# PHYS 1443 – Section 001
# Lecture #10

*Wednesday, March 2, 2011*

*Dr. Jaehoon Yu*

- Newton's Law of Universal Gravitation
- Kepler's Third Law
- Satellite Motion
- Motion in Accelerated Frames
- Work Done By A Constant Force

# Announcements

- Quiz results
  - Class average: 16.3/30
    - Equivalent to 54.3/100
  - Top score: 30/30
- Reading assignments: CH6.6 and 6.8
- Mid-term comprehensive exam
  - 19% of the total
  - 1 – 2:20pm, Monday, Mar. 7, SH103
  - Covers: CH.1.1 – CH. 6.8 plus Appendices A and B
  - Mixture of multiple choices and free response problems
    - Please be sure to explain as much as possible
    - Just the answers in free response problems are not accepted.
  - Please DO NOT miss the exam!!

# Reminder: Special Project

- Derive the formula for the gravitational acceleration ($g_{in}$) at the radius $R_{in}\left(< R_E\right)$ from the center, inside of the Earth. (10 points)
- Compute the fractional magnitude of the gravitational acceleration 1km and 500km inside the surface of the Earth with respect to that on the surface. (6 points, 3 points each)
- Due at the beginning of the class Wednesday, Mar. 9

# Reminder: Special Project

- Two protons are separated by 1m.
  - Compute the gravitational force ($F_G$) between the two protons (3 points)
  - Compute the electric force ($F_E$) between the two protons (3 points)
  - Compute the ratio of FG/FE (3 points) and explain what this tells you (1 point)
- Due: Beginning of the class, Wednesday, Mar. 23

# Gravitational Force and Weight

Gravitational Force, $\mathcal{F}_g$

*The attractive force exerted on an object by the Earth*

$$\vec{F}_G = m\vec{a} = m\vec{g}$$

Weight of an object with mass M is $W = \left|\vec{F}_G\right| = M\left|\vec{g}\right| = Mg$

What is the SI unit of weight? $\boxed{\mathrm{N}}$

*Since weight depends on the magnitude of gravitational acceleration, **g**, it varies depending on geographical location.*

*By measuring the forces one can determine masses. This is why you can measure mass using the spring scale.*

# Gravitational Acceleration

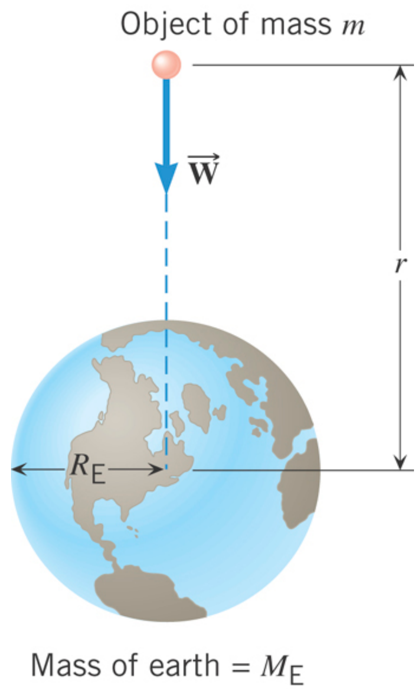
$$W = G\frac{M_E m}{r^2}$$

$$W = mg$$

$$\cancel{m}g = G\frac{M_E \cancel{m}}{r^2}$$

$$g = G\frac{M_E}{r^2}$$

Gravitational acceleration at distance r from the center of the earth!




On the surface of the Earth

$G = 6.67\times10^{-11}\,\mathrm{N\cdot m^2/kg^2}$

$M_E = 5.98\times10^{24}$ kg; $R_E = 6.38\times10^{6}$ $m$

$$g = G\frac{M_E}{R_E^2}$$

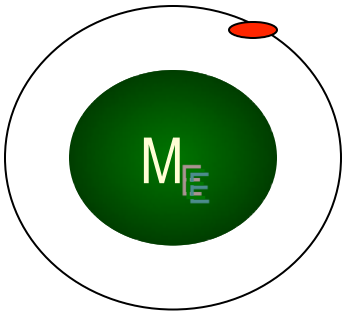
$$= \left(6.67\times10^{-11}\,\mathrm{N\cdot m^2/kg^2}\right)\frac{\left(5.98\times10^{24}\ \mathrm{kg}\right)}{\left(6.38\times10^{6}\ \mathrm{m}\right)^2}$$

$$= 9.80\ \mathrm{m/s^2}$$

What is the SI unit of g? $\boxed{\mathrm{m/s^2}}$

# Ex. 6.2 for Gravitational Force

The international space station is designed to operate at an altitude of 350km. Its designed weight (measured on the surface of the Earth) is $4.22 \times 10^6 N$. What is its weight in its orbit?




