PHYS 1444 – Section 004
Lecture #8

Monday, Feb. 13, 2012
Dr. Jae Yu

- Chapter Chapter 23 Electric Potential
  - E Determined from V
  - Electrostatic Potential
- Chapter 24 Capacitance etc..
  - Capacitors
  - Determination of Capacitance
  - Capacitors in Series or Parallel

Today's homework is homework #5, due 10pm, Tuesday, Feb. 21!!

Announcements

- First term exam
  - 5:30 – 6:50pm, Wednesday, Feb. 22
  - SH103
  - CH21.1 through what we learn on Monday, Feb. 20 plus appendices A and B
- Reading assignments
  - CH23.9
- Colloquium this week
  - 4pm Wednesday, SH101
  - Dr. Haiying Huang, UTA MAE
  - Mark your calendar on triple credit colloquium on April 4.
    - Dr. Youngkee Kim

Physics Department
The University of Texas at Arlington
COLLOQUIUM

Microwave patch antenna for battery-less wireless sensing

Dr. Haiying Huang

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4:00 pm Wednesday February 15, 2012 room 101 SH

Abstract:

Sensor technologies are the foundation of all "smart" technologies, e.g. mobile health, robotics, smart grid, environmental monitoring, structural health monitoring, etc. A major challenge for sensor research is to achieve densely distributed, wireless, low power consumption sensor networks. This talk presents our study of microwave patch antenna sensors to address this challenge. We discovered that a microstrip patch antenna can be designed to sense various physical parameters, such as strain, pressure, shear, and crack etc. Because the patch antennas serves the dual function of sensing and data transmission, battery-less wireless interrogation of the antenna sensors can be achieved. In addition, frequency-division multiplexing can be exploited to simultaneously interrogate a multi-element sensor array. These unique characteristics make microwave antenna sensor an attractive candidate for densely distributed wireless sensor networks. The operating principles of the antenna sensors will be explained firs, followed by the discussions of two wireless interrogation schemes. The applications of the wireless sensors for strain and crack monitoring will be presented.

Refreshments will be served at 3:30p.m in the Physics Library

E Determined from V

- Potential difference between two points under the electric field is $V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{l}$
- So in the differential form, we can write

$$dV = -\vec{E} \cdot d\vec{l} = -E_l dl$$

  - What are dV and E_ℓ?
    - dV is the infinitesimal potential difference between the two points separated by the distance $d\ell$
    - E_ℓ is the field component along the direction of $d\ell$.
- Thus we can write the field component E_ℓ as

$$E_l = -\frac{dV}{dl}$$

Physical Meaning?

The component of the electric field in any direction is equal to the negative rate of change of the electric potential as a function of distance in that direction.!!

E Determined from V

- The quantity dV/dℓ is called the **gradient of V** in a particular direction
  - If no direction is specified, the term gradient refers to the direction on which V changes most rapidly and this would be the direction of the field vector **E** at that point.
  - So if **E** and dℓ are parallel to each other, $E = -\frac{dV}{dl}$
- If E is written as a function x, y and z, then ℓ refers to x, y and z $E_x = -\frac{\partial V}{\partial x}$ $E_y = -\frac{\partial V}{\partial y}$ $E_z = -\frac{\partial V}{\partial z}$
- $\frac{\partial V}{\partial x}$ is the "partial derivative" of V with respect to x, with y and z held constant
- In vector form, $\vec{E} = -grad V = -\vec{\nabla} V = -\left(\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}\right)V$

$\vec{\nabla} = -\left(\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}\right)$ is called the *del* or the *gradient operator* and is a *vector operator*.

