# PHYS 1444 – Section 004
# Lecture #5

*Wednesday, Feb. 1, 2012*

*Dr. Jaehoon Yu*

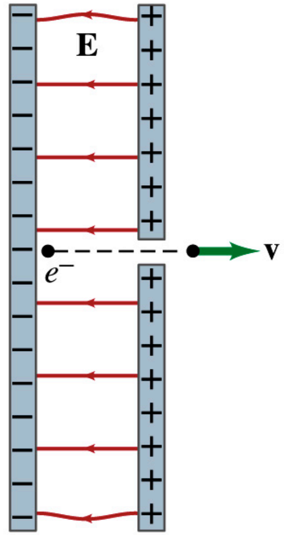
- Chapter 22 Gauss' Law
  - Gauss' Law
  - Electric Flux
  - Gauss' Law with many charges
  - What is Gauss' Law good for?
- Chapter 23 Electric Potential
  - Electric Potential Energy

# Announcements

- It was better than Monday that several of you have subscribed to the class e-mail distribution list **PHYS1444-004-SP12. Please be sure to subscribe by** clicking on the link below. https://listserv.uta.edu/cgi-bin/wa.exe?A0=PHYS1444-004-SP12
  - I now have 24!
- Quiz #2
  - Wednesday, Feb. 8
  - Beginning of the class
  - Covers: CH21.5 through what we learn on Monday, Feb. 6
- Reading assignments
  - CH21.12 and CH21.13

# Special Project

- **Particle Accelerator**. A charged particle of mass **M** with charge **-Q** is accelerated in the uniform field **E** between two parallel charged plates whose separation is **D** as shown in the figure on the right. The charged particle is accelerated from an initial speed $\mathbf{v_0}$ near the negative plate and passes through a tiny hole in the positive plate.
  - Derive the formula for the electric field E to accelerate the charged particle to a fraction $f$ of the speed of light $c$. Express E in terms of **M, Q, D,** $f$, **c** and $\mathbf{v_0}$.
  - (a) Using the Coulomb force and kinematic equations. (8 points)
  - (b) Using the work-kinetic energy theorem. ( 8 points)
  - (c) Using the formula above, evaluate the strength of the electric field E to accelerate an electron from 0.1% of the speed of light to 90% of the speed of light. You need to look up the relevant constants, such as mass of the electron, charge of the electron and the speed of light. (5 points)

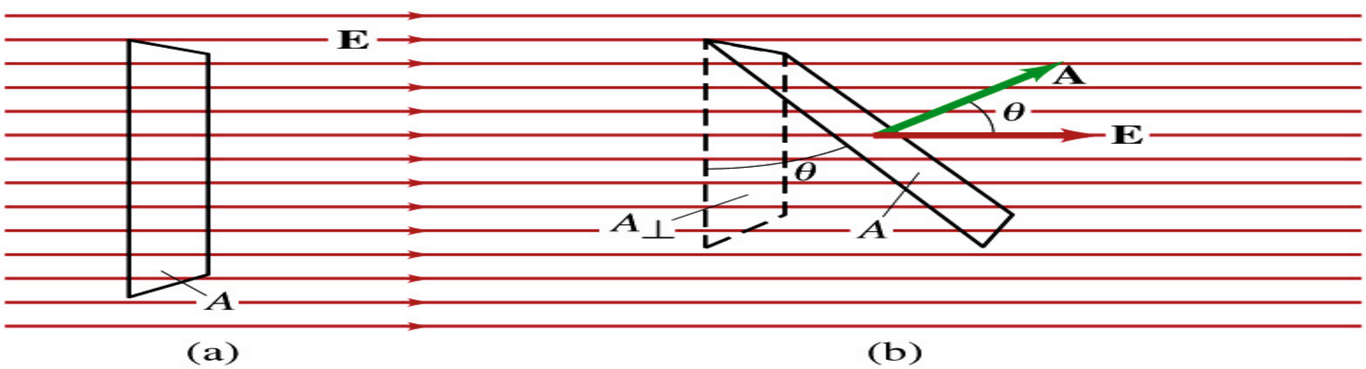



- Due beginning of the class Monday, Feb. 13

# Gauss' Law

- Gauss' law establishes the relationship between electric charge and electric field.
  - More generalized and elegant form of Coulomb's law.
- The electric field by the distribution of charges can be obtained using Coulomb's law by summing (or integrating) over the charge distributions.
- Gauss' law, however, gives an additional insight into the nature of electrostatic field and a more general relationship between the charge and the field