The total weight of the station on the surface of the Earth is

$$F_{GE} = mg = G\frac{M_E m}{R_E^{\;2}} = 4.22 \times 10^6 N$$

Since the orbit is at 350km above the surface of the Earth, the gravitational force at that altitude is

$$F_O = mg' = G\frac{M_E m}{\left(R_E + h\right)^2} = \frac{R_E^2}{\left(R_E + h\right)^2} F_{GE}$$

Therefore the weight in the orbit is

$$F_O = \frac{R_E^2}{\left(R_E + h\right)^2} F_{GE} = \frac{\left(6.37 \times 10^6\right)^2}{\left(6.37 \times 10^6 + 3.50 \times 10^5\right)^2} \times 4.22 \times 10^6 = 3.80 \times 10^6 N$$

# Example for Universal Gravitation

Using the fact that g=9.80m/s$^2$ on the Earth's surface, find the average density of the Earth.

Since the gravitational acceleration is

$$F_g = G\frac{M_E m}{R_E^2} = mg$$

**Solving for g**

$$g = G\frac{M_E}{R_E{}^2} = 6.67\times10^{-11}\frac{M_E}{R_E{}^2}$$

**Solving for $M_E$**

$$M_E = \frac{R_E{}^2 g}{G}$$

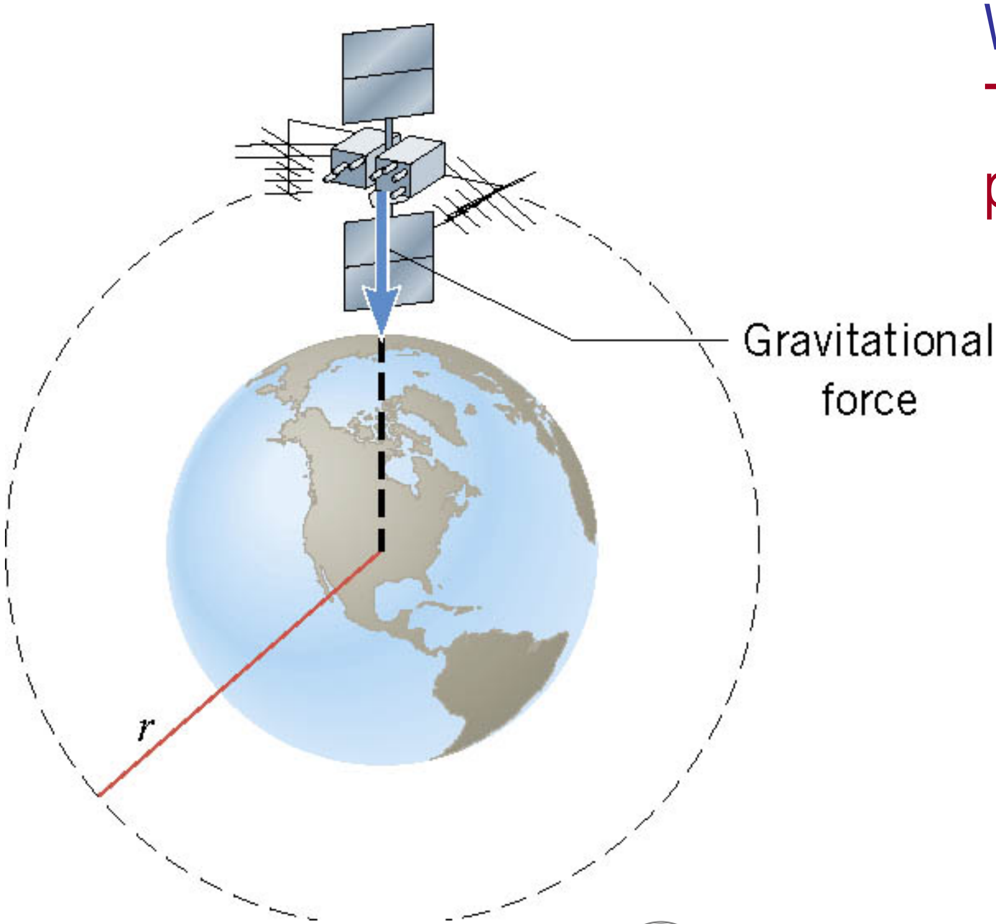
Therefore the density of the Earth is

$$\rho = \frac{M_E}{V_E} = \frac{\frac{R_E{}^2 g}{G}}{\frac{4\pi}{3}R_E{}^3} = \frac{3g}{4\pi G R_E}$$