Electrostatic Potential Energy

- Consider a case in which a point charge q is moved between points *a* and *b* where the electrostatic potential due to other charges in the system is V_a and V_b
- The change in electrostatic potential energy of q in the field by other charges is

$$\Delta U = U_b - U_a = q\left(V_b - V_a\right) = qV_{ba}$$

- Now what is the electrostatic potential energy of a system of charges?
  - Let's choose V=0 at r=∞
  - If there are no other charges around, single point charge Q_1 in isolation has no potential energy and is exerted on with no electric force

Electrostatic Potential Energy; Two charges

- If a second point charge Q_2 is brought close to Q_1 at the distance r_{12}, the potential due to Q_1 at the position of Q_2 is

$$V = \frac{Q_1}{4\pi\varepsilon_0}\frac{1}{r_{12}}$$

- The potential energy of the two charges relative to V=0 at r= ∞ is

$$U = Q_2 V = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_2}{r_{12}}$$

  - This is the work that needs to be done by an external force to bring Q_2 from infinity to a distance r_{12} from Q_1.
  - It is also the negative of the work needed to separate them to infinity.

Electrostatic Potential Energy; Three Charges

- So what do we do for three charges?
- Work is needed to bring all three charges together
  - Work needed to bring Q_1 to a certain location without the presence of any charge is 0.
  - Work needed to bring Q_2 to a distance to Q_1 is $U_{12} = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_2}{r_{12}}$
  - Work need to bring Q_3 to a distance to Q_1 and Q_2 is

$$U_3 = U_{13} + U_{23} = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_3}{r_{13}} + \frac{1}{4\pi\varepsilon_0}\frac{Q_2 Q_3}{r_{23}}$$

- So the total electrostatic potential of a three charge system is

$$U = U_{12} + U_{13} + U_{23} = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_1 Q_2}{r_{12}} + \frac{Q_1 Q_3}{r_{13}} + \frac{Q_2 Q_3}{r_{23}}\right) \quad [V = 0 \text{ at } r = \infty]$$

  - What about a four charge system or N charge system?

Electrostatic Potential Energy: electron Volt

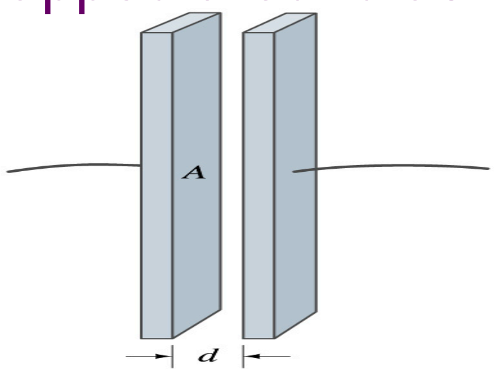
- What is the unit of the electrostatic potential energy?
  - Joules
- Joules is a very large unit in atomic scale problems, dealing with electrons, atoms or molecules
- For convenience, a new unit, electron volt (eV), is defined
  - 1 eV is defined as the energy acquired by a particle carrying the charge equal to that of an electron (q=e) when it moves across a potential difference of 1V.
  - How many Joules is 1 eV then? $1eV = 1.6\times10^{-19}C\cdot1V = 1.6\times10^{-19}J$
- eV however is **NOT a standard SI unit**. You must convert the energy to Joules for computations.
- What is the speed of an electron with kinetic energy 5000eV?

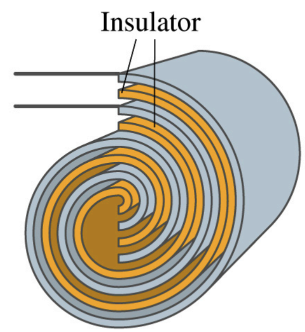
Capacitors (or Condensers)

- What is a capacitor?
  - A device that can store electric charge
  - But does not let them flow through
- What does it consist of?
  - Usually consists of two conducting objects (plates or sheets) placed near each other without touching
  - Why can't they touch each other?
    - The charge will neutralize…
- Can you give some examples?
  - Camera flash, UPS, Surge protectors, binary circuits, memory, etc…
- How is a capacitor different than a battery?
  - Battery provides potential difference by storing energy (usually chemical energy) while the capacitor stores charges but very little energy.

