# PHYS 1443 – Section 001
# Lecture #11

*Wednesday, March 9, 2011*

*Dr. Jaehoon Yu*

- Work done by a Constant Force
- Work done by a Varying Force
- Work and Kinetic Energy Theorem
- Potential Energy and the Conservative Force
  - Gravitational Potential Energy
  - Elastic Potential Energy
- Conservation of Energy

# Announcements

- Mid-term exam results
  - Class average: 67.2/94
    - Equivalent to 71.5/100
  - Top score: 91/94
- Mid-term grade discussion on Wednesday, Mar. 23
- Evaluation criteria
  - Homework: 25%
  - Two comprehensive exams: 19% each
  - Better of the two term exams: 12%
  - Lab:15%
  - Quizzes: 10%
  - Extra Credit 10%
- Change of the 2nd non-comprehensive term exam date
  - From Wednesday, Mar. 30 to Wednesday, Apr. 6
  - Final comprehensive exam: 11am, Monday, May 9
  - Mark your calendars
- Colloquium today

**Physics Department**
**The University of Texas at Arlington**

# COLLOQUIUM

***In-Situ Transmission Electron Microscopy Observations of the Charging and Discharging Processes of Lithium Ion Batteries***


**Dr. Jian Yu Huang**

**Center for Integrated Nanotechnologies, Sandia National Laboratories, Albuquerque, New Mexico**


4:00p.m Wednesday March 9, 2011
At SH Rm 101


**Abstract:**

We created the first nano-battery inside a transmission electron microscope, allowing for real time atomic scale observations of battery charging and discharging processes. The nano-battery consists of a single $SnO_2$ nanowire anode, an ionic liquid electrolyte and a bulk $LiCoO_2$ cathode [1]. Upon charging, a reaction front propagates progressively along the nanowire, causing the nanowire to swell, elongate, and spiral. The reaction front is a "Medusa zone" containing a high density of mobile dislocations, which are continuously nucleated and absorbed at the moving front. This dislocation cloud indicates large in-plane misfit stresses and is a structural precursor to electrochemically-driven solid-state amorphization. In charging Si nanowires, the nanowires swell rather than elongate. We found unexpectedly the highly anisotropic volume expansion in lithiated Si nanowires, resulting in a surprising dumbbell-shaped cross section which developed due to plastic flow and necking instability. Driven by progressive charging, the stress concentration at the neck region led to cracking, eventually splitting the single nanowire into sub-wires. These experimental results and associated theoretical models uncover the previously unknown anisotropic deformation mechanism and highlight the critical role of plastic flow in electrochemically induced failure of Si nanostructures. Because lithiation-induced volume expansion, plasticity and pulverization of electrode materials are the major mechanical effects that plague the performance and lifetime of high capacity anodes in lithium-ion batteries, our observations provide important mechanistic insight for the design of advanced batteries for powering electrical vehicles and devices.



**References**

J.Y. Huang *et al.*, **Science** 330, 1515-1520 (2010); Y. M. Chiang, **Science** 330, 1485 (2010

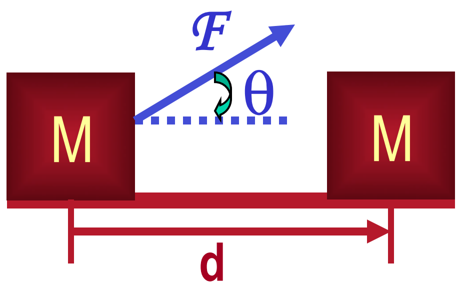

Refreshments will be served in the Physics Lounge at 3:30 pm

# Reminder: Special Project

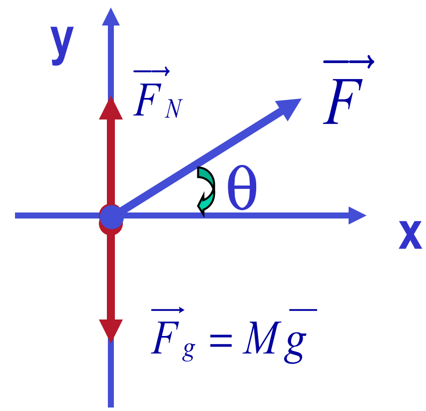
- Two protons are separated by 1m.
  - Compute the gravitational force ($F_G$) between the two protons (3 points)
  - Compute the electric force ($F_E$) between the two protons (3 points)
  - Compute the ratio of FG/FE (3 points) and explain what this tells you (1 point)
- Due: Beginning of the class, Wednesday, Mar. 23

# Work Done by the Constant Force

*A meaningful work in physics is done only when the net forces exerted on an object changes the energy of the object.*




**Free Body Diagram**




Which force did the work? Force $\vec{F}$ Why?