$$= \frac{3\times9.80}{4\pi\times6.67\times10^{-11}\times6.37\times10^{6}} = 5.50\times10^{3}\,kg/m^{3}$$

# Satellite in Circular Orbits

There is only one speed that a satellite can have if the satellite is to remain in an orbit with a fixed radius.




What is the centripetal force?

The gravitational force of the earth pulling the satellite!

$$F_c = G\frac{mM_E}{r^2} = m\frac{v^2}{r}$$

$$v^2 = \frac{GM_E}{r}$$

$$\boxed{v = \sqrt{\frac{GM_E}{r}}}$$

# Period of a Satellite in an Orbit

Speed of a satellite $$v = \sqrt{\frac{GM_E}{r}} = \frac{2\pi r}{T}$$

$$\frac{GM_E}{r} = \left(\frac{2\pi r}{T}\right)^2$$

Square either side and solve for T2

$$T^2 = \frac{(2\pi)^2 r^3}{GM_E}$$

Period of a satellite $$T = \frac{2\pi r^{3/2}}{\sqrt{GM_E}}$$

Kepler's 3rd Law

This is applicable to any satellite or even for planets and moons.

# Example of Kepler's Third Law

Calculate the mass of the Sun using the fact that the period of the Earth's orbit around the Sun is $3.16 \times 10^7$s, and its distance from the Sun is $1.496 \times 10^{11}$m.

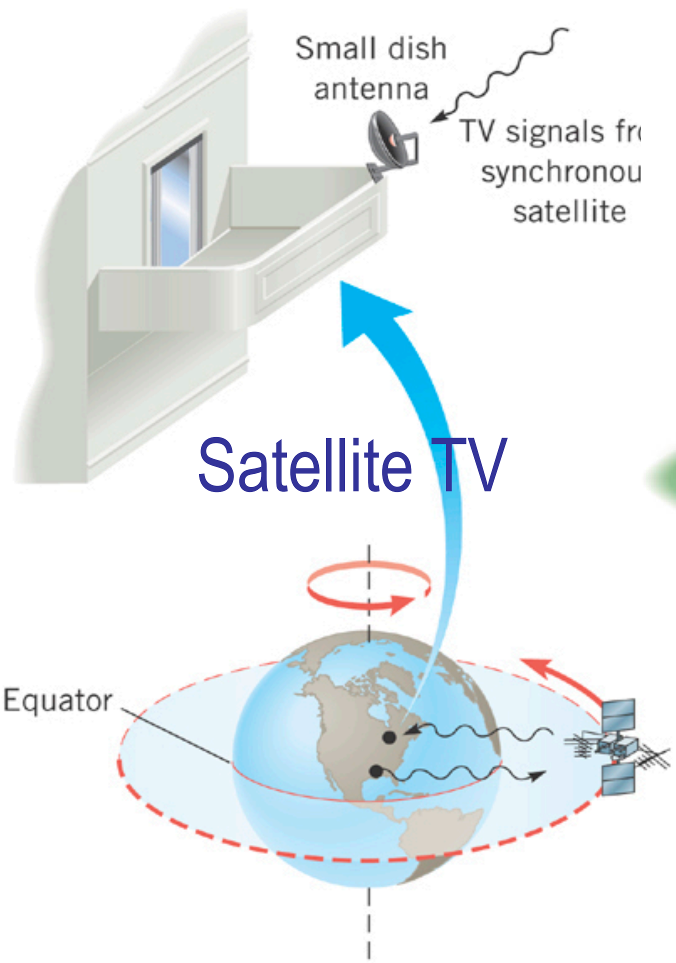
Using Kepler's third law. $$T^2 = \left(\frac{4\pi^2}{GM_s}\right) r^3 = K_s r^3$$