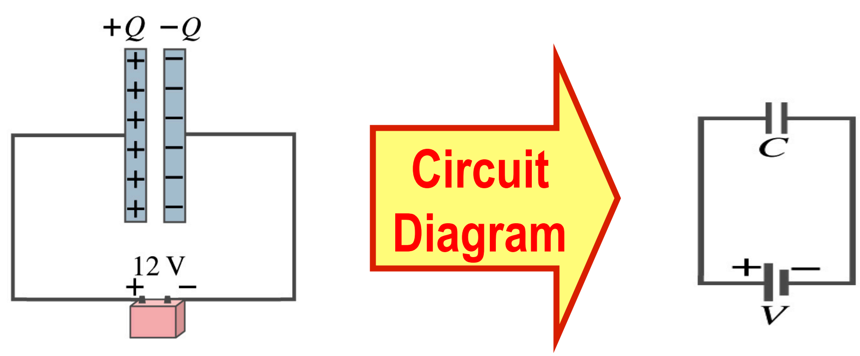
Capacitors

- A simple capacitor consists of a pair of parallel plates of area $\mathcal{A}$ separated by a distance d.
  - A cylindrical capacitors are essentially parallel plates wrapped around as a cylinder.



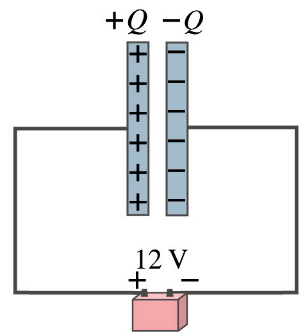


- How would you draw symbols for a capacitor and a battery?
  - Capacitor -||-
  - Battery (+) -|i- (-)



Capacitors

- What do you think will happen if a battery is connected (or the voltage is applied) to a capacitor?
  - The capacitor gets charged quickly, one plate positive and the other negative in equal amount.



(b)

- Each battery terminal, the wires and the plates are conductors. What does this mean?
  - All conductors are at the same potential. And?
  - So the full battery voltage is applied across the capacitor plates.
- So for a given capacitor, the amount of charge stored on each capacitor plate is proportional to the potential difference V_{ba} between the plates. How would you write this formula?

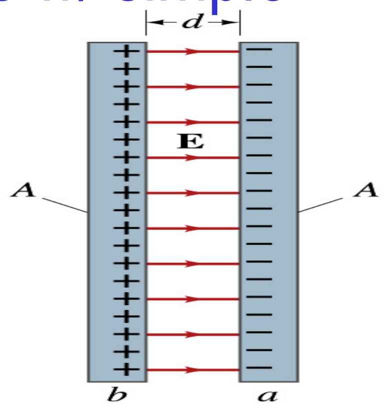
$$Q = CV_{ba}$$

C is the property of a capacitor so does not depend on Q or V.

  - C is a proportionality constant, called capacitance of the device.
  - What is the unit? C/V or Farad (F) Normally use μF or pF.

Determination of Capacitance

- C can be determined analytically for capacitors w/ simple geometry and air in between.



- Let's consider a parallel plate capacitor.
  - Plates have area A each and separated by d.
    - d is smaller than the length, and so E is uniform.
  - E for parallel plates is $E=\sigma/\varepsilon_0$, σ is the surface charge density.
- E and V are related $V_{ba}=-\int_a^b \vec{E}\cdot d\vec{l}$
- Since we take the integral from lower potential (a) to higher potential (b) along the field line, we obtain
- $V_{ba}=V_b-V_a=-\int_a^b Edl\cos 180^\circ=+\int_a^b Edl=\int_a^b \frac{\sigma}{\varepsilon_0}dl=\int_a^b \frac{Q}{\varepsilon_0 A}dl=\frac{Q}{\varepsilon_0 A}\int_a^b dl=\frac{Q}{\varepsilon_0 A}(b-a)=\frac{Qd}{\varepsilon_0 A}$
- So from the formula: $C=\frac{Q}{V_{ba}}=\frac{Q}{Qd/\varepsilon_0 A}=\frac{\varepsilon_0 A}{d}$
  - What do you notice?

C only depends on the area and the distance of the plates and the permittivity of the medium between them.

Example 24 – 1

Capacitor calculations: (a) Calculate the capacitance of a capacitor whose plates are 20cmx3.0cm and are separated by a 1.0mm air gap. (b) What is the charge on each plate if the capacitor is connected to a 12-V battery? (c) What is the electric field between the plates? (d) Estimate the area of the plates needed to achieve a capacitance of 1F, given the same air gap.

