PHYS 1443 – Section 001
Lecture #15

Monday, April 4, 2011

Dr. Jaehoon Yu

- Linear Momentum and Forces
- Linear Momentum Conservation
- Collisions and Impulse
- Collisions – Elastic and Inelastic Collisions
- Center of Mass

Announcements

- Second non-comprehensive term exam date
  - Time: 1 – 2:20pm, Wednesday, Apr. 6
  - Location: SH103
  - Covers: CH6.4 – CH9.3
  - Please do NOT miss the exam!!
  - The worse of the two non-comprehensive term exams will be dropped!
- Colloquium Wednesday at 4pm in SH101

Physics Department
The University of Texas at Arlington
COLLOQUIUM

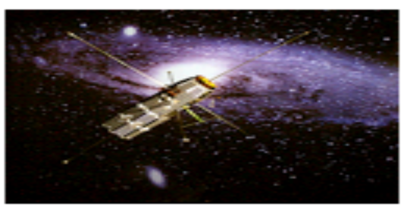
Measurements of Linked Plasma-Neutral Structure and Dynamics in the Ionosphere

Dr. Greg Earle
University of Texas at Dallas

4:00 pm Wednesday April 06, 2011 room 101 SH

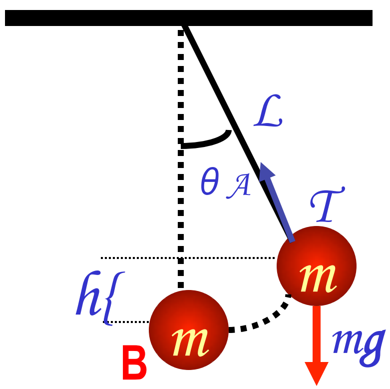
Abstract:

Interactions between the co-located "oceans" of ionized and neutral gas that surround the Earth create a host of interesting phenomena that have real-world consequences for satellite and terrestrial communication and navigation systems. These gaseous media and the physics governing their behavior have been studied over the last 50 years using a combination of rocket probes, satellites, radars, lidars, and computational modeling. This talk will describe both new and time-tested measurement techniques for in-situ ion and neutral measurements aboard orbital and sub-orbital spacecraft, and show examples of how these measurements reveal details of plasma physics processes in the near-Earth space environment. Specific examples of measurements relevant to linked plasma-neutral structures and dynamics will be presented, including gravity wave-induced plasma oscillations, ionized sporadic layer formation, and localized instability development as observed from a wide variety of missions and instruments.



Refreshments will be served at 3:30p.m in the Physics Library

Reminder: Special Project



A ball of mass m is attached to a light cord of length L, making up a pendulum. The ball is released from rest when the cord makes an angle θ_A which is greater than 90 degrees with the vertical, and the pivoting point P is frictionless. Find the speed of the ball when it is at the lowest point, B. 10 points

Due: beginning of the class Monday, Apr. 11

Extra-Credit Special Project

- Derive the formula for the final velocity of two objects which underwent an elastic collision as a function of known quantities m_1, m_2, v_{01} and v_{02} in page 20 of this lecture note in a far greater detail than the note.
  - 20 points extra credit
- Show mathematically what happens to the final velocities if $m_1=m_2$ and describe in words the resulting motion.
  - 5 point extra credit
- Due: Start of the class Wednesday, Apr. 13

Linear Momentum

The principle of energy conservation can be used to solve problems that are harder to solve just using Newton's laws. It is used to describe motion of an object or a system of objects.

A new concept of linear momentum can also be used to solve physical problems, especially the problems involving collisions of objects.

*Linear momentum of an object whose mass is m and is moving at the velocity of **v** is defined as*

$$\vec{p} \equiv m\vec{v}$$

What can you tell from this definition about momentum?

1. *Momentum is a vector quantity.*
2. *The heavier the object the higher the momentum*
3. *The higher the velocity the higher the momentum*
4. *Its unit is kg.m/s*

What else can use see from the definition? Do you see force?

The change of momentum in a given time interval

$$\frac{\Delta\vec{p}}{\Delta t} = \frac{m\vec{v} - m\vec{v}_0}{\Delta t} = \frac{m(\vec{v} - \vec{v}_0)}{\Delta t} = m\frac{\Delta\vec{v}}{\Delta t} = m\vec{a} = \sum\vec{F}$$

Linear Momentum and Forces

$$\sum \vec{F} = \lim_{\Delta t \to 0} \frac{\Delta \vec{p}}{\Delta t} = \frac{d\vec{p}}{dt}$$

What can we learn from this force-momentum relationship?