The mass of the Sun, $M_s$, is $$M_s = \left(\frac{4\pi^2}{GT^2}\right) r^3$$

$$= \left(\frac{4\pi^2}{6.67 \times 10^{-11} \times \left(3.16 \times 10^7\right)^2}\right) \times \left(1.496 \times 10^{11}\right)^3$$

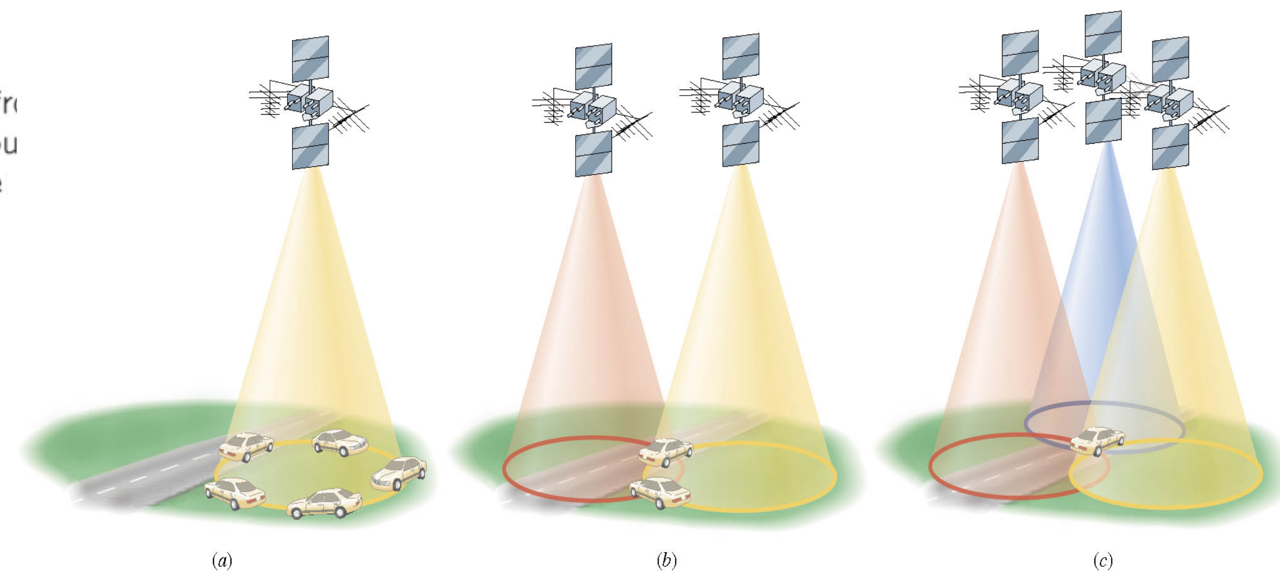
$$= 1.99 \times 10^{30}\, kg$$

# Geo-synchronous Satellites




Satellite TV

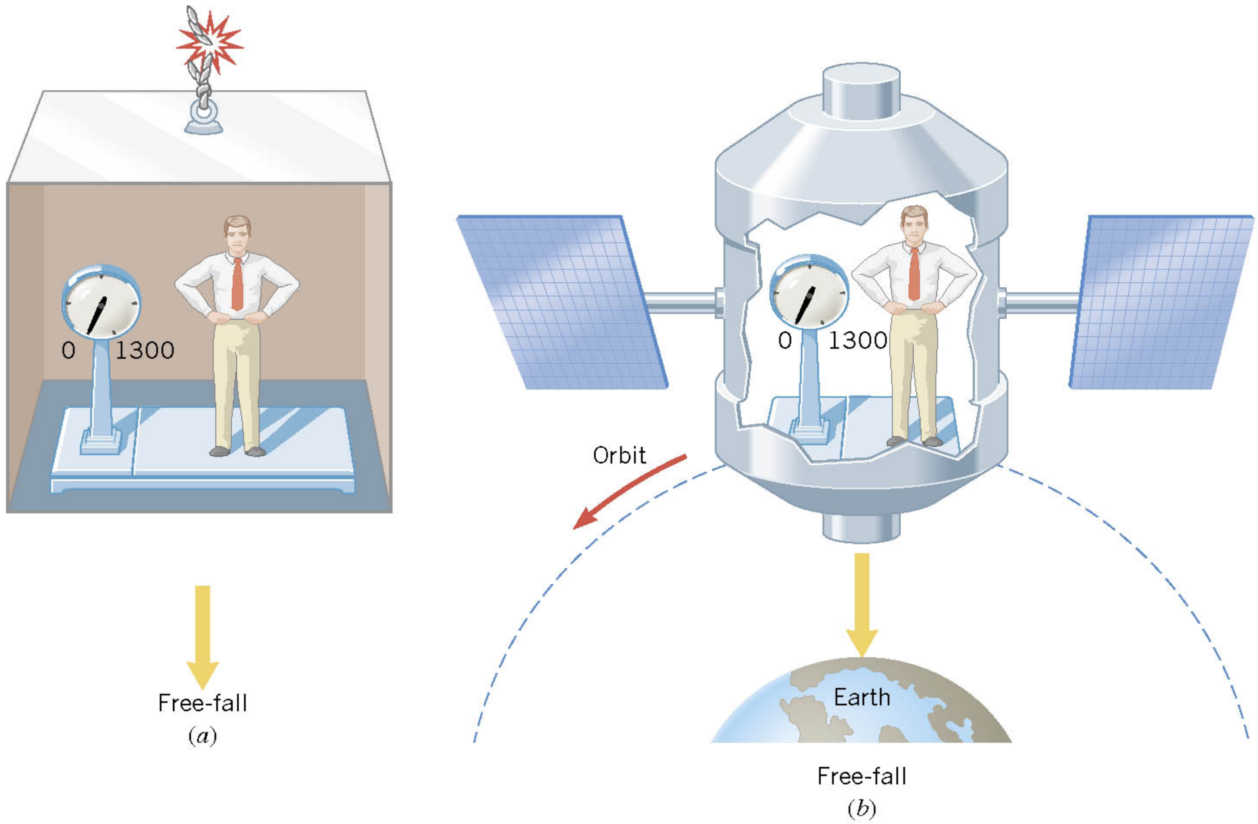
Global Positioning System (GPS)