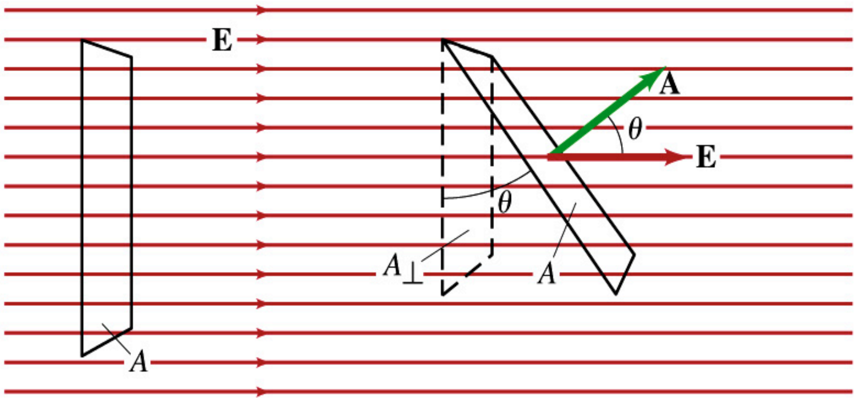
# Electric Flux




- Let's imagine a surface of area A through which a uniform electric field E passes
- The electric flux $\Phi_E$ is defined as
  - $\Phi_E$=EA, if the field is perpendicular to the surface
  - $\Phi_E$=EAcosθ, if the field makes an angle θ to the surface
- So the electric flux is defined as $\Phi_E = \vec{E} \cdot \vec{A}$.
- How would you define the electric flux in words?
  - The total number of field lines passing through the unit area perpendicular to the field. $N_E \propto EA_{\perp} = \Phi_E$

# Example 22 – 1

- **Electric flux**. (a) Calculate the electric flux through the rectangle in the figure (a). The rectangle is 10cm by 20cm and the electric field is uniform with magnitude 200N/C. (b) What is the flux in figure if the angle is 30 degrees?



(a) (b)

The electric flux is defined as

$$\Phi_E = \vec{E} \cdot \vec{A} = EA\cos\theta$$

So when (a) θ=0, we obtain

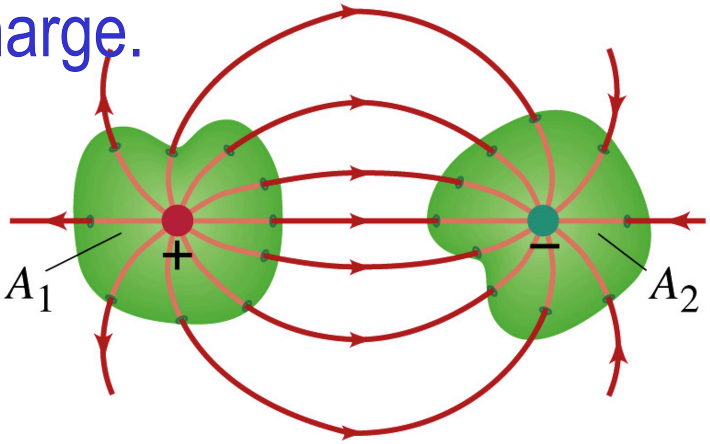
$$\Phi_E = EA\cos\theta = EA = (200N/C)\cdot\left(0.1\times 0.2m^2\right) = 4.0\,\mathrm{N}\cdot\mathrm{m}^2/C$$

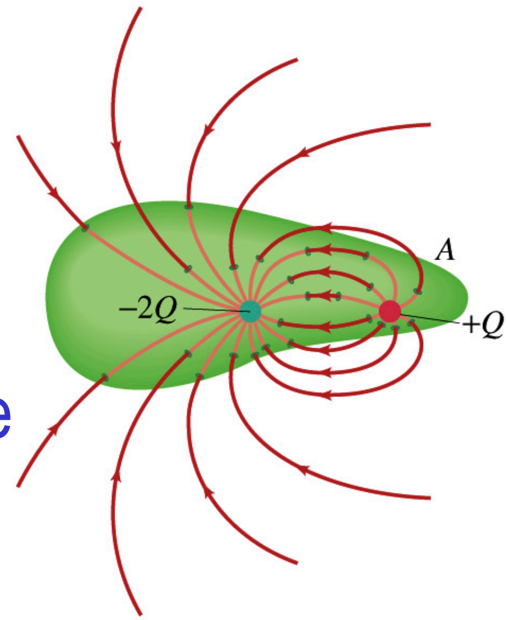
And when (b) θ=30 degrees, we obtain

$$\Phi_E = EA\cos 30^\circ = (200N/C)\cdot\left(0.1\times 0.2m^2\right)\cos 30^\circ = 3.5\,\mathrm{N}\cdot\mathrm{m}^2/C$$