What kind? Scalar

How much work did it do? $W = \left(\sum \vec{F}\right) \cdot \vec{d} = Fd\cos\theta$

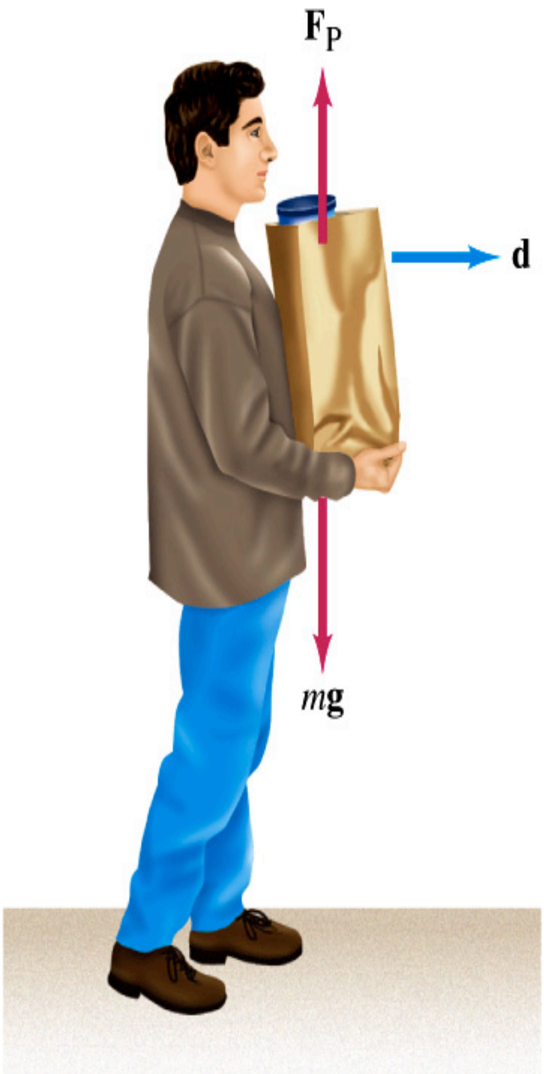
Unit? $N \cdot m$ $= J$ (for Joule)

What does this mean?

**Physically meaningful work is done only by the component of the force along the movement of the object.**

**Work is an energy transfer!!**

# Let's think about the meaning of work!




- A person is holding a grocery bag and walking at a constant velocity.
- Is he doing any work ON the bag?
  - No
  - Why not?
  - Because the force he exerts on the bag, $F_p$, is perpendicular to the displacement!!
  - This means that he is not adding any energy to the bag.
- So what does this mean?
  - In order for a force to perform any meaningful work, the energy of the object the force exerts on must change!!
- What happened to the person?
  - He spends his energy just to keep the bag up but did not perform any work on the bag.

# Scalar Product of Two Vectors

- Product of magnitude of the two vectors and the cosine of the angle between them $\vec{A}\cdot\vec{B} \equiv \left|\vec{A}\right|\left|\vec{B}\right|\cos\theta$
- Operation is commutative $\vec{A}\cdot\vec{B} = \left|\vec{A}\right|\left|\vec{B}\right|\cos\theta = \left|\vec{B}\right|\left|\vec{A}\right|\cos\theta = \vec{B}\cdot\vec{A}$
- Operation follows the distribution law of multiplication $\vec{A}\cdot\left(\vec{B}+\vec{C}\right) = \vec{A}\cdot\vec{B} + \vec{A}\cdot\vec{C}$
- Scalar products of Unit Vectors $\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1 \quad \hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0$
- How does scalar product look in terms of components?

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k} \qquad \vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$$

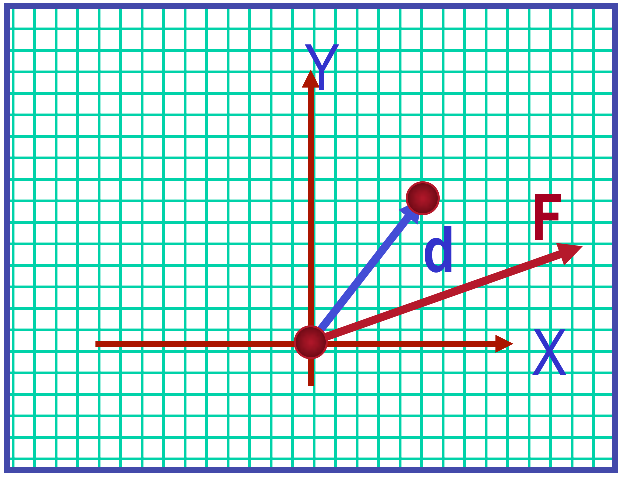
$$\vec{A}\cdot\vec{B} = \left(A_x\hat{i} + A_y\hat{j} + A_z\hat{k}\right)\cdot\left(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}\right) = \left(A_xB_x\,\hat{i}\cdot\hat{i} + A_yB_y\,\hat{j}\cdot\hat{j} + A_zB_z\,\hat{k}\cdot\hat{k}\right) + \text{cross terms}$$

cross terms **=0**

$$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$$

# Example of Work by Scalar Product

A particle moving on the xy plane undergoes a displacement **d**=(2.0**i**+3.0**j**)m as a constant force **F**=(5.0**i**+2.0**j**) N acts on the particle.