What period should these satellites have?

The same as the earth!! 24 hours

# Ex. Apparent Weightlessness and Free Fall




0 0

In each case, what is the weight recorded by the scale?

# Ex. Artificial Gravity

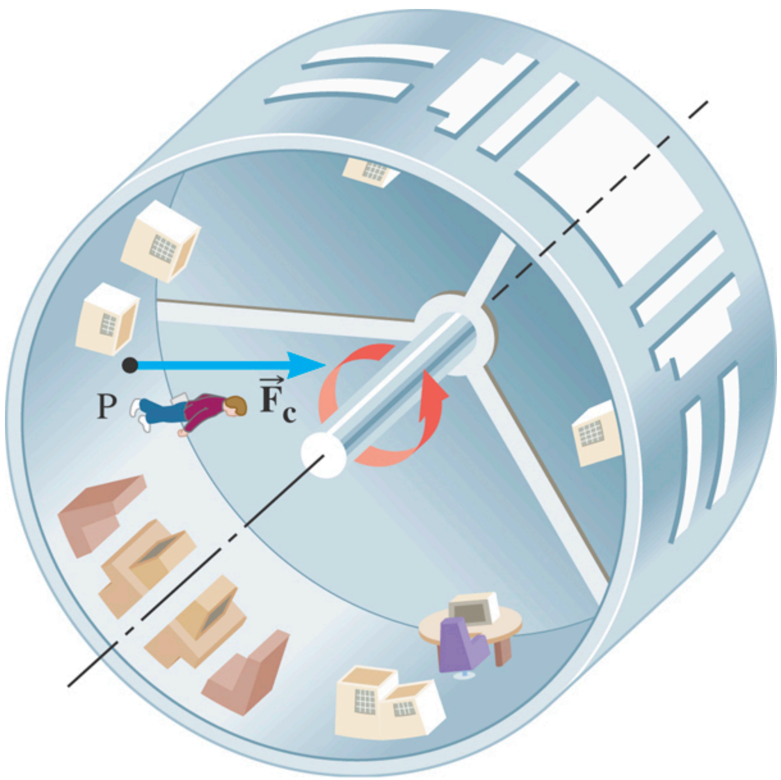
At what speed must the surface of the space station move so that the astronaut experiences a push on his feet equal to his weight on earth? The radius is 1700 m.

