PHYS 1444 – Section 004
Lecture #17

Wednesday, April 4, 2012
Dr. Jaehoon Yu

- Solenoid and Toroidal Magnetic Field
- Biot-Savart Law
- Magnetic Materials
- Hysteresis
- Induced EMF and EM Induction
- Faraday's Law of Induction

Announcements

- Quiz #4
  - Beginning of the class Monday, Apr. 9
  - Covers Ch 28.1 through what we finish today (CH29.2)
- Reading assignments
  - CH28.8, 9 and 10

Special Project #5

B due to current I in a straight wire. For the field near a long straight wire carrying a current I, show that

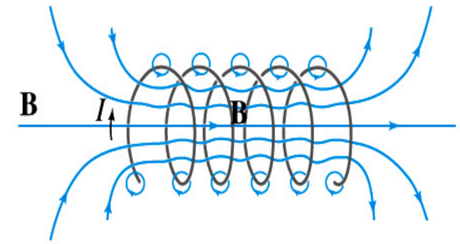
(a) the Ampere's law gives the same result as the simple long straight wire, $B=\mu_0 I/2\pi R$. (10 points)

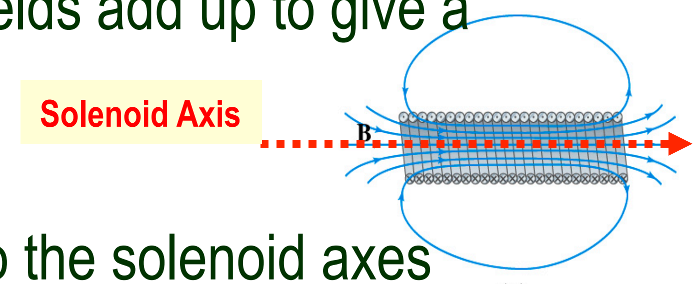
(b) That Biot-Savarat law gives the same result as the simple long straight wire, $B=\mu_0 I/2\pi R$. (10 points)

- Must be your OWN work. No credit will be given for for copying straight out of the book, lecture notes or from your friends' work.
- Due is at the beginning of the exam on Wednesday, Apr. 18.

Solenoid and Its Magnetic Field

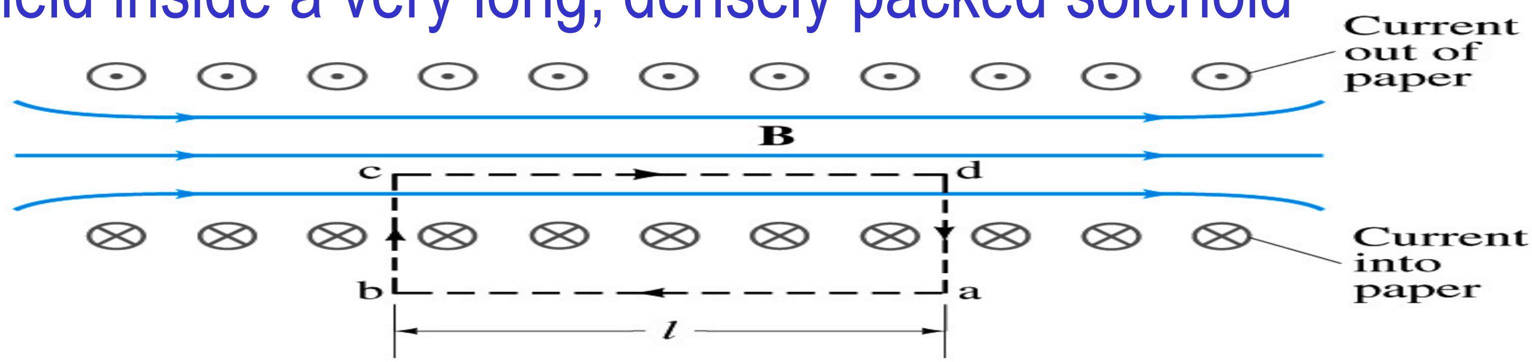
- What is a solenoid?
  - A long coil of wire consisting of many loops
  - If the space between loops are wide
    - The field near the wires are nearly circular
    - Between any two wires, the fields due to each loop cancel
    - Toward the center of the solenoid, the fields add up to give a field that can be fairly large and uniform
  - For a long, densely packed loops
    - The field is nearly uniform and parallel to the solenoid axes within the entire cross section
    - The field outside the solenoid is very small compared to the field inside, except the ends
      - The same number of field lines spread out to an open space





