

PHYS 1444 – Section 004

Lecture #9

Wednesday, Feb. 15, 2012

Dr. Jae Yu

- Capacitors in Series or Parallel
- Electric Energy Storage
- Effect of Dielectric
- Molecular description of Dielectric Material



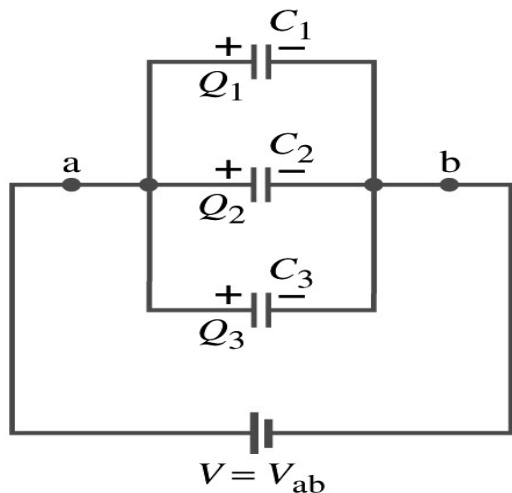
Announcements

- First term exam
 - 5:30 – 6:50pm, Wednesday, Feb. 22
 - SH103
 - CH21.1 through CH24.6 plus appendices A and B
- Quiz results:
 - Class average: 46.7/70
 - Equivalent to 66.7/100
 - Previous quiz: 37.6/100
 - Top score:
- Reading assignment
 - CH24.6



Capacitors in Series or Parallel

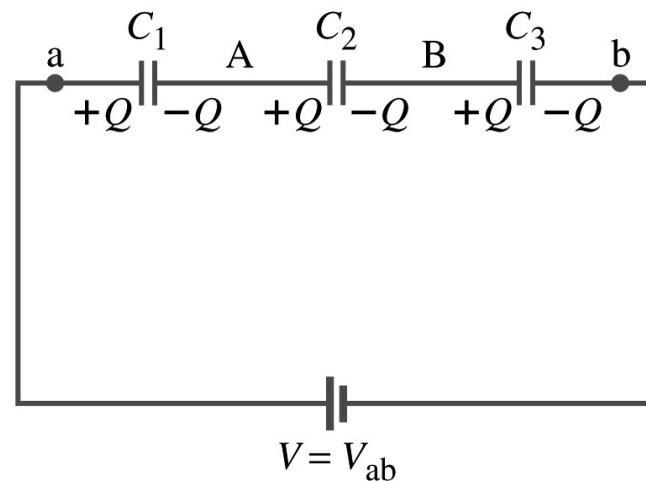
- Capacitors are used in many electric circuits
- What is an electric circuit?
 - A closed path of conductors, usually wires connecting capacitors and other electrical devices, in which
 - charges can flow
 - And includes a voltage source such as a battery
- Capacitors can be connected in various ways.
 - In parallel, in series or in combination



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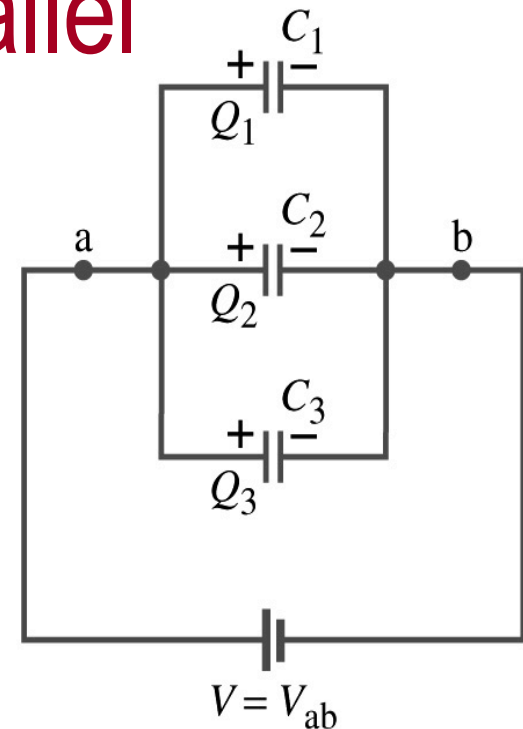


Capacitors in Parallel

- Parallel arrangement provides the same voltage across all the capacitors.