a) Calculate the magnitude of the displacement and that of the force.

$$\left|\vec{d}\right| = \sqrt{d_x^2 + d_y^2} = \sqrt{(2.0)^2 + (3.0)^2} = 3.6m$$

$$\left|\vec{F}\right| = \sqrt{F_x^2 + F_y^2} = \sqrt{(5.0)^2 + (2.0)^2} = 5.4N$$

b) Calculate the work done by the force F.

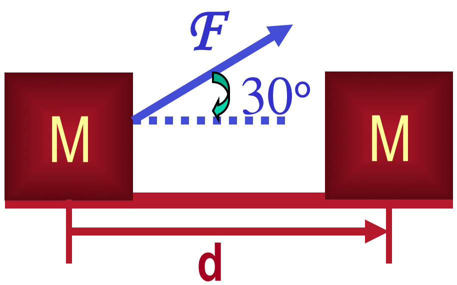
$$W = \vec{F}\cdot\vec{d} = \left(2.0\,\hat{i} + 3.0\,\hat{j}\right)\cdot\left(5.0\,\hat{i} + 2.0\,\hat{j}\right) = 2.0\times5.0\,\hat{i}\cdot\hat{i} + 3.0\times2.0\,\hat{j}\cdot\hat{j} = 10 + 6 = 16(J)$$

Can you do this using the magnitudes and the angle between **d** and **F**?

$$W = \vec{F}\cdot\vec{d} = \left|\vec{F}\right|\left|\vec{d}\right|\cos\theta$$

# Example of Work w/ Constant Force

A man cleaning a floor pulls a vacuum cleaner with a force of magnitude F=50.0N at an angle of 30.0° with East. Calculate the work done by the force on the vacuum cleaner as the vacuum cleaner is displaced by 3.00m to East.




$$W = \left(\sum \vec{F}\right) \cdot \vec{d} = \left|\left(\sum \vec{F}\right)\right| \left|\vec{d}\right| \cos\theta$$

$$W = 50.0 \times 3.00 \times \cos 30^{\circ} = 130 J$$

Does work depend on mass of the object being worked on?

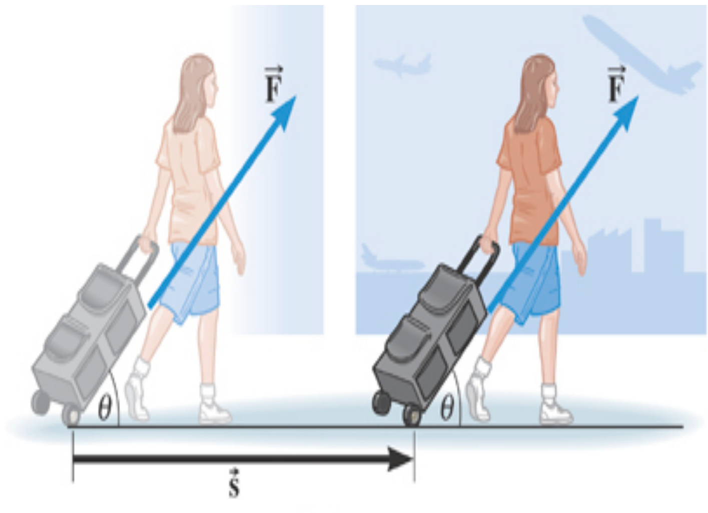
No

Why ?

This is because the work done by the force bringing the object to a displacement **d** is constant independent of the mass of the object being worked on. The only difference would be the acceleration and the final speed of each of the objects after the completion of the work!!

# Ex. Pulling A Suitcase-on-Wheel

Find the work done by a 45.0N force in pulling the suitcase in the figure at an angle 50.0$^o$ for a distance s=75.0m.



(a)

$$W = \left(\sum \vec{F}\right) \cdot \vec{d} = \left|\left(\sum \vec{F}\right)\cos\theta\right|\left|\vec{d}\right|$$

$$= \left(45.0 \cdot \cos 50^\circ\right) \cdot 75.0 = 2170J$$

Does work depend on mass of the object being worked on?