$$F_c = m\frac{v^2}{r} = mg$$

$$v = \sqrt{rg}$$

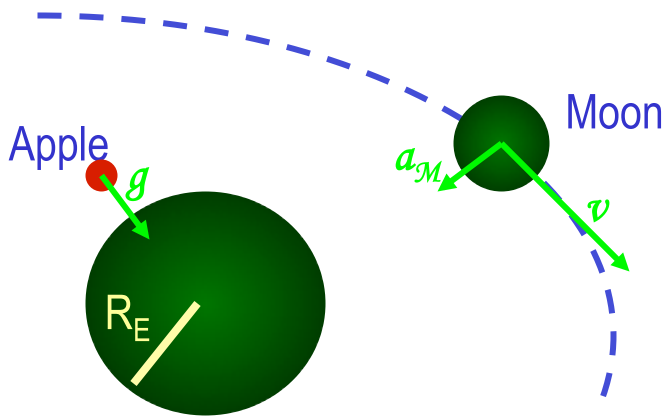
$$= \sqrt{(1700\ \text{m})(9.80\,\text{m/s}^2)}$$

$$= 130\, m/s$$



# The Law of Gravity and Motions of Planets

- Newton assumed that the law of gravitation applies the same whether it is to the apple or to the Moon.
- The interacting bodies are assumed to be point like objects.




Newton predicted that the ratio of the Moon's acceleration $a_M$ to the apple's acceleration $g$ would be

$$\frac{a_M}{g} = \frac{(1/r_M)^2}{(1/R_E)^2} = \left(\frac{R_E}{r_M}\right)^2 = \left(\frac{6.37\times10^6}{3.84\times10^8}\right)^2 = 2.75\times10^{-4}$$

Therefore the centripetal acceleration of the Moon, $a_M$, is

$$a_M = 2.75\times10^{-4}\times9.80 = 2.70\times10^{-3}\,m/s^2$$

Newton also calculated the Moon's orbital acceleration $a_M$ from the knowledge of its distance from the Earth and its orbital period, T=27.32 days=2.36x10$^6$s

$$a_M = \frac{v^2}{r_M} = \frac{(2\pi r_M/T)^2}{r_M} = \frac{4\pi^2 r_M}{T^2} = \frac{4\pi^2\times3.84\times10^8}{\left(2.36\times10^6\right)^2} = 2.72\times10^{-3}\,m/s^2 \approx \frac{9.80}{(60)^2}$$

This means that the distance to the Moon is about 60 times that of the Earth's radius, and its acceleration is reduced by the square of the ratio. This proves that the inverse square law is valid.