- All left hand plates are at V_a and all right hand plates are at V_b
- So each capacitor plate acquires charges given by the formula

- $Q_1 = C_1 V$, $Q_2 = C_2 V$, and $Q_3 = C_3 V$



- The total charge Q that must leave the battery is then
 - $Q = Q_1 + Q_2 + Q_3 = V(C_1 + C_2 + C_3)$
- Consider that the three capacitors behave like an equivalent one
 - $Q = C_{eq} V = V(C_1 + C_2 + C_3)$
- Thus the equivalent capacitance in parallel is $C_{eq} = C_1 + C_2 + C_3$

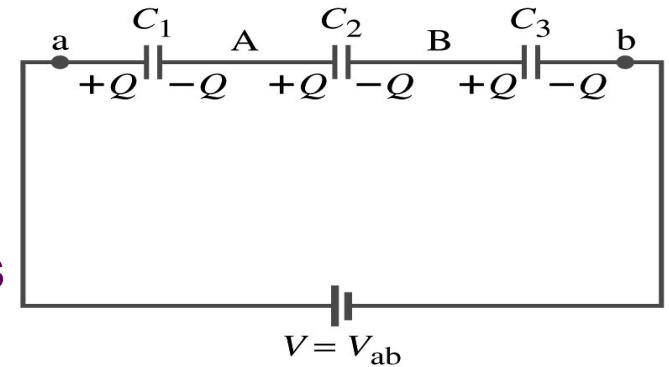
W
20 What is the net effect?

HY The capacitance increases!!!

Capacitors in Series

- Series arrangement is more interesting

- When battery is connected, $+Q$ flows to the left plate of C_1 and $-Q$ flows to the right plate of C_3 .
- Since the in between were originally neutral, charges get induced to neutralize the ones in the middle.



- So the charge on each capacitor plate is the same value, Q . (**Same charge**)

- Consider that the three capacitors behave like an equivalent one

- $Q = C_{eq} V$

- The total voltage V across the three capacitors in series must be equal to the sum of the voltages across each capacitor.

- $V = V_1 + V_2 + V_3 = Q/C_1 + Q/C_2 + Q/C_3$

- Putting all these together, we obtain:

- $V = Q/C_{eq} = Q(1/C_1 + 1/C_2 + 1/C_3)$

- Thus the equivalent capacitance is

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

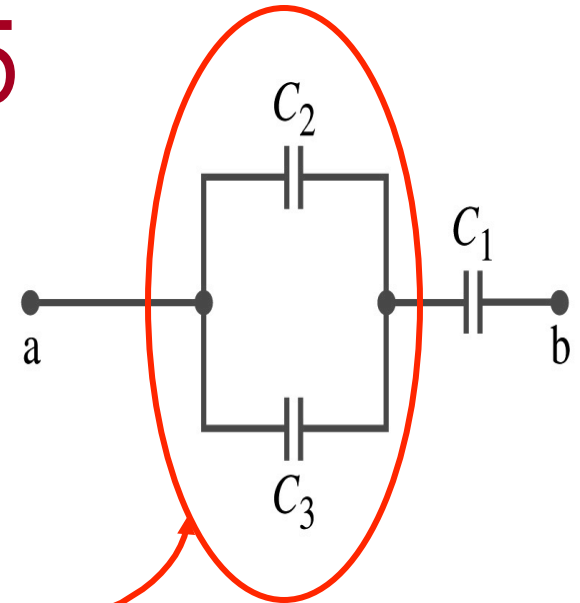
What is the net effect?



The capacitance smaller than the smallest C !!!

Example 24 – 5

Equivalent Capacitor: Determine the capacitance of a single capacitor that will have the same effect as the combination shown in the figure. Take $C_1 = C_2 = C_3 = C$.



We should do these first!!

How? These are in parallel so the equivalent capacitance is:

$$C_{eq1} = C_1 + C_2 = 2C$$

Now the equivalent capacitor is in series with C1.

$$\frac{1}{C_{eq}} = \frac{1}{C_{eq1}} + \frac{1}{C_2} = \frac{1}{2C} + \frac{1}{C} = \frac{3}{2C} \quad \text{Solve for } C_{eq} \quad C_{eq} = \frac{2C}{3}$$

Electric Energy Storage

- A charged capacitor stores energy.
 - The stored energy is the amount of the work done to charge it.
- The net effect of charging a capacitor is removing one type of charge from a plate and put them on to the other.
 - Battery does this when it is connected to a capacitor.
- Capacitors do not get charged immediately.
 - Initially when the capacitor is uncharged, no work is necessary to move the first bit of charge. Why?
 - Since there is no charge, there is no field that the external work needs to overcome.
 - When some charge is on each plate, it requires work to add more charge of the same sign due to the electric repulsion.