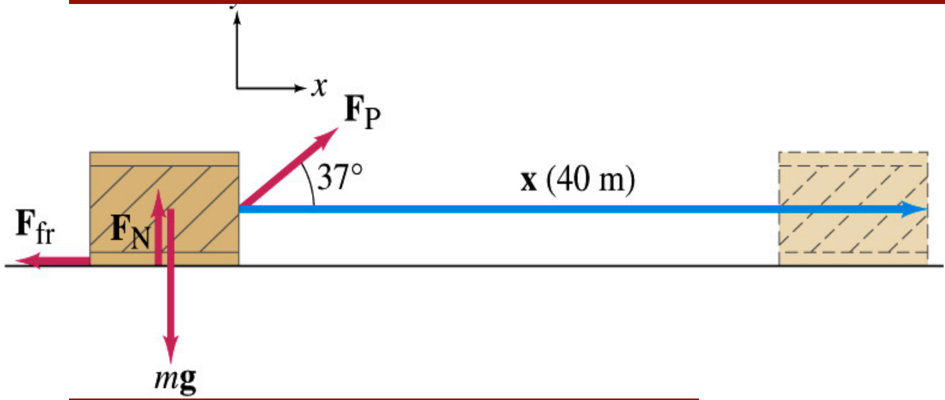
No

Why ?

This is because the work done by the force bringing the object to a displacement **d** is constant independent of the mass of the object being worked on. The only difference would be the acceleration and the final speed of each of the objects after the completion of the work!!

# Ex. 7.1 Work done on a crate

A person pulls a 50kg crate 40m along a horizontal floor by a constant force $F_p$=100N, which acts at a 37° angle as shown in the figure. The floor is rough and exerts a friction force $F_{fr}$=50N. Determine (a) the work done by each force and (b) the net work done on the crate.




What are the forces exerting on the crate?

$F_p$ | $F_{fr}$ | $F_G$=-mg | $F_N$=+mg

Which force performs the work on the crate?

$F_p$ | $F_{fr}$

Work done on the crate by $F_G$

$$W_G = \vec{F}_G \cdot \vec{x} = -mg\cos\left(-90^\circ\right)\cdot\left|\vec{x}\right| = 0J$$

Work done on the crate by $F_N$

$$W_N = \vec{F}_N \cdot \vec{x} = mg\cos 90^\circ \cdot \left|\vec{x}\right| = 100\cdot\cos 90^\circ \cdot 40 = 0J$$

Work done on the crate by $F_p$:

$$W_p = \vec{F}_p \cdot \vec{x} = \left|\vec{F}_p\right|\cos 37^\circ \cdot \left|\vec{x}\right| = 100\cdot\cos 37^\circ \cdot 40 = 3200J$$

Work done on the crate by $F_{fr}$:

$$W_{fr} = \vec{F}_{fr} \cdot \vec{x} = \left|\vec{F}_{fr}\right|\cos 180^\circ \cdot \left|\vec{x}\right| = 50\cdot\cos 180^\circ \cdot 40 = -2000J$$

So the net work on the crate

$$W_{net} = W_N + W_G + W_p + W_{fr} = 0 + 0 + 3200 - 2000 = 1200(J)$$

This is the same as

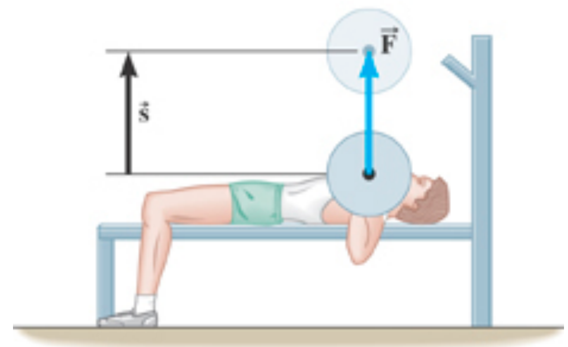
$$W_{net} = \sum\left(\vec{F}\cdot\vec{x}\right) = \left(\vec{F}_N\cdot\vec{x} + \vec{F}_G\cdot\vec{x} + \vec{F}_p\cdot\vec{x} + \vec{F}_{fr}\cdot\vec{x}\right)$$

# Ex. Bench Pressing and The Concept of Negative Work

A weight lifter is bench-pressing a barbell whose weight is 710N a distance of 0.65m above his chest. Then he lowers it the same distance. The weight is raised and lowered at a constant velocity. Determine the work in the two cases.

(a)

What is the angle between the force and the displacement?



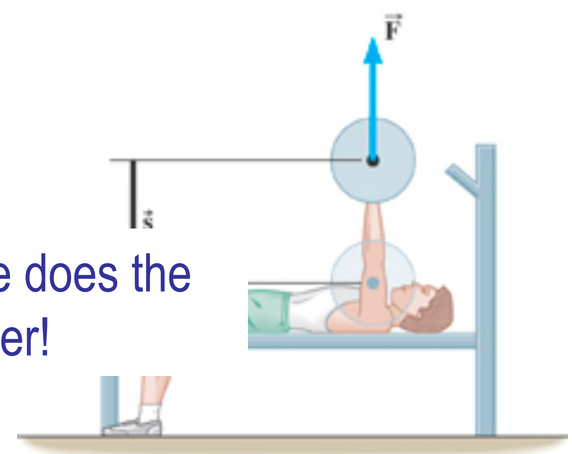

(b)