Solenoid Magnetic Field

- Now let's use Ampere's law to determine the magnetic field inside a very long, densely packed solenoid

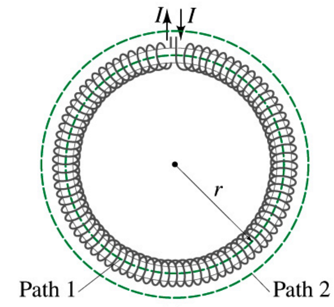


- Let's choose the path *abcd*, far away from the ends
  - We can consider four segments of the loop for integral
  - $\oint \vec{B} \cdot d\vec{l} = \int_a^b \vec{B} \cdot d\vec{l} + \int_b^c \vec{B} \cdot d\vec{l} + \int_c^d \vec{B} \cdot d\vec{l} + \int_d^a \vec{B} \cdot d\vec{l}$
  - The field outside the solenoid is negligible. So the integral on $a \rightarrow b$ is 0.
  - Now the field B is perpendicular to the *bc* and *da* segments. So these integrals become 0, also.

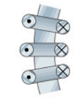
Solenoid Magnetic Field

- So the sum becomes: $\oint \vec{B}\cdot d\vec{l} = \int_c^d \vec{B}\cdot d\vec{l} = Bl$
- If the current I flows in the wire of the solenoid, the total current enclosed by the closed path is $\mathcal{N}I$
  - Where $\mathcal{N}$ is the number of loops (or turns of the coil) enclosed
- Thus Ampere's law gives us $Bl = \mu_0 NI$
- If we let $n = \mathcal{N}/l$ be the number of loops per unit length, the magnitude of the magnetic field within the solenoid becomes
- $$B = \mu_0 nI$$
  - B depends on the number of loops per unit length, n, and the current I
    - Does not depend on the position within the solenoid but uniform inside it, like a bar magnet

Example 28 – 10



(a)


(b)

Toroid. Use Ampere's law to determine the magnetic field (a) inside and (b) outside a toroid, which is like a solenoid bent into the shape of a circle.

(a) How do you think the magnetic field lines inside the toroid look?

Since it is a bent solenoid, it should be a circle concentric with the toroid. If we choose the path of integration one of these field lines of radius r inside the toroid, path 1, to use the symmetry of the situation, making B the same at all points on the path, we obtain from Ampere's law

$$\oint \vec{B}\cdot d\vec{l} = B\left(2\pi r\right) = \mu_0 I_{encl} = \mu_0 NI$$

Solving for B

$$B = \frac{\mu_0 NI}{2\pi r}$$

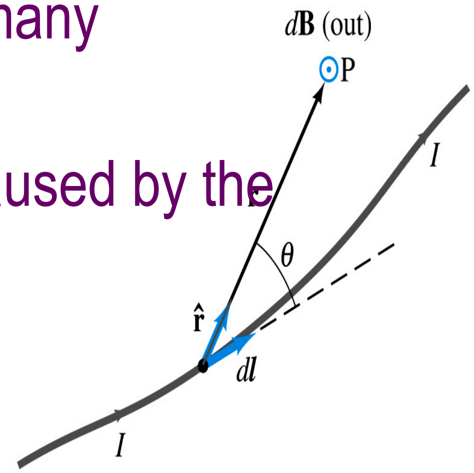
So the magnetic field inside a toroid is not uniform. It is larger on the inner edge. However, the field will be uniform if the radius is large and the toroid is thin. The field in this case is $B = \mu_0 n I$.

(b) Outside the solenoid, the field is 0 since the net enclosed current is 0.

Biot-Savart Law

- Ampere's law is useful in determining magnetic field utilizing symmetry
- But sometimes it is useful to have another method of using infinitesimal current segments for B field
  - Jean Baptiste Biot and Feilx Savart developed a law that a current I flowing in any path can be considered as many infinitesimal current elements
  - The infinitesimal magnetic field, d**B,** at distance r, caused by the infinitesimal length $d\ell$ that carries current I is
  - $$d\vec{B} = \frac{\mu_0 I}{4\pi}\frac{d\vec{l} \times \hat{r}}{r^2}$$