# Generalization of the Electric Flux

- The field line starts or ends only on a charge.
- Sign of the net flux on the surface $A_1$?
  - The net outward flux (positive flux)
- How about $A_2$?
  - Net inward flux (negative flux)
- What is the flux in the bottom figure?
  - There should be a net inward flux (negative flux) since the total charge inside the volume is negative.
- The net flux that crosses an enclosed surface is proportional to the total charge inside the surface. ➔ This is the crux of Gauss' law.

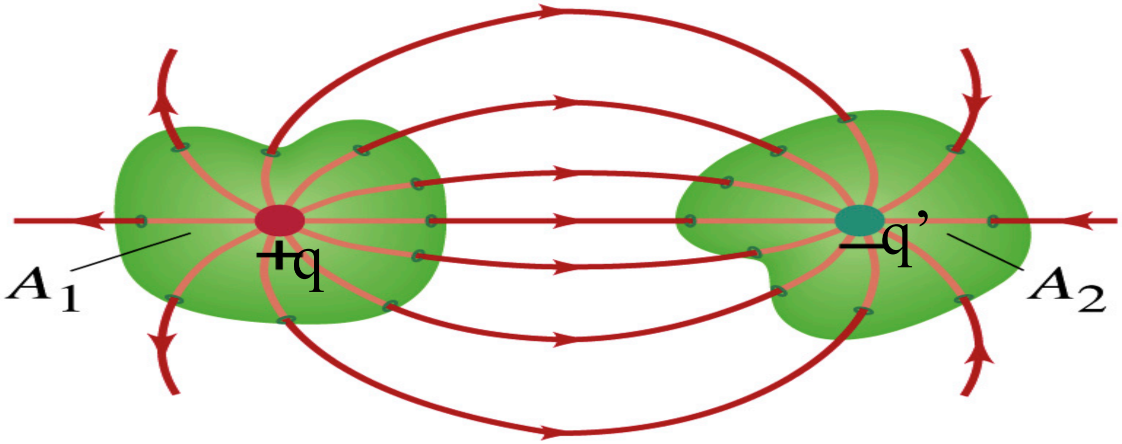


# Gauss' Law

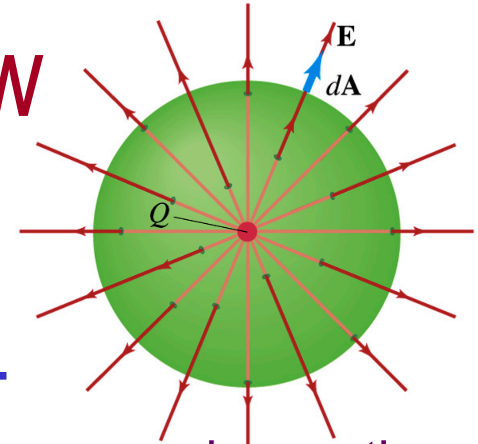
- The precise relationship between flux and the enclosed charge is given by Gauss' Law $$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_0}$$
    - $\varepsilon_0$ is the permittivity of free space in the Coulomb's law
- A few important points on Gauss' Law
  - Freedom to choose!!
    - The integral is performed over the value of **E** on a closed surface of our choice in any given situation.
  - Test of the existence of the electrical charge!!
    - The charge $Q_{encl}$ is the net charge enclosed by the arbitrary closed surface of our choice.
  - This law is universal!
    - It does NOT matter where or how much charge is distributed inside the surface.
  - The charge outside the surface does not contribute to $Q_{encl}$. Why?
    - The charge outside the surface might impact field lines but not the total number of lines entering or leaving the surface

# Gauss' Law




- Let's consider the case in the above figure.
- What are the results of the closed integral of the Gaussian surfaces $A_1$ and $A_2$?
  - For $A_1$ $\oint \vec{E} \cdot d\vec{A} = \frac{+q}{\varepsilon_0}$
  - For $A_2$ $\oint \vec{E} \cdot d\vec{A} = \frac{-q'}{\varepsilon_0}$

# Coulomb's Law from Gauss' Law




- Let's consider a charge Q enclosed inside our imaginary gaussian surface of sphere of radius r.
  - Since we can choose any surface enclosing the charge, we choose the simplest possible one! ☺
- The surface is symmetric about the charge.
  - What does this tell us about the field E?
    - Must have the same magnitude (uniform) at any point on the surface
    - Points radially outward / inward parallel to the surface vector d**A**.
- The gaussian integral can be written as

