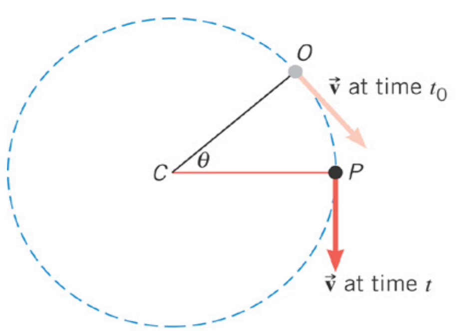
From the geometry

$$\sin\theta/2 = \frac{\Delta v/\not{2}}{v} = \frac{v\Delta t/\not{2}}{r}$$

$$\frac{\Delta v}{\Delta t} = \frac{v^2}{r}$$

$$a_c = \frac{v^2}{r}$$

Centripetal Acceleration

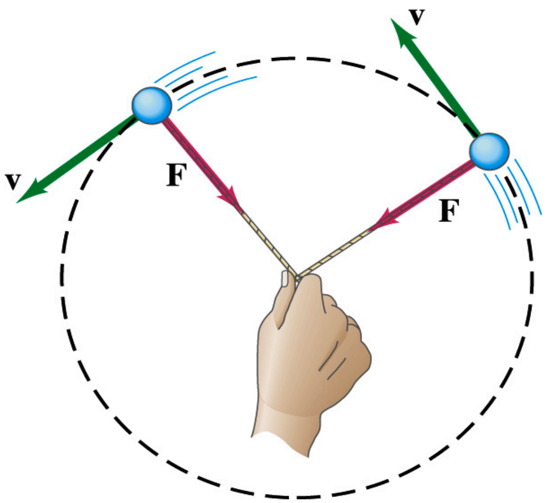


(a) (b)

What is the direction of a_c?

Always toward the center of circle!

Newton's Second Law & Uniform Circular Motion



The ***centripetal**** *acceleration is always perpendicular to the velocity vector,* ***v****, and points to the center of the axis (radial direction) in a uniform circular motion.*

$$a_r = \frac{v^2}{r}$$

Are there forces in this motion? If so, what do they do?

The force that causes the centripetal acceleration acts toward the center of the circular path and causes the change in the direction of the velocity vector. This force is called the **centripetal force.**

$$\sum F_r = ma_r = m\frac{v^2}{r}$$

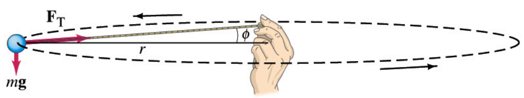
What do you think will happen to the ball if the string that holds the ball breaks?

The external force no longer exist. Therefore, based on Newton's 1st law, the ball will continue its motion without changing its velocity and will fly away along the tangential direction to the circle.

***Mirriam Webster: Proceeding or acting in the direction toward the center or axis**

Ex. 5.11 of Uniform Circular Motion

A ball of mass 0.500kg is attached to the end of a 1.50m long cord. The ball is moving in a horizontal circle. If the string can withstand the maximum tension of 50.0 N, what is the maximum speed the ball can attain before the cord breaks?



Centripetal acceleration:

$$a_r = \frac{v^2}{r}$$

When does the string break?

$$\sum F_r = ma_r = m\frac{v^2}{r} > T$$

when the required centripetal force is greater than the sustainable tension.

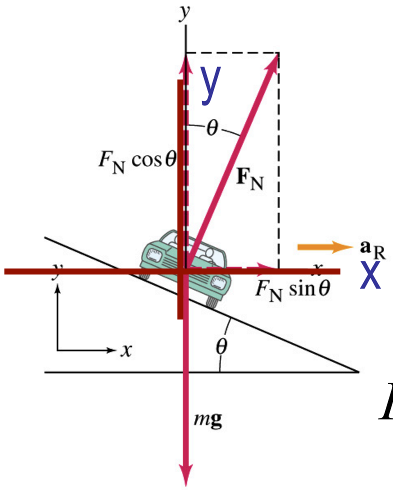
$$m\frac{v^2}{r} = T \qquad v = \sqrt{\frac{Tr}{m}} = \sqrt{\frac{50.0\times 1.5}{0.500}} = 12.2\,(m/s)$$

Calculate the tension of the cord when speed of the ball is 5.00m/s.

$$T = m\frac{v^2}{r} = 0.500\times\frac{(5.00)^2}{1.5} = 8.33\,(N)$$

Example 5.15: Banked Highway

(a) For a car traveling with speed *v* around a curve of radius *r*, determine the formula for the angle at which the road should be banked so that no friction is required to keep the car from skidding.