(a) Using the formula for a parallel plate capacitor, we obtain

$$C = \frac{\varepsilon_0 A}{d} =$$

$$= \left(8.85\times10^{-12}\, C^2/N\cdot m^2\right)\frac{0.2\times0.03m^2}{1\times10^{-3}m} = 53\times10^{-12}\, C^2/N\cdot m = 53pF$$

(b) From Q=CV, the charge on each plate is

$$Q = CV = \left(53\times10^{-12}\, C^2/N\cdot m\right)\left(12V\right) = 6.4\times10^{-10}\, C = 640pC$$

Example 24 – 1

(C) Using the formula for the electric field in two parallel plates

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0} = \frac{6.4\times10^{-10}\,C}{6.0\times10^{-3}\,m^2 \times 8.85\times10^{-12}\,C^2/N\cdot m^2} = 1.2\times10^4\,N/C = 1.2\times10^4\,V/m$$

Or, since $V = Ed$ we can obtain $E = \frac{V}{d} = \frac{12V}{1.0\times10^{-3}\,m} = 1.2\times10^4\,V/m$

(d) Solving the capacitance formula for A, we obtain

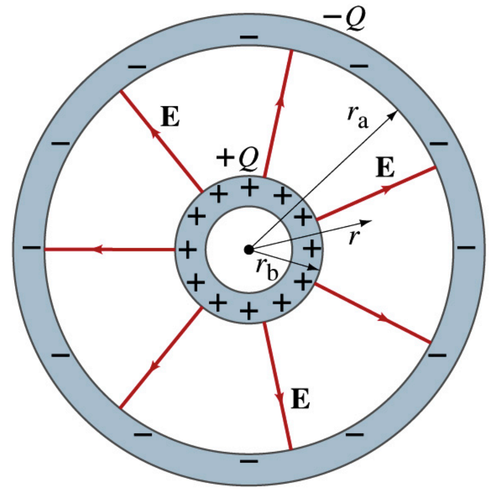
$$C = \frac{\varepsilon_0 A}{d}$$

Solve for A

$$A = \frac{Cd}{\varepsilon_0} = \frac{1F\cdot 1\times10^{-3}\,m}{\left(9\times10^{-12}\,C^2/N\cdot m^2\right)} \approx 10^8\,m^2 \approx 100km^2$$

About 40% the area of Arlington ($256km^2$).

Example 24 – 3



Spherical capacitor: A spherical capacitor consists of two thin concentric spherical conducting shells, of radius r_a and r_b, as in the figure. The inner shell carries a uniformly distributed charge Q on its surface and the outer shell and equal but opposite charge –Q. Determine the capacitance of the two shells.

Using Gauss' law, the electric field outside a uniformly charged conducting sphere is

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

So the potential difference between a and b is

$$V_{ba} = -\int_a^b \vec{E}\cdot d\vec{l} =$$

$$= -\int_a^b E\cdot dr = -\int_a^b \frac{Q}{4\pi\varepsilon_0 r^2}dr = -\frac{Q}{4\pi\varepsilon_0}\int_a^b \frac{dr}{r^2} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r}\right)_{r_a}^{r_b} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_b}-\frac{1}{r_a}\right) = \frac{Q}{4\pi\varepsilon_0}\left(\frac{r_a - r_b}{r_b r_a}\right)$$

Thus capacitance is

$$C = \frac{Q}{V} = \frac{Q}{\frac{Q}{4\pi\varepsilon_0}\left(\frac{r_a - r_b}{r_b r_a}\right)} = \frac{4\pi\varepsilon_0 r_b r_a}{r_a - r_b}$$

Capacitor Cont'd

- A single isolated conductor can be said to have a capacitance, C.
- C can still be defined as the ratio of the charge to absolute potential V on the conductor.
  - So Q=CV.
- The potential of a single conducting sphere of radius r_b can be obtained as

$$V = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_b} - \frac{1}{r_a}\right) = \frac{Q}{4\pi\varepsilon_0 r_b} \quad \text{where} \quad r_a \to \infty$$

- So its capacitance is $C = \frac{Q}{V} = 4\pi\varepsilon_0 r_b$