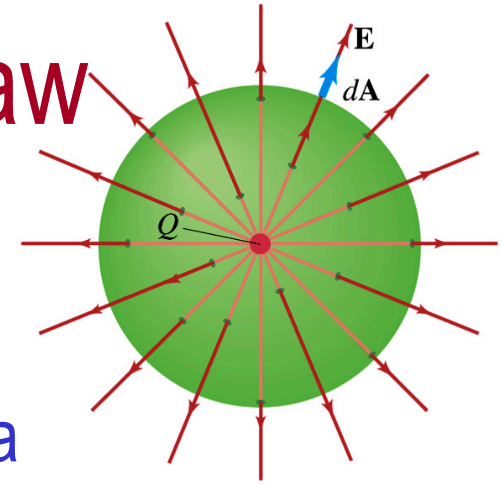
$$\oint \vec{E} \cdot d\vec{A} = \oint E dA = E \oint dA = E\left(4\pi r^2\right) = \frac{Q_{encl}}{\varepsilon_0} = \frac{Q}{\varepsilon_0}$$

**Solve for E**

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

**Electric Field of Coulomb's Law**

# Gauss' Law from Coulomb's Law




- Let's consider a single static point charge Q surrounded by an imaginary spherical surface.
- Coulomb's law tells us that the electric field at a spherical surface is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

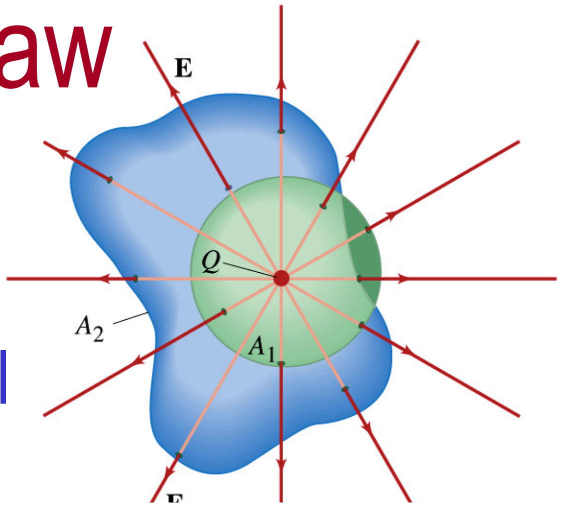
- Performing a closed integral over the surface, we obtain

$$\oint \vec{E}\cdot d\vec{A} = \oint \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}\cdot d\vec{A} = \oint \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}dA$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\oint dA = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\left(4\pi r^2\right) = \frac{Q}{\varepsilon_0}$$

**Gauss' Law**

# Gauss' Law from Coulomb's Law Irregular Surface




- Let's consider the same single static point charge Q surrounded by a symmetric spherical surface $A_1$ and a randomly shaped surface $A_2$.
- What is the difference in the number of field lines due to the charge Q, passing through the two surfaces?
  - None. What does this mean?
    - The total number of field lines passing through the surface is the same no matter what the shape of the enclosed surface is.
  - So we can write: $\oint_{A_1} \vec{E} \cdot d\vec{A} = \oint_{A_2} \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0}$
  - What does this mean?
    - The flux due to the given enclosed charge is the same no matter what the shape of the surface enclosing it is. ➔ Gauss' law, $\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0}$, is valid for any surface surrounding a single point charge Q. ➔ Freedom to choose!

# Gauss' Law w/ more than one charge

- Let's consider several charges inside a closed surface.
- For each charge, $Q_i$, inside the chosen closed surface,

$$\oint \vec{E}_i \cdot d\vec{A} = \frac{Q_i}{\varepsilon_0}$$

**What is $\vec{E}_i$?**

**The electric field produced by $Q_i$ alone!**

- Since electric fields can be added vectorially, following the superposition principle, the total field **E** is equal to the sum of the fields due to each charge $\vec{E} = \sum \vec{E}_i$ plus any external field. So

$$\oint \vec{E} \cdot d\vec{A} = \oint \left(\vec{E}_{ext} + \sum \vec{E}_i\right) \cdot d\vec{A} = \frac{\sum Q_i}{\varepsilon_0} = \frac{Q_{encl}}{\varepsilon_0}$$

**What is $Q_{encl}$?**

**The total enclosed charge!**

- The value of the flux depends on the charge enclosed in the surface!! ➔ Gauss' law.