x comp. $\sum F_x = F_N \sin\theta = ma_r = \frac{mv^2}{r}$

$$F_N \sin\theta = \frac{mv^2}{r}$$

y comp. $\sum F_y = F_N \cos\theta - mg = 0 \qquad F_N \cos\theta = mg$

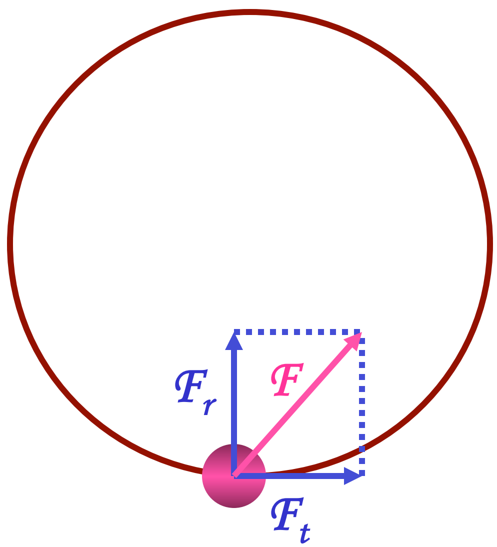
$$F_N = \frac{mg}{\cos\theta} \Rightarrow F_N \sin\theta = \frac{mg\sin\theta}{\cos\theta} = mg\tan\theta = \frac{mv^2}{r}$$

$$\tan\theta = \frac{v^2}{gr}$$

(b) What is this angle for an expressway off-ramp curve of radius 50m at a design speed of 50km/h?

$$v = 50km/hr = 14m/s \qquad \tan\theta = \frac{(14)^2}{50 \times 9.8} = 0.4 \qquad \theta = \tan^{-1}(0.4) = 22°$$

Forces in Non-uniform Circular Motion



The object has both tangential and radial accelerations.

What does this statement mean?

The object is moving under both tangential and radial forces.

$$\vec{F} = \vec{F}_r + \vec{F}_t$$

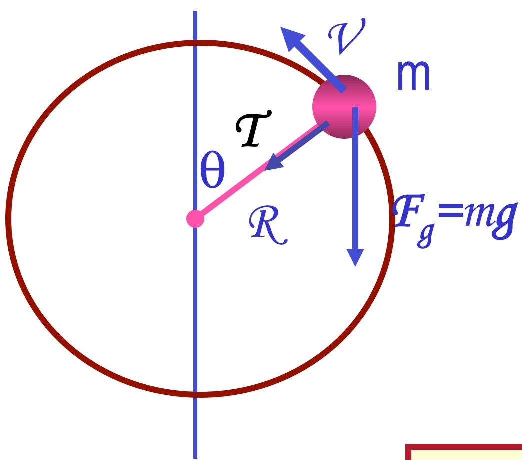
These forces cause not only the velocity but also the speed of the ball to change. The object undergoes a curved motion in the absence of constraints, such as a string.

What is the magnitude of the net acceleration?

$$a = \sqrt{a_r^2 + a_t^2}$$

Ex. 5.12 for Non-Uniform Circular Motion

A ball of mass m is attached to the end of a cord of length R. The ball is moving in a vertical circle. Determine the tension of the cord at any instance in which the speed of the ball is v and the cord makes an angle θ with vertical.



What are the forces involved in this motion?

- The gravitational force $\mathbf{F_g}$
- The radial force, **T**, providing the tension.

tangential comp.

$$\sum F_t = mg\sin\theta = ma_t$$

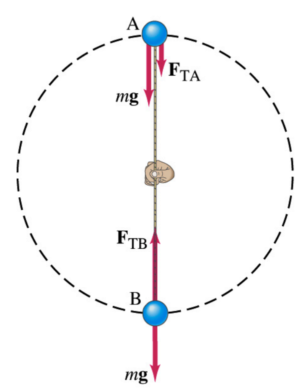
$$a_t = g\sin\theta$$

Radial comp.

$$\sum F_r = T + mg\cos\theta = ma_r = m\frac{v^2}{R}$$

$$T = m\left(\frac{v^2}{R} - g\cos\theta\right)$$

At what angles the tension becomes the maximum and the minimum. What are the tensions?



Motion in Resistive Forces

Medium can exert resistive forces on an object moving through it due to viscosity or other types frictional properties of the medium.

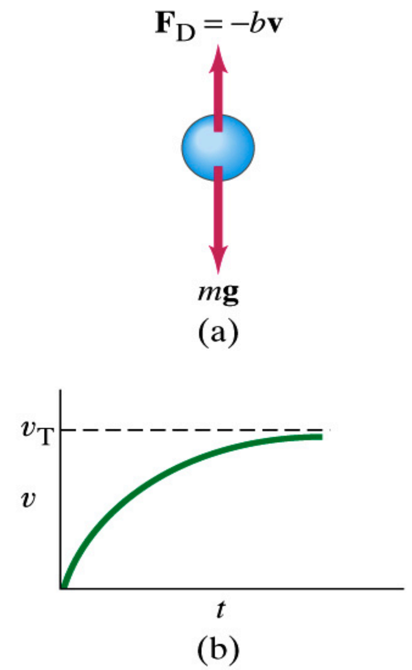
Some examples? Air resistance, viscous force of liquid, etc

These forces are exerted on moving objects in opposite direction of the movement.