- The rate of the change of particle's momentum is the same as the net force exerted on it.
- When the net force is 0, the particle's linear momentum is a constant as a function of time.
- If a particle is isolated, the particle experiences no net force. Therefore its momentum does not change and is conserved.

Something else we can do with this relationship. What do you think it is?

The relationship can be used to study the case where the mass changes as a function of time.

Can you think of a few cases like this?

$$\sum \vec{F} = \frac{d\vec{p}}{dt} = \frac{d\left(m\vec{v}\right)}{dt} = \frac{dm}{dt}\vec{v} + m\frac{d\vec{v}}{dt}$$

Motion of a meteorite

Motion of a rocket

Conservation of Linear Momentum in a Two Particle System

Consider an isolated system with two particles that do not have any external forces exerting on it. What is the impact of Newton's 3rd Law?

If particle #1 exerts force on particle #2, there must be another force that the particle #2 exerts on #1 as the reaction force. Both the forces are internal forces, and the net force in the entire SYSTEM is ***still 0.***

Now how would the momenta of these particles look like?

Let say that the particle #1 has momentum $\mathbf{p}_1$ *and #2 has* $\mathbf{p}_2$ *at some point of time.*

Using momentum-force relationship

$$\vec{F}_{21} = \frac{d\vec{p}_1}{dt} \quad \text{and} \quad \vec{F}_{12} = \frac{d\vec{p}_2}{dt}$$

And since net force of this system is 0

$$\sum \vec{F} = \vec{F}_{12} + \vec{F}_{21} = \frac{d\vec{p}_2}{dt} + \frac{d\vec{p}_1}{dt} = \frac{d}{dt}\left(\vec{p}_2 + \vec{p}_1\right) = 0$$

Therefore $\vec{p}_2 + \vec{p}_1 = const$

The total linear momentum of the system is conserved!!!

More on Conservation of Linear Momentum in a Two Body System

From the previous slide we've learned that the total momentum of the system is conserved if no external forces are exerted on the system.

$$\sum \vec{p} = \vec{p}_2 + \vec{p}_1 = const$$

What does this mean?

As in the case of energy conservation, this means that the total vector sum of all momenta in the system is the same before and after any interactions

Mathematically this statement can be written as

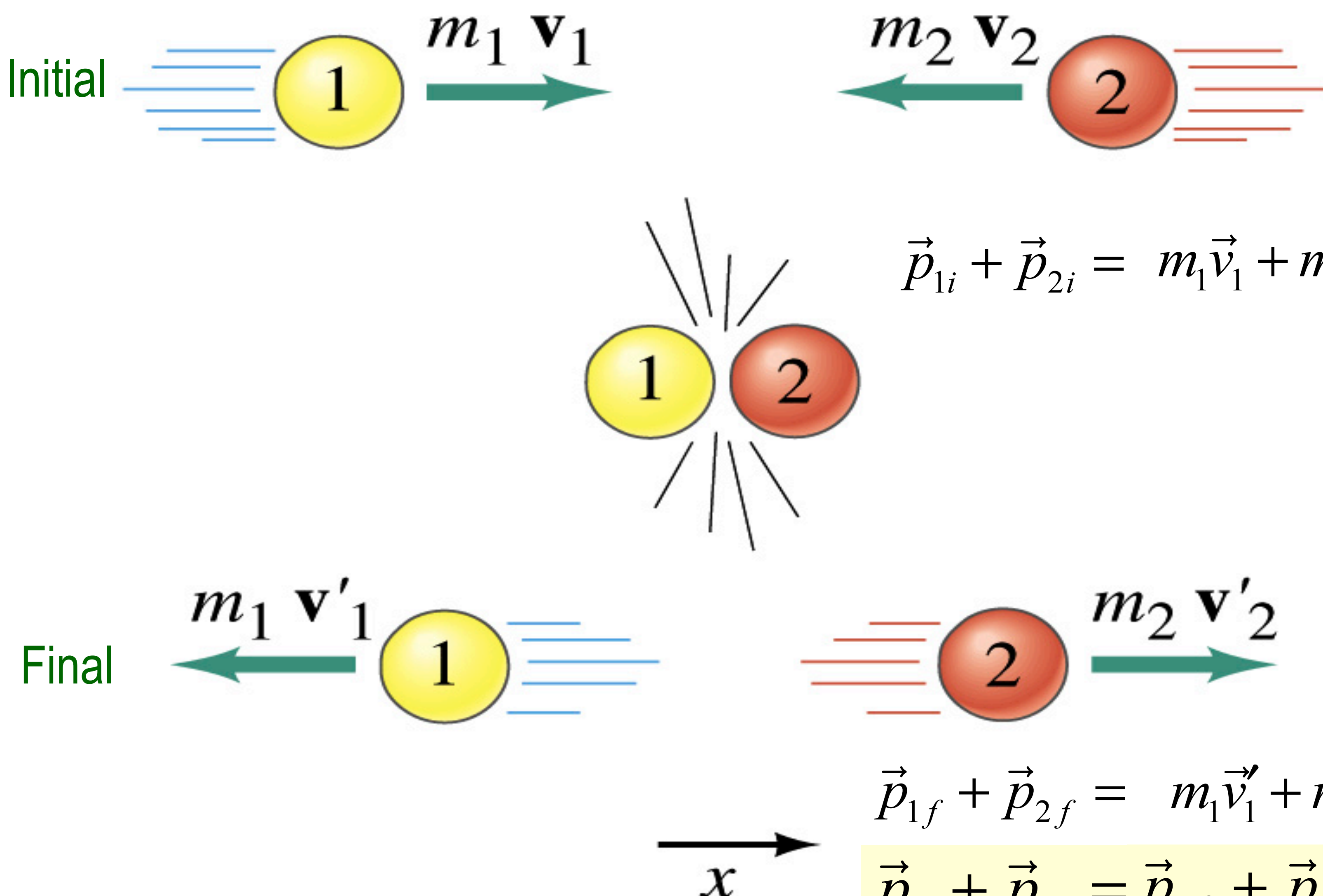
$$\vec{p}_{2i} + \vec{p}_{1i} = \vec{p}_{2f} + \vec{p}_{1f}$$