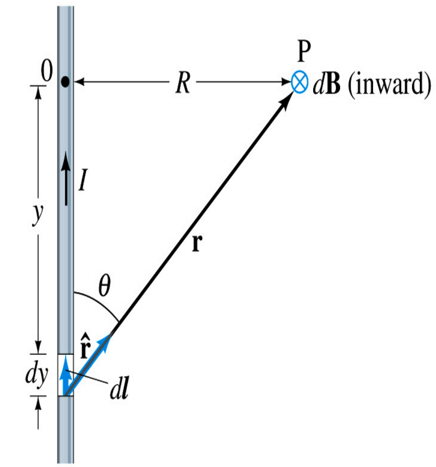
Biot-Savart Law



- **r** is the displacement vector from the element $d\ell$ to the point P
- Biot-Savart law is the magnetic equivalent to Coulomb's law

B field in Biot-Savart law is only by the current, nothing else.

Example 28 – 11



B due to current I in a straight wire. For the field near a long straight wire carrying a current I, show that the Biot-Savarat law gives the same result as the simple long straight wire, $B=\mu_0 I/2\pi R$.

What is the direction of the field **B** at point P? Going into the page.

All d**B** at point P has the same direction based on right-hand rule.

The magnitude of **B** using Biot-Savart law is

$$B=\oint dB=\frac{\mu_0 I}{4\pi}\int_{-\infty}^{+\infty}\frac{\left|d\vec{l}\times\hat{r}\right|}{r^2}=\frac{\mu_0 I}{4\pi}\int_{y=-\infty}^{+\infty}\frac{dy\sin\theta}{r^2}$$

Where $dy=dl$ and $r^2=R^2+y^2$ and since $y=-R\cot\theta$ we obtain

$$dy=+R\csc^2\theta d\theta=\frac{Rd\theta}{\sin^2\theta}=\frac{Rd\theta}{(R/r)^2}=\frac{r^2 d\theta}{R}$$

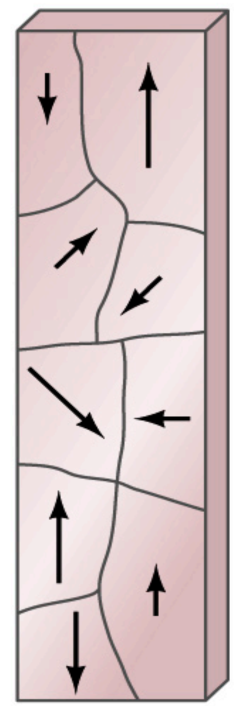
Integral becomes

$$B=\frac{\mu_0 I}{4\pi}\int_{y=-\infty}^{+\infty}\frac{dy\sin\theta}{r^2}=\frac{\mu_0 I}{4\pi}\frac{1}{R}\int_{\theta=0}^{\pi}\sin\theta d\theta=-\frac{\mu_0 I}{4\pi}\frac{1}{R}\cos\theta\Big|_0^{\pi}=\frac{\mu_0 I}{2\pi}\frac{1}{R}$$

 The same as the simple, long straight wire!! It works!!

Magnetic Materials - Ferromagnetism

- Iron is a material that can turn into a strong magnet
  - This kind of material is called **ferromagnetic** material
- In a microscopic sense, ferromagnetic materials consist of many tiny regions called **domains**
  - Domains are like little magnets usually smaller than 1mm in length or width
- What do you think the alignment of domains are like when they are not magnetized?
  - Randomly arranged
- What if they are magnetized?
  - The size of the domains aligned with the external magnetic field direction grows while those of the domains not aligned reduce
  - This gives magnetization to the material
- How do we demagnetize a bar magnet?
  - Hit the magnet hard or heat it over the Curie temperature



B in Magnetic Materials

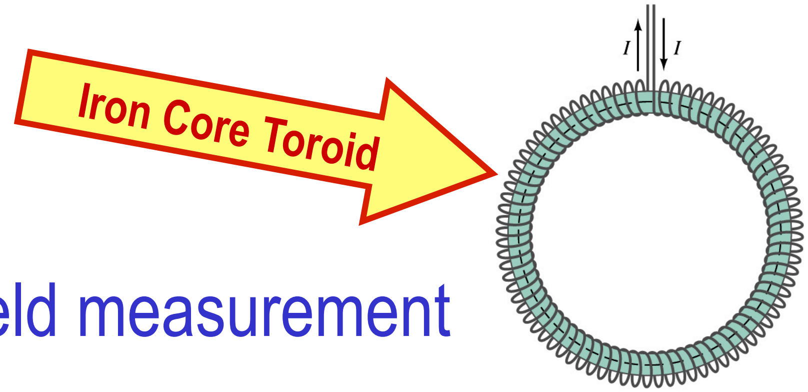
- What is the magnetic field inside a solenoid?
- $$B_0 = \mu_0 n I$$
  - Magnetic field in a long solenoid is directly proportional to the current.
  - This is valid only if air is inside the coil
- What do you think will happen to B if we have something other than the air inside the solenoid?
  - It will be increased dramatically, when the current flows
    - Especially if a ferromagnetic material such as an iron is put inside, the field could increase by several orders of magnitude
- Why?
  - Since the domains in the iron aligns permanently by the external field.
  - The resulting magnetic field is the sum of that due to current and due to the iron