These forces are proportional to such factors as speed. They almost always increase with increasing speed.

Two different cases of proportionality:

1. Forces linearly proportional to speed: Slowly moving or very small objects
2. Forces proportional to square of speed: Large objects w/ reasonable speed



Newton's Law of Universal Gravitation

People have been very curious about the stars in the sky, making observations for a long time. The data people collected, however, have not been explained until Newton has discovered the law of gravitation.

Every object in the universe attracts every other object with a force that is directly **proportional to the product of their masses** and **inversely proportional to the square of the distance** between them.

How would you write this law mathematically?

$$F_g \propto \frac{m_1 m_2}{r_{12}^2} \quad \xrightarrow{\text{With } G} \quad F_g = G\frac{m_1 m_2}{r_{12}^2}$$

G is the universal gravitational constant, and its value is

$$G = 6.673 \times 10^{-11}$$

Unit? $N \cdot m^2 / kg^2$

This constant is not given by the theory but must be measured by experiments.

This form of forces is known as **the inverse-square law**, because the magnitude of the force is inversely proportional to the square of the distances between the objects.

Free Fall Acceleration & Gravitational Force

The weight of an object with mass m is mg. Using the force exerting on a particle of mass m on the surface of the Earth, one can obtain

$$mg = G\frac{M_E m}{R_E^2}$$

$$g = G\frac{M_E}{R_E^2}$$

What would the gravitational acceleration be if the object is at an altitude h above the surface of the Earth?

$$F_g = mg' = G\frac{M_E m}{r^2} = G\frac{M_E m}{(R_E + h)^2}$$

$$g' = G\frac{M_E}{(R_E + h)^2}$$

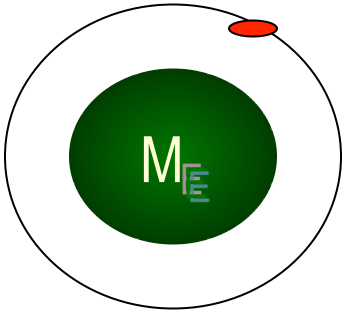
Distance from the center of the Earth to the object at the altitude h.

What do these tell us about the gravitational acceleration?

- The gravitational acceleration is independent of the mass of the object
- The gravitational acceleration decreases as the altitude increases
- If the distance from the surface of the Earth gets infinitely large, the weight of the object approaches 0.

Ex. 6.2 for Gravitational Force

The international space station is designed to operate at an altitude of 350km. Its designed weight (measured on the surface of the Earth) is 4.22×10^6N. What is its weight in its orbit?



The total weight of the station on the surface of the Earth is

$$F_{GE} = mg = G\frac{M_E m}{R_E^{\ 2}} = 4.22 \times 10^6 \, N$$

Since the orbit is at 350km above the surface of the Earth, the gravitational force at that altitude is

$$F_O = mg' = G\frac{M_E m}{\left(R_E + h\right)^2} = \frac{R_E^2}{\left(R_E + h\right)^2} F_{GE}$$

Therefore the weight in the orbit is

$$F_O = \frac{R_E^2}{\left(R_E + h\right)^2} F_{GE} = \frac{\left(6.37 \times 10^6\right)^2}{\left(6.37 \times 10^6 + 3.50 \times 10^5\right)^2} \times 4.22 \times 10^6 = 3.80 \times 10^6 \, N$$

Example for Universal Gravitation

Using the fact that $g=9.80m/s^2$ on the Earth's surface, find the average density of the Earth.

Since the gravitational acceleration is

$$F_g = G\frac{M_E m}{R_E^2} = mg$$

Solving for g

$$g = G\frac{M_E}{R_E^{\;2}} = 6.67\times10^{-11}\frac{M_E}{R_E^{\;2}}$$

Solving for M_E

$$M_E = \frac{R_E^{\;2} g}{G}$$

Therefore the density of the Earth is

$$\rho = \frac{M_E}{V_E} = \frac{\frac{R_E^{\;2} g}{G}}{\frac{4\pi}{3}R_E^{\;3}} = \frac{3g}{4\pi G R_E}$$

$$= \frac{3\times9.80}{4\pi\times6.67\times10^{-11}\times6.37\times10^{6}} = 5.50\times10^{3}\,kg/m^{3}$$