$$\sum_{system} P_{xi} = \sum_{system} P_{xf} \qquad \sum_{system} P_{yi} = \sum_{system} P_{yf} \qquad \sum_{system} P_{zi} = \sum_{system} P_{zf}$$

This can be generalized into conservation of linear momentum in many particle systems.

*Whenever two or more particles in an **isolated system** interact, the total momentum of the system remains constant.*

Linear Momentum Conservation



$$\vec{p}_{1i} + \vec{p}_{2i} = m_1\vec{v}_1 + m_2\vec{v}_2$$

$$\vec{p}_{1f} + \vec{p}_{2f} = m_1\vec{v}'_1 + m_2\vec{v}'_2$$

$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f}$$

Example 9.4: Rifle Recoil

Calculate the recoil velocity of a 5.0kg rifle that shoots a 0.020kg bullet at a speed of 620m/s.

From momentum conservation, we can write

$$\vec{p}_i = 0 = \vec{p}_f = \vec{P}_R + \vec{P}_B = m_R \vec{v}_R + m_B \vec{v}_B$$

The x-comp $P_R + P_B = m_R v_R + m_B v_B = 0$

Solving the above for v_R and using the rifle's mass and the bullet's mass, we obtain

$$v_R = \frac{m_B}{m_R} v_B = \frac{0.020}{5.0} \cdot 620 = -2.5 m/s$$

$$\vec{v}_R = -2.5\vec{i}\,(m/s)$$

Example for Linear Momentum Conservation

Estimate an astronaut's (M=70kg) resulting velocity after he throws his book (m=1kg) to a direction in the space to move to another direction.