Electric Energy Storage

- The work needed to add a small amount of charge, dq , when a potential difference across the plate is V : $dW=Vdq$.
- Since $V=q/C$, the work needed to store total charge Q is

$$W = \int_0^Q V dq = \frac{1}{C} \int_0^Q q dq = \frac{Q^2}{2C}$$

- Thus, the energy stored in a capacitor when the capacitor carries charges $+Q$ and $-Q$ is

$$U = \frac{Q^2}{2C}$$

- Since $Q=CV$, we can rewrite

$$U = \frac{Q^2}{2C} = \frac{1}{2} CV^2 = \frac{1}{2} QV$$



Example 24 – 8

Energy store in a capacitor: A camera flash unit stores energy in a $150\mu\text{F}$ capacitor at 200V . How much electric energy can be stored?

Using the formula for stored energy. Umm.. Which one?

What do we know from the problem? C and V

So we use the one with C and V: $U = \frac{1}{2}CV^2$

$$U = \frac{1}{2}CV^2 = \frac{1}{2}(150 \times 10^{-6} \text{ F})(200 \text{ V})^2 = 3.0 \text{ J}$$

How do we get J from FV^2 ? $FV^2 = \left(\frac{C}{V}\right)V^2 = CV = C\left(\frac{J}{C}\right) = J$



Electric Energy Density

- The energy stored in a capacitor can be considered as being stored in the electric field between the two plates
- For a uniform field E between two plates, $V=Ed$ and $C=\epsilon_0 A/d$
- Thus the stored energy is

$$U = \frac{1}{2} CV^2 = \frac{1}{2} \left(\frac{\epsilon_0 A}{d} \right) (Ed)^2 = \frac{1}{2} \epsilon_0 E^2 \textcircled{Ad}$$

- Since Ad is the gap volume V , we can obtain the energy density, stored energy per unit volume, as

$$u = \frac{1}{2} \epsilon_0 E^2$$

**Valid for any space
that is vacuum**

Electric energy stored per unit volume in any region of space is proportional to the square of E in that region.

Dielectrics

- Capacitors have an insulating sheet of material, called dielectric, between the plates to
 - Increase breakdown voltage than that in the air
 - Higher voltage can be applied without the charge passing across the gap
 - Allow the plates get closer together without touching
 - Increases capacitance (recall $C = \epsilon_0 A/d$)
 - Also increases the capacitance by the dielectric constant
$$C = K C_0$$
 - Where C_0 is the intrinsic capacitance when the gap is vacuum



Dielectrics

- The value of dielectric constant varies depending on material (Table 24 – 1)
 - K for vacuum is 1.0000
 - K for air is 1.0006 (this is why the permittivity of air and vacuum are used interchangeably.)
- Maximum electric field before breakdown occurs is called the dielectric strength. What is its unit?
 - V/m
- The capacitance of a parallel plate capacitor with a dielectric (K) filling the gap is

$$C = K C_0 = K \epsilon_0 \frac{A}{d}$$



Dielectrics

- A new quantity of the permittivity of dielectric is defined as $\epsilon = K\epsilon_0$
- The capacitance of a parallel plate capacitor with a dielectric medium filling the gap is

$$C = \epsilon \frac{A}{d}$$

- The energy density stored in an electric field E in a dielectric is

$$u = \frac{1}{2} K \epsilon_0 E^2 = \frac{1}{2} \epsilon E^2$$

Effect of a Dielectric Material

- Let's consider the two cases below:

Case #1 :
constant V

$$\begin{array}{ccc}
 V_0 \text{ --- } \boxed{\text{no dielectric}} \text{ --- } & +Q_0 & -Q_0 \\
 & C_0 = \frac{Q_0}{V_0} & \longrightarrow \\
 & V_0 \text{ --- } \boxed{\text{with dielectric}} \text{ --- } & +Q & -Q \\
 & & = +KQ_0 & = -KQ_0 \\
 & & C = \frac{Q}{V_0} = KC_0 &
 \end{array}$$