$$W = (F\cos 0)s = Fs$$

$$= 710 \cdot 0.65 = +460(J)$$

$$W = (F\cos 180)s = -Fs$$

$$= -710 \cdot 0.65 = -460(J)$$



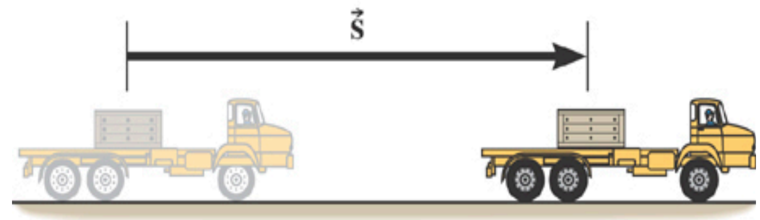

(c)

What does the negative work mean?

The gravitational force does the work on the weight lifter!

# Ex. Accelerating a Crate

The truck is accelerating at a rate of +1.50 $m/s^2$. The mass of the crate is 120-kg and it does not slip. The magnitude of the displacement is 65 m. What is the total work done on the crate by all of the forces acting on it?



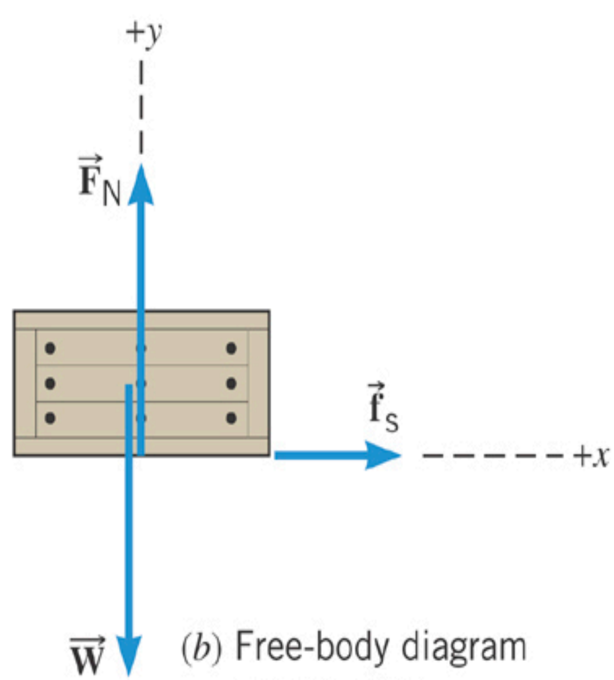
(a)

What are the forces acting in this motion?

Gravitational force on the crate, weight, **W** or $\mathbf{F_g}$

Normal force force on the crate, $\mathbf{F_N}$

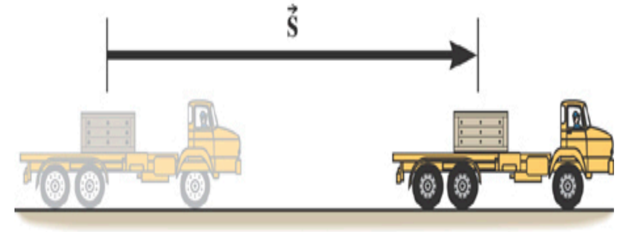
Static frictional force on the crate, $\mathbf{f_s}$



(b) Free-body diagram for the crate

# Ex. Continued…

Let's figure what the work done by each force in this motion is.



(a)

Work done by the gravitational force on the crate, **W** or **$F_g$**

$$W = \left(F_g \cos\left(-90^o\right)\right)s = 0$$

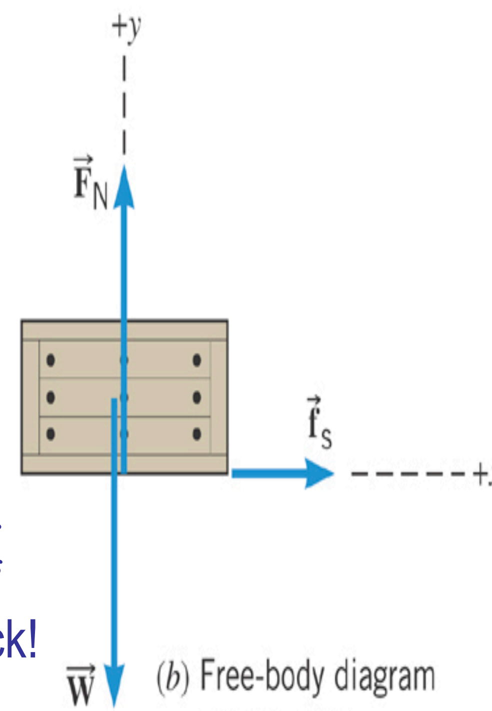
Work done by Normal force force on the crate, **$F_N$**

$$W = \left(F_N \cos\left(+90^o\right)\right)s = 0$$

Work done by the static frictional force on the crate, $f_s$

$$f_s = ma = \left(120 \text{ kg}\right)\left(1.5 \text{ m/s}^2\right) = 180\text{N}$$

$$W = f_s \cdot s = \left[\left(180\text{N}\right)\cos 0\right]\left(65 \text{ m}\right) = 1.2 \times 10^4 \text{ J}$$



(b) Free-body diagram for the crate

Which force did the work? Static frictional force on the crate, $f_s$

How? By holding on to the crate so that it moves with the truck!

