

PHYS 1443 – Section 001

Lecture #20

Monday, April 25, 2011

Dr. Jaehoon Yu

- Angular Momentum Conservation
- Similarity between Linear and Angular Quantities
- Conditions for Equilibrium
- How to Solve Equilibrium Problems?
- Equilibrium Problem Exercises
- Elastic Properties of Solids
- Density and Specific Gravity

Today's homework is homework #11, due 10pm, Friday, May 6!!

Announcements

- Planetarium extra credit sheets
 - Tape ONLY one corner of the ticket stub on a sheet with your name and ID on it
 - I must be able to see the initial of the start lecturer of the show
 - Bring it by the beginning of the class Monday, May 2
- Last quiz next Wednesday, May 4
 - Covers from Ch. 11 through what we finish this Wednesday
- Final comprehensive exam
 - Time: 11am, Monday, May 9
 - Location: SH103
 - Covers: Chapter 1.1 through what we finish Monday, May 2+ appendices
 - A 100 problem package will be distributed in class this Wednesday
 - Review: Wednesday, May 4th, in the class after the quiz
 - Attendance will be taken
- Reading assignments: 12 – 5 and 12 – 6

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Conservation of Angular Momentum

Remember under what condition the linear momentum is conserved?

Linear momentum is conserved when the net external force is 0. $\sum \vec{F} = 0 = \frac{d\vec{p}}{dt}$
 $\vec{p} = \text{const}$

By the same token, the angular momentum of a system is constant in both magnitude and direction, if the resultant external torque acting on the system is 0.

$$\sum \vec{\tau}_{ext} = \frac{d\vec{L}}{dt} = 0$$

$$\vec{L} = \text{const}$$

What does this mean?

Angular momentum of the system before and after a certain change is the same.

$$\vec{L}_i = \vec{L}_f = \text{constant}$$

Three important conservation laws for isolated system that does not get affected by external forces

$$K_i + U_i = K_f + U_f$$

$$\vec{p}_i = \vec{p}_f$$

$$\vec{L}_i = \vec{L}_f$$

Mechanical Energy

Linear Momentum

Angular Momentum



Ex. 11 – 3 Neutron Star

A star rotates with a period of 30 days about an axis through its center. After the star undergoes a supernova explosion, the stellar core, which had a radius of $1.0 \times 10^4 \text{ km}$, collapses into a neutron star of radius 3.0 km. Determine the period of rotation of the neutron star.

What is your guess about the answer?

The period will be significantly shorter, because its radius got smaller.

Let's make some assumptions:

1. There is no external torque acting on it
2. The shape remains spherical
3. Its mass remains constant

Using angular momentum conservation

$$L_i = L_f$$

$$I_i \omega_i = I_f \omega_f$$

The angular speed of the star with the period T is

$$\omega = \frac{2\pi}{T}$$

Thus
$$\omega_f = \frac{I_i \omega_i}{I_f} = \frac{mr_i^2}{mr_f^2} \frac{2\pi}{T_i}$$

$$T_f = \frac{2\pi}{\omega_f} = \left(\frac{r_f^2}{r_i^2} \right) T_i = \left(\frac{3.0}{1.0 \times 10^4} \right)^2 \times 30 \text{ days} = 2.7 \times 10^{-6} \text{ days} = 0.23 \text{ s}$$

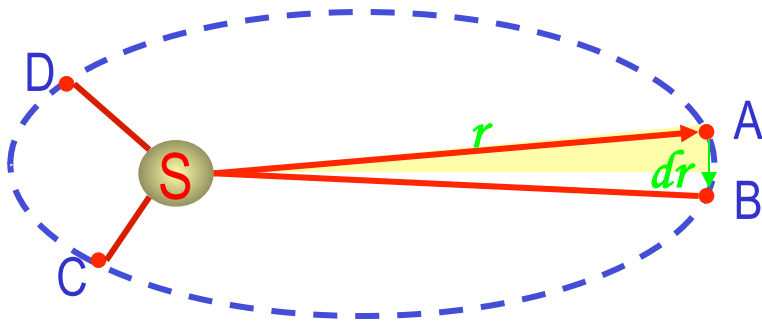
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Kepler's Second Law and Angular Momentum Conservation

Consider a planet of mass M_p moving around the Sun in an elliptical orbit.