(a) Voltage constant

Case #2 :
constant Q

$$\begin{array}{ccc}
 V_0 \text{ --- } \boxed{\text{no dielectric}} \text{ --- } & +Q_0 & -Q_0 \\
 & C_0 = \frac{Q_0}{V_0} & \longrightarrow \\
 & \begin{array}{c} +Q_0 \\ \text{---} \\ -Q_0 \end{array} V_0, C_0 = \frac{Q_0}{V_0} & \longrightarrow \\
 & \text{battery disconnected} & \\
 & \begin{array}{c} +Q_0 \\ \text{---} \\ -Q_0 \end{array} V = \frac{V_0}{K} & \\
 & \text{dielectric inserted} & C = KC_0
 \end{array}$$

(b) Charge constant

- Constant voltage: Experimentally observed that the total charge on the each plates of the capacitor increases by K as the dielectric material is inserted between the gap $\rightarrow Q=KQ_0$
 - The capacitance increased to $C=Q/V_0=KQ_0/V_0=KC_0$
- Constant charge: Voltage found to drop by a factor K $\rightarrow V=V_0/K$
 - The capacitance increased to $C=Q_0/V=KQ_0/V_0=KC_0$

Effect of a Dielectric Material on Field

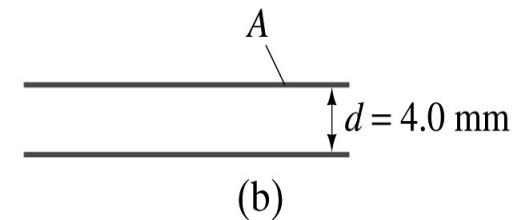
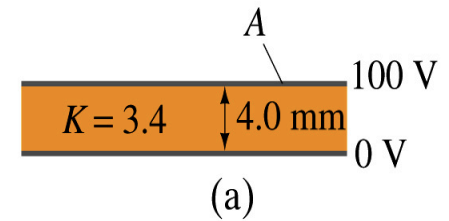
- What happens to the electric field within a dielectric?
- Without a dielectric, the field is $E_0 = \frac{V_0}{d}$
 - What are V_0 and d ?
 - V_0 : Potential difference between the two plates
 - d : separation between the two plates
- For the constant voltage, the electric field remains the same
- For the constant charge: the voltage drops to $V=V_0/K$, thus the field in the dielectric is
 - The field in the dielectric is reduced.

$$E = E_D = \frac{V}{d} = \frac{V_0}{dK} = \frac{E_0}{K}$$

$$E_D = \frac{E_0}{K}$$

Example 24 – 11

Dielectric Removal: A parallel-plate capacitor, filled with a dielectric of $K=3.4$, is connected to a 100-V battery. After the capacitor is fully charged, the battery is disconnected. The plates have area $A=4.0\text{m}^2$, and are separated by $d=4.0\text{mm}$. (a) Find the capacitance, the charge on the capacitor, the electric field strength, and the energy stored in the capacitor. (b) The dielectric is carefully removed, without changing the plate separation nor does any charge leave the capacitor. Find the new value of capacitance, electric field strength, voltage between the plates and the energy stored in the capacitor.



$$(a) \quad C = \frac{\epsilon A}{d} = \frac{K\epsilon_0 A}{d} = \left(3.4 \times 8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2\right) \frac{4.0\text{m}^2}{4.0 \times 10^{-3} \text{ m}} = 3.0 \times 10^{-8} \text{ F} = 30\text{nF}$$

$$Q = CV = (3.0 \times 10^{-8} \text{ F}) \times 100\text{V} = 3.0 \times 10^{-6} \text{ C} = 3.0\mu\text{C}$$

$$E = \frac{V}{d} = \frac{100\text{V}}{4.0 \times 10^{-3} \text{ m}} = 2.5 \times 10^4 \text{ V/m}$$

$$U = \frac{1}{2} CV^2 = \frac{1}{2} (3.0 \times 10^{-8} \text{ F}) (100\text{V})^2 = 1.5 \times 10^{-4} \text{ J}$$

Example 24 – 11 cont'd

- (b) Since the dielectric has been removed, the effect of dielectric constant must be removed as well.