B in Magnetic Materials

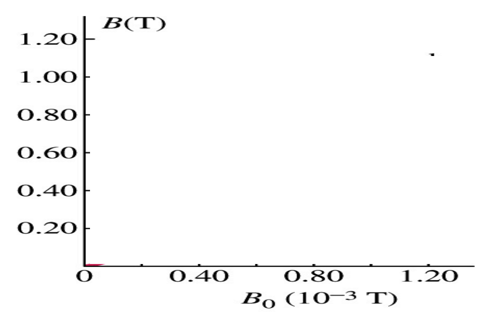
- It is sometimes convenient to write the total field as the sum of two terms
- $\vec{B} = \vec{B}_0 + \vec{B}_M$
  - $\mathbf{B_0}$ is the field due only to the current in the wire, namely the external field
    - The field that would be present without a ferromagnetic material
  - $\mathbf{B_M}$ is the additional field due to the ferromagnetic material itself; often $\mathbf{B_M} >> \mathbf{B_0}$
- The total field in this case can be written by replacing μ_0 with another proportionality constant μ, the magnetic permeability of the material $B = \mu n I$
  - μ is a property of a magnetic material
  - μ is not a constant but varies with the external field

Hysteresis

- What is a toroid?
  - A solenoid bent into a shape

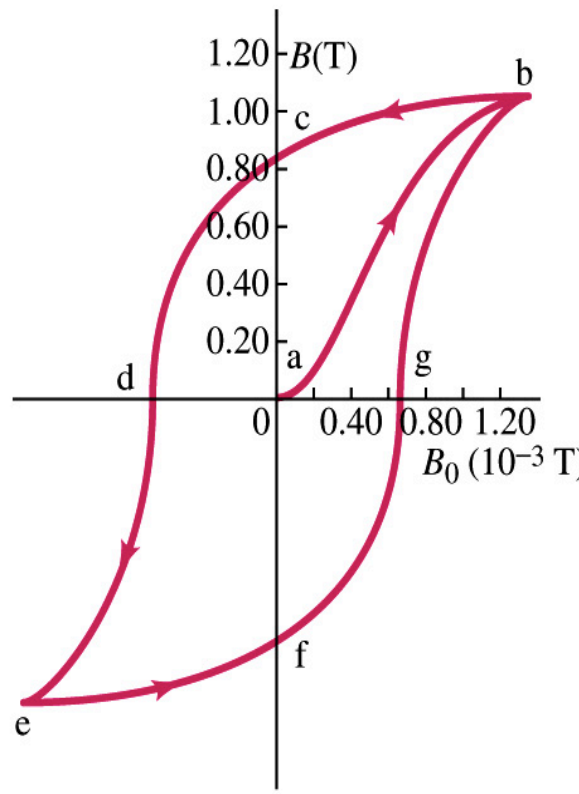


- Toroid can be used for magnetic field measurement
  - Why?
  - Since it does not leak magnetic field outside of itself, it fully contains all the magnetic field created within it.
- Consider an un-magnetized iron core toroid, without any current flowing in the wire
  - What do you think will happen if the current slowly increases?
  - B_0 increases linearly with the current.
  - And B increases also but follows the curved line shown in the graph
  - As B_0 increases, the domains become more aligned until nearly all are aligned (point b on the graph)
    - The iron is said to be approaching saturation
    - Point b is typically at 70% of the max