Since the gravitational force acting on the planet is always toward radial direction, it is a *central force*. Therefore the torque acting on the planet by this force is always 0.

$$\vec{\tau} = \vec{r} \times \vec{F} = \vec{r} \times F\hat{r} = 0$$

Since torque is the time rate change of angular momentum \mathcal{L} , the angular momentum is constant.

$$\vec{\tau} = \frac{d\vec{L}}{dt} = 0 \quad \vec{L} = \text{const}$$

Because the gravitational force exerted on a planet by the Sun results in no torque, the angular momentum L of the planet is constant.

$$\vec{L} = \vec{r} \times \vec{p} = \vec{r} \times M_p \vec{v} = M_p \vec{r} \times \vec{v} = \text{const}$$

Since the area swept by the motion of the planet is

$$dA = \frac{1}{2} |\vec{r} \times d\vec{r}| = \frac{1}{2} |\vec{r} \times \vec{v} dt| = \frac{L}{2M_p} dt \quad \Rightarrow \quad \frac{dA}{dt} = \frac{L}{2M_p} = \text{const}$$

This is Kepler's second law which states that the radius vector from the Sun to a planet sweeps out equal areas in equal time intervals.

Similarity Between Linear and Rotational Motions

All physical quantities in linear and rotational motions show striking similarity.

Quantities	Linear	Rotational
Mass	Mass M	Moment of Inertia $I = mr^2$
Length of motion	Distance L	Angle θ (Radian)
Speed	$v = \frac{\Delta r}{\Delta t}$	$\omega = \frac{\Delta \theta}{\Delta t}$
Acceleration	$a = \frac{\Delta v}{\Delta t}$	$\alpha = \frac{\Delta \omega}{\Delta t}$
Force	Force $\vec{F} = m\vec{a}$	Torque $\vec{\tau} = I\vec{\alpha}$
Work	Work $W = \vec{F} \cdot \vec{d}$	Work $W = \tau\theta$
Power	$P = \vec{F} \cdot \vec{v}$	$P = \tau\omega$
Momentum	$\vec{p} = m\vec{v}$	$\vec{L} = I\vec{\omega}$
Kinetic Energy	Kinetic $K = \frac{1}{2}mv^2$	Rotational $K_R = \frac{1}{2}I\omega^2$

Conditions for Equilibrium

What do you think the term “An object is at its equilibrium” means?

An object is either at rest (Static Equilibrium) or its center of mass is moving at a constant velocity (Dynamic Equilibrium).

When do you think an object is at its equilibrium?

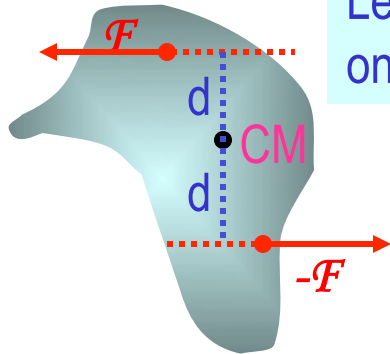
Translational Equilibrium: Equilibrium in linear motion

$$\sum \vec{F} = 0$$

Is this it?

The above condition is sufficient for a point-like object to be at its translational equilibrium. For an object with size, however, this is not sufficient. One more condition is needed. What is it?

Let's consider two forces equal in magnitude but in opposite direction acting on a rigid object as shown in the figure. What do you think will happen?