# Work Done by the Varying Force

- If the force depends on the position of the object in motion,

$\rightarrow$ one must consider the work in small segments of the displacement where the force can be considered constant

$$\Delta W = F_x \cdot \Delta x$$

– Then add all the work-segments throughout the entire motion ($x_i \rightarrow x_f$)

$$W \approx \sum_{x_i}^{x_f} F_x \cdot \Delta x$$

In the limit where $\Delta x \rightarrow 0$

$$\lim_{\Delta x \rightarrow 0} \sum_{x_i}^{x_f} F_x \cdot \Delta x = \int_{x_i}^{x_f} F_x dx = W$$

– If more than one force is acting, the net work done by the net force is

$$W(net) = \int_{x_i}^{x_f} \left(\sum F_{ix}\right) dx$$

One of the position dependent forces is the force by the spring $F_s = -kx$

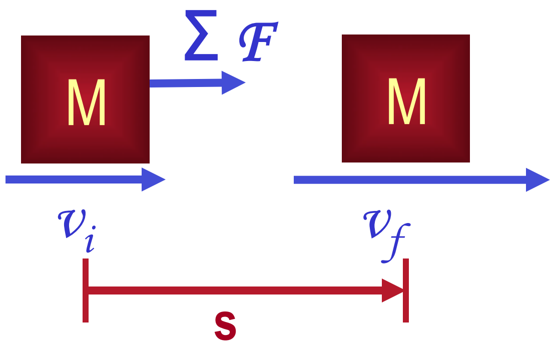
**Hooke's Law**

The work done by the spring force is

$$W = \int_{-x_{max}}^{0} F_s dx = \int_{-x_{max}}^{0} (-kx) dx = \frac{1}{2} k x_{max}^2$$

# Kinetic Energy and Work-Kinetic Energy Theorem

- Some problems are hard to solve using Newton's second law
  - If forces exerting on an object during the motion are complicated
  - Relate the work done on the object by the net force to the change of the speed of the object




Suppose net force $\Sigma\mathcal{F}$ was exerted on an object for displacement d to increase its speed from $v_i$ to $v_f$.

The work on the object by the net force $\Sigma\mathcal{F}$ is

$$W = \left(\sum \vec{F}\right)\cdot\vec{s} = \left(ma\cos 0\right)s = \left(ma\right)s$$

Using the kinematic equation of motion

$$2as = v_f^2 - v_0^2 \Rightarrow as = \frac{v_f^2 - v_0^2}{2}$$

Work $$W = \left(ma\right)s = \frac{1}{2}m\left(v_f^2 - v_0^2\right) = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2$$

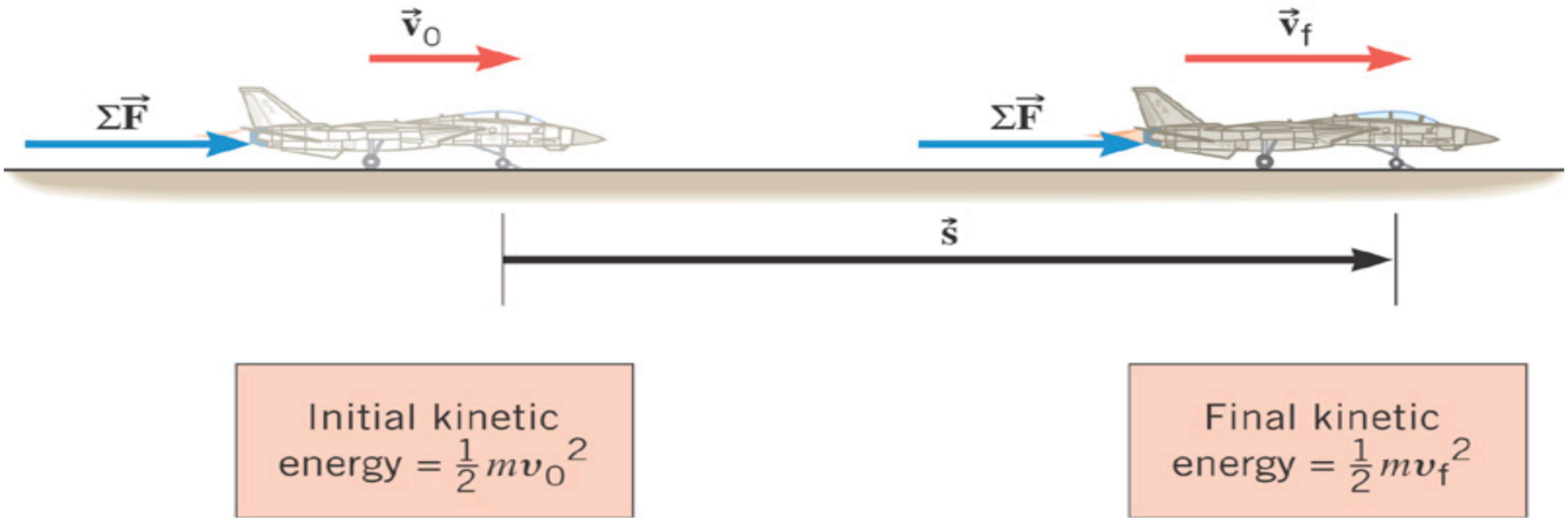
Kinetic Energy $$KE \equiv \frac{1}{2}mv^2$$