From momentum conservation, we can write

$$\vec{p}_i = 0 = \vec{p}_f = m_A \vec{v}_A + m_B \vec{v}_B$$

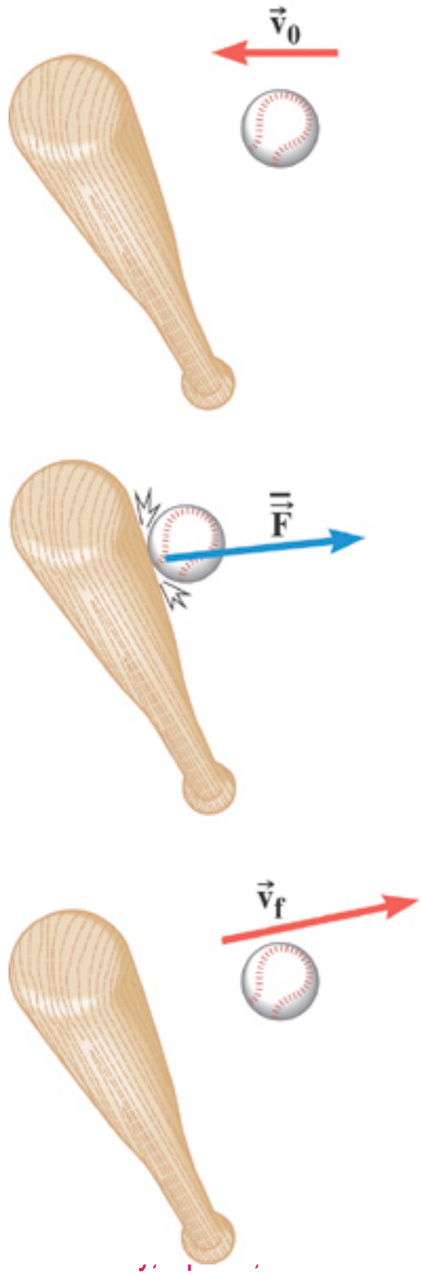
Assuming the astronaut's mass is 70kg, and the book's mass is 1kg and using linear momentum conservation

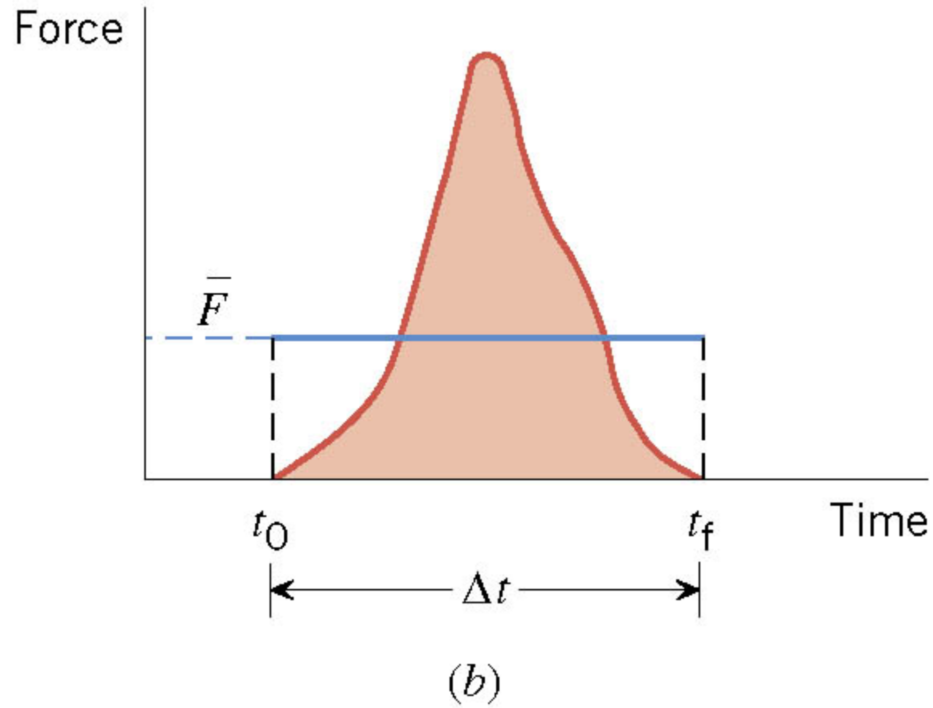
$$v_A = -\frac{m_B v_B}{m_A} = -\frac{1}{70} v_{\mathbf{B}}$$

Now if the book gained a velocity of 20 m/s in +x-direction, the Astronaut's velocity is

$$\vec{v}_A = -\frac{1}{70}\left(20\vec{i}\right) = -0.3\vec{i}\ \left(m/s\right)$$