The object will rotate about the CM. The net torque acting on the object about any axis must also be 0.

$$\sum \vec{\tau} = 0$$

For an object to be at its *static equilibrium*, the object should not have linear or angular speed.

$$v_{CM} = 0 \quad \omega = 0$$

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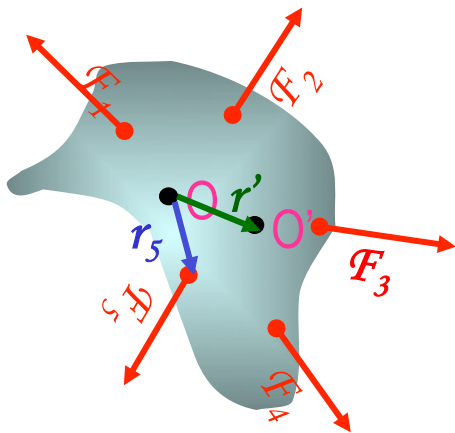
More on Conditions for Equilibrium

To simplify the problem, we will only deal with forces acting on x-y plane, giving torque only along z-axis. What do you think the conditions for equilibrium are in this case?

The six possible equations from the two vector equations turns to three equations.

$$\sum \vec{F} = 0 \Rightarrow \begin{matrix} \sum F_x = 0 \\ \sum F_y = 0 \end{matrix} \text{ AND } \sum \vec{\tau} = 0 \Rightarrow \sum \tau_z = 0$$

What happens if there are many forces exerting on an object?



If an object is at its translational static equilibrium, and if the net torque acting on the object is 0 about one axis, the net torque must be 0 about any arbitrary axis.

Why is this true?

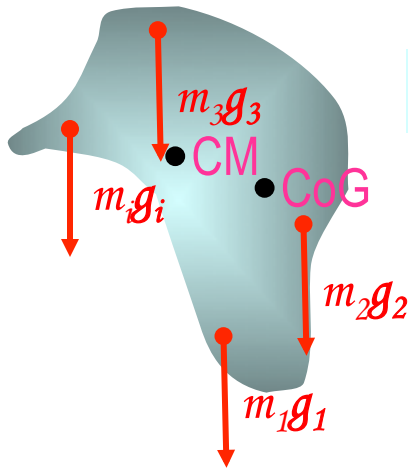
Because the object is not moving in the first place, no matter where the rotational axis is, there should not be any motion. This simply is a matter of mathematical manipulation.

Center of Gravity Revisited

When is the center of gravity of a rigid body the same as the center of mass?

Under the uniform gravitational field throughout the body of the object.

Let's consider an arbitrary shaped object



The center of mass of this object is at

$$x_{CM} = \frac{\sum m_i x_i}{\sum m_i} = \frac{\sum m_i x_i}{M}$$

$$y_{CM} = \frac{\sum m_i y_i}{\sum m_i} = \frac{\sum m_i y_i}{M}$$

Let's now examine the case that the gravitational acceleration on each point is g_i

Since the CoG is the point as if all the gravitational force is exerted on, the torque due to this force becomes

$$(m_1 g_1 + m_2 g_2 + \dots) x_{CoG} = m_1 g_1 x_1 + m_2 g_2 x_2 + \dots$$

Generalized expression for different g throughout the body

If g is uniform throughout the body

$$(m_1 + m_2 + \dots) g x_{CoG} = (m_1 x_1 + m_2 x_2 + \dots) g$$

$$x_{CoG} = \frac{\sum m_i x_i}{\sum m_i} = x_{CM}$$

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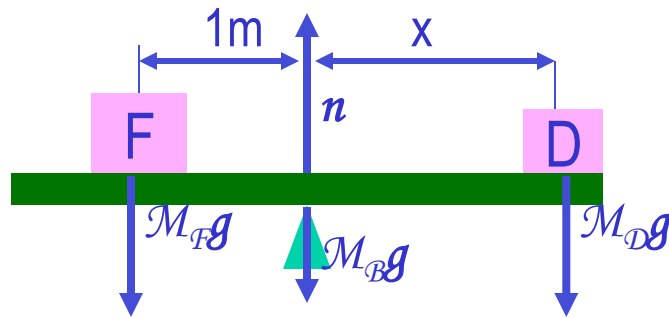
How do we solve equilibrium problems?