Work $$W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = KE_f - KE_i = \Delta KE$$

**Work done by the net force causes change in the object's kinetic energy.**

**Work-Kinetic Energy Theorem**

# Work-Kinetic Energy Theorem




When a net external force by the jet engine does work on and object, the kinetic energy of the object changes according to

$$W = \mathrm{KE_f} - \mathrm{KE_o} = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_o^2$$

# Work and Kinetic Energy

*A meaningful work in physics is done only when the sum of the forces exerted on an object made a motion to the object.*

*What does this mean?*

*However much tired your arms feel, if you were just holding an object without moving it you have not done any physical work to the object.*

*Mathematically, the work is written as the product of magnitudes of the net force vector, the magnitude of the displacement vector and the angle between them.*

$$W = \sum\left(\vec{F}_i\right)\bullet\vec{d} = \left|\sum\left(\vec{F}_i\right)\right|\left|\vec{d}\right|\cos\theta$$

*Kinetic Energy is the energy associated with the motion and capacity to perform work.*
*Work causes change of energy after the completion* ← **Work-Kinetic energy theorem**

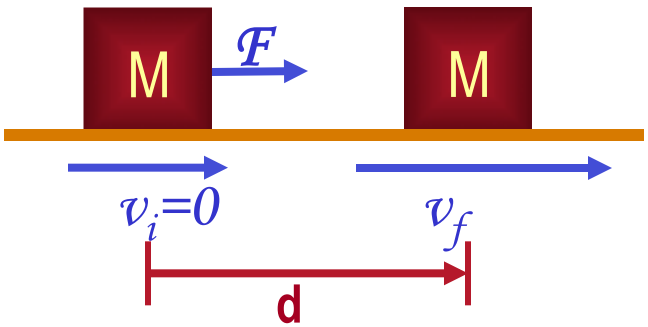
$$K = \frac{1}{2}mv^2$$

$$\sum W = K_f - K_i = \Delta K$$

*Nm=Joule*

# Example for Work-KE Theorem

A 6.0kg block initially at rest is pulled to East along a horizontal, frictionless surface by a constant horizontal force of 12N. Find the speed of the block after it has moved 3.0m.




Work done by the force $F$ is

$$W = \vec{F}\cdot\vec{d} = \left|\vec{F}\right|\left|\vec{d}\right|\cos\theta = 12\times 3.0\cos 0 = 36\,(J)$$

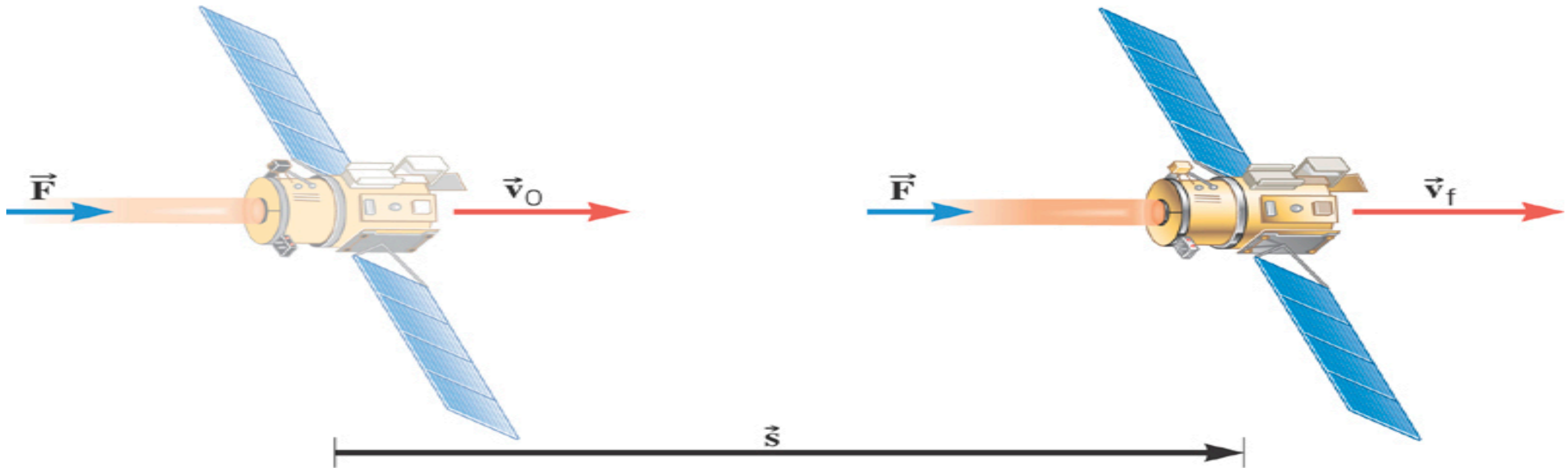
From the work-kinetic energy theorem, we know $W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