$$C_0 = \frac{C}{K} = \left(8.85 \times 10^{-12} \text{ C}^2 / \text{N} \cdot \text{m}^2\right) \frac{4.0 \text{ m}^2}{4.0 \times 10^{-3} \text{ m}} = 8.8 \times 10^{-9} \text{ F} = 8.8 \text{ nF}$$

Since charge is the same ($Q_0 = Q$) before and after the removal of the dielectric, we obtain

$$V_0 = Q/C_0 = K Q/C = K V = 3.4 \times 100 \text{ V} = 340 \text{ V}$$

$$E_0 = \frac{V_0}{d} = \frac{340 \text{ V}}{4.0 \times 10^{-3} \text{ m}} = 8.5 \times 10^4 \text{ V/m} = 84 \text{ kV/m}$$

$$U_0 = \frac{1}{2} C_0 V_0^2 = \frac{1}{2} \frac{C}{K} (K V)^2 = \frac{1}{2} K C V^2 = K U = 3.4 \times 1.5 \times 10^{-4} \text{ J} = 5.1 \times 10^{-4} \text{ J}$$

Where did the extra energy come from?

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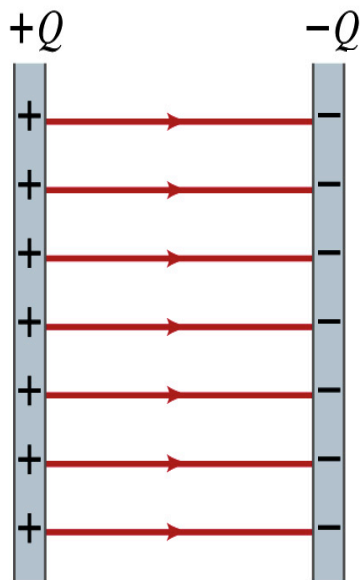
~~The energy conservation law is violated in electrostatics??~~

Wrong!
Wrong!
Wrong!

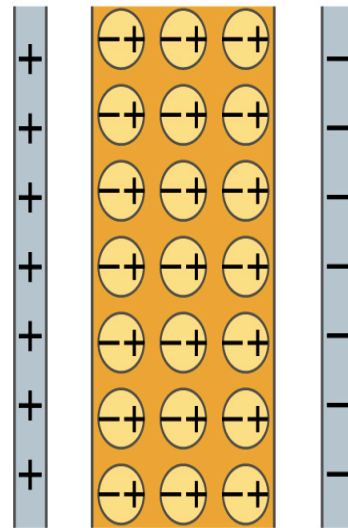
External force has done the work of $3.6 \times 10^{-4} \text{ J}$ on the system to remove dielectric!!

Molecular Description of Dielectric

- So what in the world makes dielectrics behave the way they do?
- We need to examine this in a microscopic scale.
- Let's consider a parallel plate capacitor that is charged up $+Q$ ($=C_0V_0$) and $-Q$ with air in between.
 - Assume there is no way any charge can flow in or out



(a)

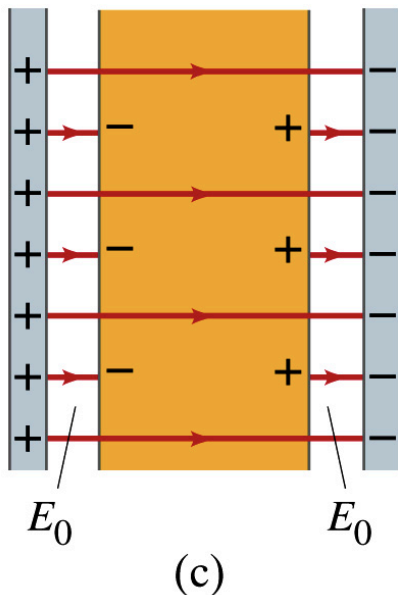


(b)

- Now insert a dielectric
 - Dielectric can be polar → could have permanent dipole moment. What will happen?
- Due to electric field molecules may be aligned.

Molecular Description of Dielectric

- OK. Then what happens?
- Then effectively, there will be some negative charges close to the surface of the positive plate and positive charge on the negative plate
 - Some electric field do not pass through the whole dielectric but stops at the negative charge



- So the field inside dielectric is smaller than the air
- Since electric field is smaller, the force is smaller
 - The work need to move a test charge inside the dielectric is smaller
 - Thus the potential difference across the dielectric is smaller than across the air

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PHYS 1444-004, Spring 2012 Dr.
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