# Motion in Accelerated Frames

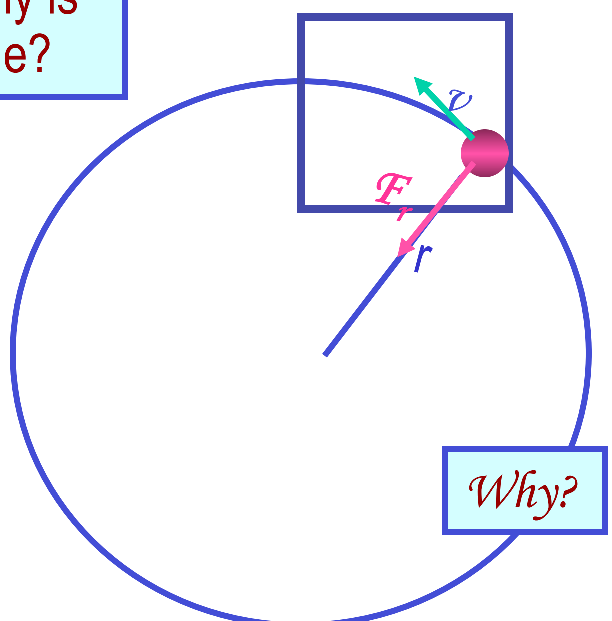
Newton's laws are valid only when observations are made in an inertial frame of reference. What happens in a non-inertial frame?

*Fictitious forces are needed to apply Newton's second law in an accelerated frame.*

*This force does not exist when the observations are made in an inertial reference frame.*

What does this mean and why is this true?

Let's consider a free ball inside a box under uniform circular motion.




How does this motion look like in an inertial frame (or frame outside a box)?

*We see that the box has a radial force exerted on it but none on the ball directly*

How does this motion look like in the box?

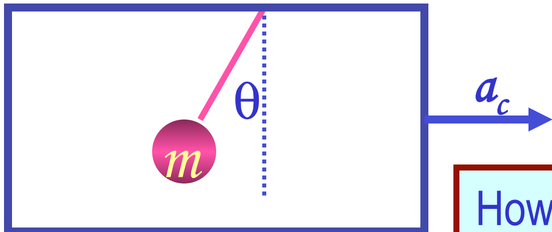
*The ball is tumbled over to the wall of the box and feels that it is getting force that pushes it toward the wall.*

*Why?*

According to Newton's first law, the ball wants to continue on its original movement but since the box is turning, the ball feels like it is being pushed toward the wall relative to everything else in the box.

# Example of Motion in Accelerated Frames

A ball of mass m is hung by a cord to the ceiling of a boxcar that is moving with an acceleration *a*. What do the inertial observer at rest and the non-inertial observer traveling inside the car conclude? How do they differ?

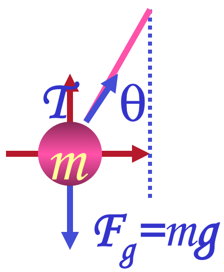



*This is how the ball looks like no matter which frame you are in.*

How do the free-body diagrams look for two frames?

How do the motions interpreted in these two frames? Any differences?

*Inertial Frame*




$$\sum \vec{F} = \vec{F}_g + \vec{T}$$

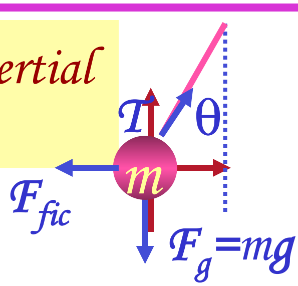
$$\sum F_x = ma_x = ma_c = T\sin\theta$$

$$\sum F_y = T\cos\theta - mg = 0$$

$$\boxed{T = \frac{mg}{\cos\theta}} \quad \boxed{a_c = g\tan\theta}$$

For an inertial frame observer, the forces being exerted on the ball are only **T** and **F<sub></sub>**

*Non-Inertial Frame*




$$\sum \vec{F} = \vec{F}_g + \vec{T} + \vec{F}_{fic}$$

$$\sum F_x = T\sin\theta - F_{fic} = 0 \quad \boxed{F_{fic} = ma_{fic} = T\sin\theta}$$

$$\sum F_y = T\cos\theta - mg = 0$$

$$\boxed{T = \frac{mg}{\cos\theta}} \quad \boxed{a_{fic} = g\tan\theta}$$

For an inertial frame observer, the forces being exerted on the ball are only **T** and **$F_g$**. The acceleration of the ball is the same as that of the box car and is provided by the x component of the tension force.

In the non-inertial frame observer, the forces being exerted on the ball are **T**, **$F_g$**, and **$F_{fic}$**. For some reason the ball is under a force, **$F_{fic}$**, that provides acceleration to the ball.

While the mathematical expression of the acceleration of the ball is identical to that of inertial frame observer's, the cause of the force is dramatically different.