1. Identify all the forces and their directions and locations
2. Draw a free-body diagram with forces indicated on it with their directions and locations properly noted
3. Write down the force equation for each x and y component with proper signs
4. Select a rotational axis for torque calculations → Selecting the axis such that the torque of one of the unknown forces become 0 makes the problem easier to solve
5. Write down the torque equation with proper signs
6. Solve the equations for unknown quantities



Ex. 12 – 3: Seesaw Balancing

A uniform 40.0 N board supports the father and the daughter each weighing 800 N and 350 N, respectively, and is not moving. If the support (or fulcrum) is under the center of gravity of the board, and the father is 1.00 m from CoG, what is the magnitude of the normal force n exerted on the board by the support?



Since there is no linear motion, this system is in its translational equilibrium

$$\sum F_x = 0$$

$$\sum F_y = n - M_B g - M_F g - M_D g = 0$$

Therefore the magnitude of the normal force $n = 40.0 + 800 + 350 = 1190 \text{ N}$

Determine where the child should sit to balance the system.

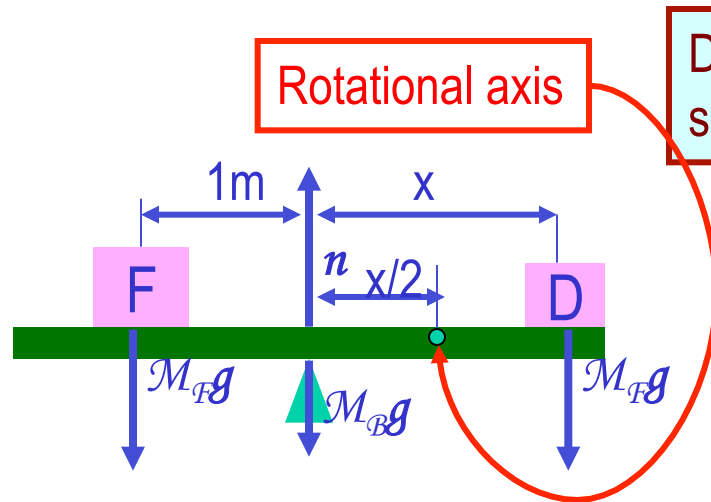
The net torque about the fulcrum by the three forces are

Therefore to balance the system the daughter must sit

$$\tau = M_B g \cdot 0 + n \cdot 0 + M_F g \cdot 1.00 - M_D g \cdot x = 0$$

$$x = \frac{M_F g}{M_D g} \cdot 1.00 \text{ m} = \frac{800}{350} \cdot 1.00 \text{ m} = 2.29 \text{ m}$$

Seesaw Example Cont'd



Determine the position of the child to balance the system for different position of axis of rotation.

The net torque about the axis of rotation by all the forces are

$$\tau = M_B g \cdot x/2 + M_F g \cdot (1.00 + x/2) - n \cdot x/2 - M_D g \cdot x/2 = 0$$

Since the normal force is $n = M_B g + M_F g + M_D g$

The net torque can be rewritten

$$\begin{aligned} \tau &= M_B g \cdot x/2 + M_F g \cdot (1.00 + x/2) \\ &\quad - (M_B g + M_F g + M_D g) \cdot x/2 - M_D g \cdot x/2 \\ &= M_F g \cdot 1.00 - M_D g \cdot x = 0 \end{aligned}$$

What do we learn?

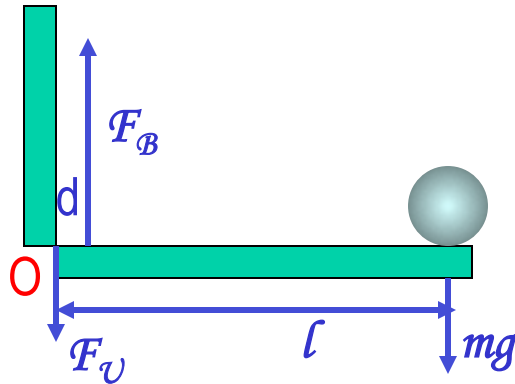
Therefore

$$x = \frac{M_F g}{M_D g} \cdot 1.00m = \frac{800}{350} \cdot 1.00m = 2.29m$$

No matter where the rotation axis is, net effect of the torque is identical.