Since initial speed is 0, the above equation becomes $W = \frac{1}{2}mv_f^2$

Solving the equation for $v_f$, we obtain $v_f = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2\times 36}{6.0}} = 3.5m/s$

# Ex. Deep Space 1

The mass of the space probe is 474-kg and its initial velocity is 275 m/s. If the 56.0-mN force acts on the probe parallel through a displacement of $2.42\times10^9$m, what is its final speed?




$$\left[\left(\sum F\right)\cos\theta\right]s = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_o^2$$

Solve for $v_f$

$$v_f = \sqrt{v_o^2 + 2\left(\sum F\cos\theta\right)s/m} = \sqrt{(275\,\text{m/s})^2 + 2\left(5.60\times10^{-2}\text{N}\right)\cos 0^\circ\left(2.42\times10^9\,\text{m}\right)/474}$$

$$v_f = 805\,\text{m/s}$$

# Ex. Satellite Motion and Work By the Gravity

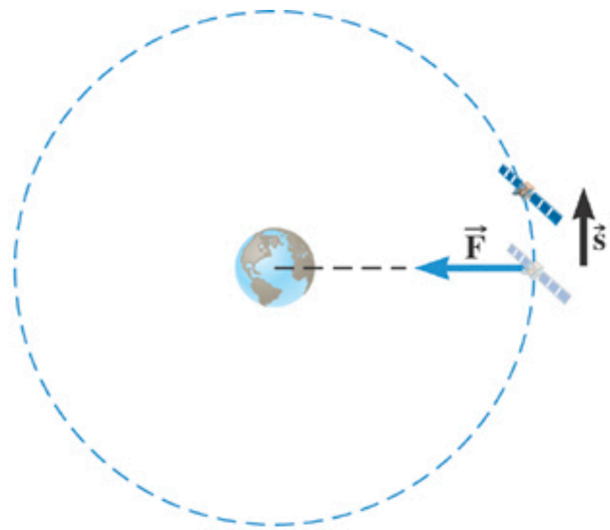
A satellite is moving about the earth in a circular orbit and an elliptical orbit. For these two orbits, determine whether the kinetic energy of the satellite changes during the motion.

For a circular orbit   No change!   Why not?

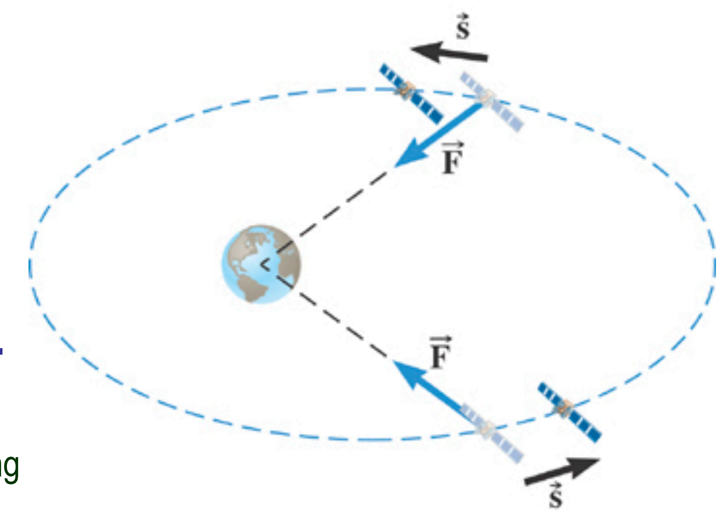
Gravitational force is the only external force but it is perpendicular to the displacement. So no work.

For an elliptical orbit   Changes!   Why?

Gravitational force is the only external force but its angle with respect to the displacement varies. So it performs work.




(a)




(b)