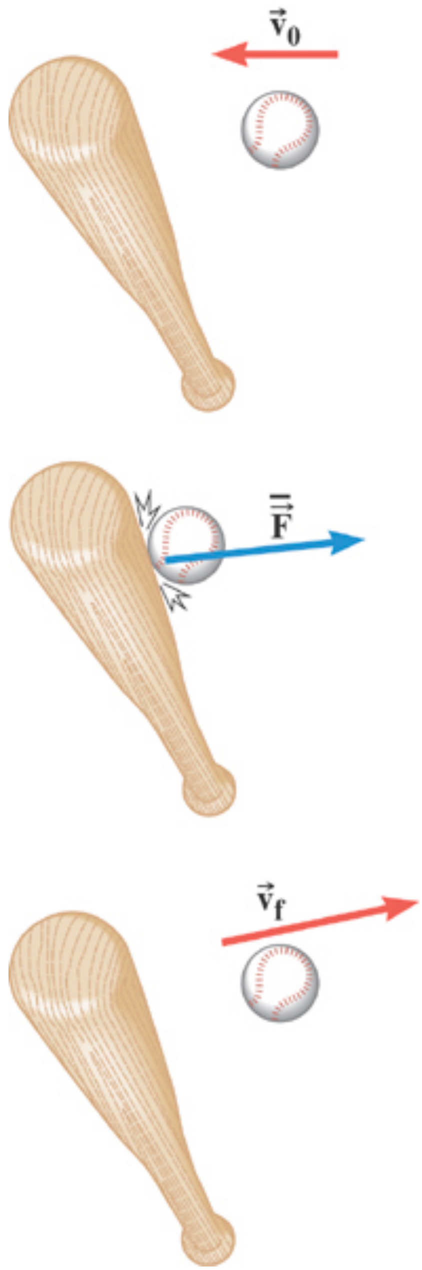
Impulse





There are many situations when the force on an object is not constant and in fact quite complicated!!

Ball Hit by a Bat



$$\overline{\vec{\mathbf{a}}} = \frac{\vec{\mathbf{v}}_f - \vec{\mathbf{v}}_o}{\Delta t}$$

$$\sum \overline{\vec{\mathbf{F}}} = m\overline{\vec{\mathbf{a}}}$$

$$\sum \overline{\vec{\mathbf{F}}} = \frac{m\vec{\mathbf{v}}_f - m\vec{\mathbf{v}}_o}{\Delta t}$$

Multiply either side by Δt

$$\left(\sum \overline{\vec{\mathbf{F}}}\right)\Delta t = m\vec{\mathbf{v}}_f - m\vec{\mathbf{v}}_o = \vec{J}$$

Impulse and Linear Momentum

Net force causes change of momentum → Newton's second law

$$\vec{F} = \frac{d\vec{p}}{dt} \Rightarrow d\vec{p} = \vec{F}dt$$

By integrating the above equation in a time interval t_i to t_f, one can obtain impulse I.

$$\int_{t_i}^{t_f} d\vec{p} = \vec{p}_f - \vec{p}_i = \Delta\vec{p} = \int_{t_i}^{t_f} \vec{F}\,dt = \vec{J}$$

So what do you think an impulse is?

Effect of the force F acting on an object over the time interval $\Delta t = t_f - t_i$ is equal to the change of the momentum of the object caused by that force. Impulse is the degree of which an external force changes an object's momentum.

The above statement is called the impulse-momentum theorem and is equivalent to Newton's second law.

What are the dimension and unit of Impulse? What is the direction of an impulse vector?

Defining a time-averaged force

$$\bar{\vec{F}} \equiv \frac{1}{\Delta t}\sum_i \vec{F}_i \Delta t$$

Impulse can be rewritten

$$\vec{J} \equiv \bar{\vec{F}}\Delta t$$

If force is constant

$$\vec{J} \equiv \vec{F}\Delta t$$

It is generally assumed that the impulse force acts on a short time but much greater than any other forces present.

An Example for Impulse

In a crash test, an automobile of mass 1500kg collides with a wall. The initial and final velocities of the automobile are v_i= -15.0i m/s and v_f=2.60i m/s. If the collision lasts for 0.150 seconds, what would be the impulse caused by the collision and the average force exerted on the automobile?

Let's assume that the force involved in the collision is a lot larger than any other forces in the system during the collision. From the problem, the initial and final momentum of the automobile before and after the collision is

$$\vec{p}_i = m\vec{v}_i = 1500\times(-15.0)\vec{i} = -22500\vec{i}\ kg\cdot m/s$$