Ex. 12 – 4 for Mechanical Equilibrium

A person holds a 50.0N sphere in his hand. The forearm is horizontal. The biceps muscle is attached 3.00 cm from the joint, and the sphere is 35.0cm from the joint. Find the upward force exerted by the biceps on the forearm and the downward force exerted by the upper arm on the forearm and acting at the joint. Neglect the weight of forearm.



Since the system is in equilibrium, from the translational equilibrium condition

$$\sum F_x = 0$$

$$\sum F_y = F_B - F_U - mg = 0$$

From the rotational equilibrium condition $\sum \tau = F_U \cdot 0 + F_B \cdot d - mg \cdot l = 0$

Thus, the force exerted by the biceps muscle is

$$F_B \cdot d = mg \cdot l$$

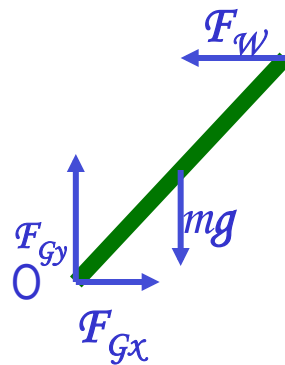
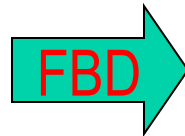
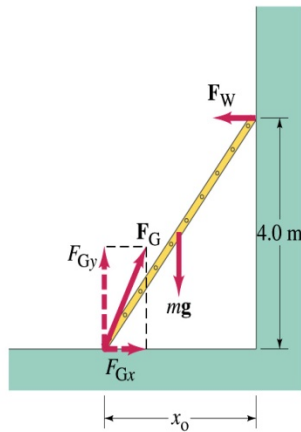
$$F_B = \frac{mg \cdot l}{d} = \frac{50.0 \times 35.0}{3.00} = 583 \text{ N}$$

Force exerted by the upper arm is

$$F_U = F_B - mg = 583 - 50.0 = 533 \text{ N}$$

Example 12 – 6

A 5.0 m long ladder leans against a wall at a point 4.0m above the ground. The ladder is uniform and has mass 12.0kg. Assuming the wall is frictionless (but ground is not), determine the forces exerted on the ladder by the ground and the wall.



First the translational equilibrium, using components

$$\sum F_x = F_{Gx} - F_W = 0$$

$$\sum F_y = -mg + F_{Gy} = 0$$

Thus, the y component of the force by the ground is

$$F_{Gy} = mg = 12.0 \times 9.8 N = 118 N$$

The length x_0 is, from Pythagorean theorem

$$x_0 = \sqrt{5.0^2 - 4.0^2} = 3.0 m$$

Example 12 – 6 cont'd

From the rotational equilibrium $\sum \tau_O = -mg x_0/2 + F_W 4.0 = 0$

Thus the force exerted on the ladder by the wall is

$$F_W = \frac{mg x_0/2}{4.0} = \frac{118 \cdot 1.5}{4.0} = 44N$$

The x component of the force by the ground is

$$\sum F_x = F_{Gx} - F_W = 0 \quad \text{Solve for } F_{Gx} \quad F_{Gx} = F_W = 44N$$

Thus the force exerted on the ladder by the ground is

$$F_G = \sqrt{F_{Gx}^2 + F_{Gy}^2} = \sqrt{44^2 + 118^2} \approx 130N$$

The angle between the ground force to the floor

$$\theta = \tan^{-1} \left(\frac{F_{Gy}}{F_{Gx}} \right) = \tan^{-1} \left(\frac{118}{44} \right) = 70^\circ$$