# So what is Gauss' Law good for?

- Derivation of Gauss' law from Coulomb's law is only valid for **static electric charge**.
- Electric field can also be produced by changing magnetic fields.
  - Coulomb's law cannot describe this field while Gauss' law is still valid ➔ can describe electric field in this situation also!
- Gauss' law is more general than Coulomb's law.
  - Can be used to obtain electric field, force or charges

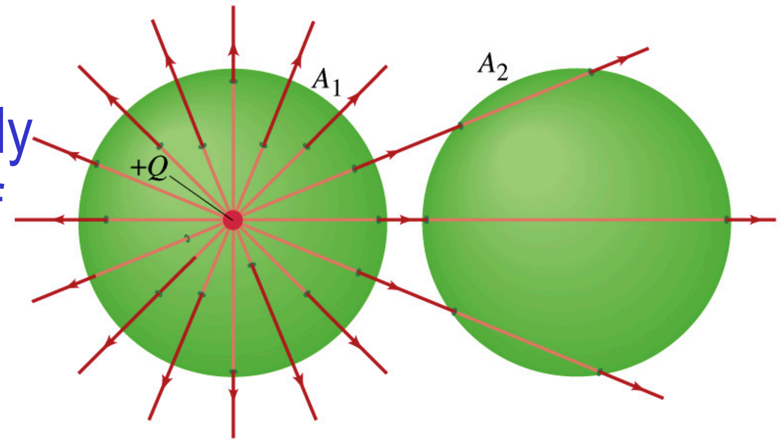
Gauss' Law: Any **differences** between the input and output flux of the electric field over any enclosed surface is due to the charge within that surface!!!

# Solving problems with Gauss' Law

- Identify the symmetry of the charge distributions
- Draw the appropriate gaussian surface, making sure it passes through the point you want to know the electric field
- Use the symmetry of charge distribution to determine the direction of E at the point of gaussian surface
- Evaluate the flux
- Calculate the charge enclosed by the gaussian surface
  - Ignore all the charges outside the gaussian surface
- Equate the flux to the enclosed charge and solve for E

# Example 22 – 2

**Flux from Gauss' Law**: Consider two gaussian surfaces, $A_1$ and $A_2$, shown in the figure. The only charge present is the charge +Q at the center of surface $A_1$. What is the net flux through each surface $A_1$ and $A_2$?




- The surface $A_1$ encloses the charge +Q, so from Gauss' law we obtain the total net flux

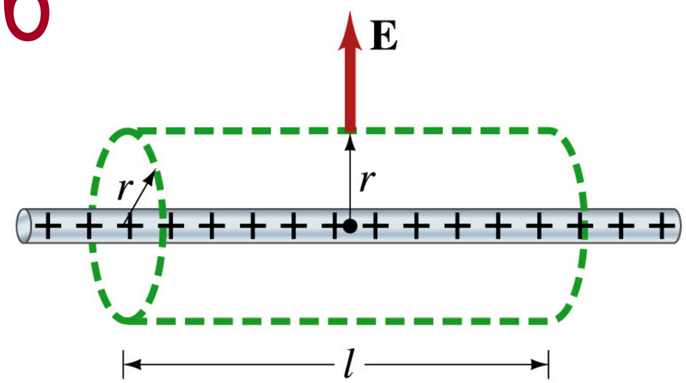
$$\oint \vec{E} \cdot d\vec{A} = \frac{+Q}{\varepsilon_0}$$

- The surface $A_2$ the charge, +Q, is outside the surface, so the total net flux is 0.

$$\oint \vec{E} \cdot d\vec{A} = \frac{0}{\varepsilon_0} = 0$$

# Example 22 – 6

**Long uniform line of charge**: A very long straight wire possesses a uniform positive charge per unit length, $\lambda$. Calculate the electric field at the points near but outside the wire, far from the ends.




- Which direction do you think the field due to the charge on the wire is?
  - Radially outward from the wire, the direction of radial vector **r**.
- Due to cylindrical symmetry, the field is the same on the gaussian surface of a cylinder surrounding the wire.
  - The end surfaces do not contribute to the flux at all. Why?
    - Because the field vector **E** is perpendicular to the surface vector d**A**.
- From Gauss' law

$$\oint \vec{E} \cdot d\vec{A} = E \oint dA = E\left(2\pi r l\right) = \frac{Q_{encl}}{\varepsilon_0} = \frac{\lambda l}{\varepsilon_0}$$

Solving for E

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$