$$\vec{p}_f = m\vec{v}_f = 1500\times(2.60)\vec{i} = 3900\vec{i}\ kg\cdot m/s$$

Therefore the impulse on the automobile due to the collision is

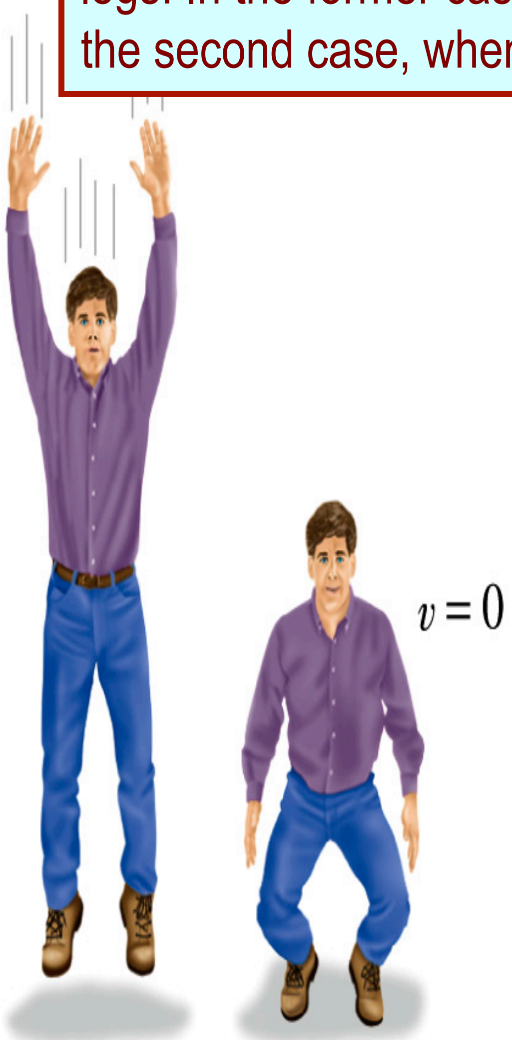
$$\vec{J} = \Delta\vec{p} = \vec{p}_f - \vec{p}_i = (3900+22500)\vec{i}\ kg\cdot m/s$$
$$= 26400\vec{i}\ kg\cdot m/s = 2.64\times10^4\vec{i}\ kg\cdot m/s$$

The average force exerted on the automobile during the collision is

$$\overline{\vec{F}} = \frac{\Delta\vec{p}}{\Delta t} = \frac{2.64\times10^4}{0.150}\vec{i}$$
$$= 1.76\times10^5\vec{i}\ kg\cdot m/s^2 = 1.76\times10^5\vec{i}\ \mathrm{N}$$

Another Example for Impulse

(a) Calculate the impulse experienced when a 70 kg person lands on firm ground after jumping from a height of 3.0 m. Then estimate the average force exerted on the person's feet by the ground, if the landing is (b) stiff-legged and (c) with bent legs. In the former case, assume the body moves 1.0cm during the impact, and in the second case, when the legs are bent, about 50 cm.



We don't know the force. How do we do this?

Obtain velocity of the person before striking the ground.

$$KE = -\Delta PE \qquad \frac{1}{2}mv^2 = -mg\left(y - y_i\right) = mgy_i$$

Solving the above for velocity v, we obtain

$$v = \sqrt{2gy_i} = \sqrt{2 \cdot 9.8 \cdot 3} = 7.7 m/s$$

Then as the person strikes the ground, the momentum becomes 0 quickly giving the impulse

$$\vec{J} = \vec{\overline{F}}\Delta t = \Delta\vec{p} = \vec{p}_f - \vec{p}_i = 0 - m\vec{v} =$$

$$= -70kg \cdot 7.7m/s\,\vec{j} = -540\vec{j}N \cdot s$$

Example cont'd

In coming to rest, the body decelerates from 7.7m/s to 0m/s in a distance d=1.0cm=0.01m.

The average speed during this period is $\bar{v} = \frac{0+v_i}{2} = \frac{7.7}{2} = 3.8m/s$

The time period the collision lasts is $\Delta t = \frac{d}{\bar{v}} = \frac{0.01m}{3.8m/s} = 2.6\times10^{-3}s$

Since the magnitude of impulse is $\left|\vec{J}\right| = \left|\vec{\bar{F}}\Delta t\right| = 540N\cdot s$

The average force on the feet during this landing is $\bar{F} = \frac{J}{\Delta t} = \frac{540}{2.6\times10^{-3}} = 2.1\times10^{5}N$

How large is this average force? $Weight = 70kg\cdot 9.8m/s^2 = 6.9\times10^2 N$

$$\bar{F} = 2.1\times10^5 N = 304\times6.9\times10^2 N = 304\times Weight$$

If landed in stiff legged, the feet must sustain 300 times the body weight. The person will likely break his leg.

For bent legged landing: $\Delta t = \frac{d}{\bar{v}} = \frac{0.50m}{3.8m/s} = 0.13s$

$$\bar{F} = \frac{540}{0.13} = 4.1\times10^3 N = 5.9Weight$$