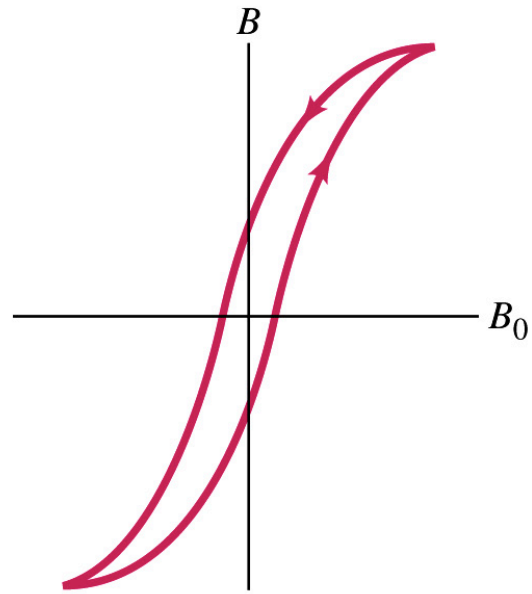
Hysteresis

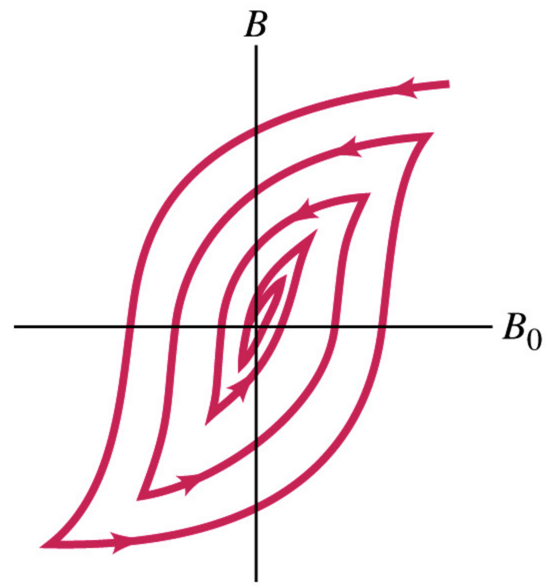
- What do you think will happen to B if the external field B_0 is reduced to 0 by decreasing the current in the coil?
  - ~~Of course it goes to 0!!~~
  - Wrong! Wrong! Wrong! They do not go to 0. Why not?
  - The domains do not completely return to the random alignment state
- Now if the current direction is reversed, the external magnetic field direction is reversed, causing the total field B pass 0, and the direction reverses to the opposite side
  - If the current is reversed again, the total field B will increase but never goes through the origin
- This kind of curve whose path does not retrace themselves and does not go through the origin is called the **Hysteresis**.



Magnetically Soft Material

- In a hysteresis cycle, much energy is transformed to thermal energy. Why?
  - Due to the microscopic friction between domains as they change directions to align with the external field
- The energy dissipated in the hysteresis cycle is proportional to the area of the hysteresis loop
- Ferromagnetic material with large hysteresis area is called magnetically hard while the small ones are called soft
  - Which one do you think are preferred in electromagnets or transformers?
    - Soft. Why?
    - Since the energy loss is small and much easier to switch off the field
- Then how do we demagnetize a ferromagnetic material?
  - Keep repeating the Hysteresis loop, reducing the range of B_0.



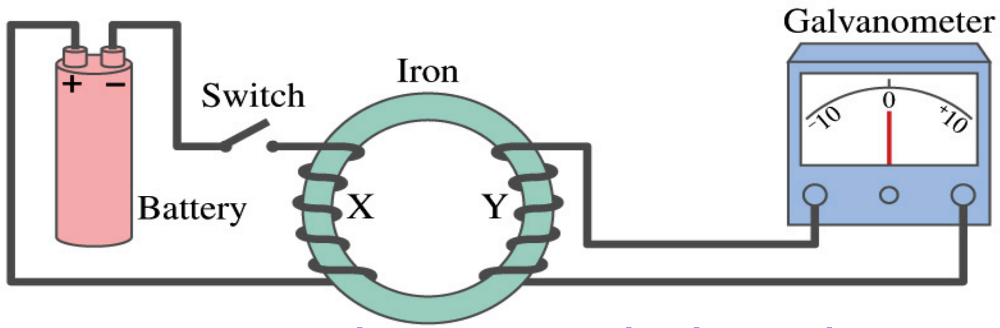


Induced EMF

- It has been discovered by Oersted and company in early 19th century that
  - Magnetic field can be produced by the electric current
  - Magnetic field can exert force on the electric charge
- So if you were scientists at that time, what would you wonder?
  - Yes, you are absolutely right! You would wonder if the magnetic field can create the electric current.
  - An American scientist Joseph Henry and an English scientist Michael Faraday independently found that it was possible
    - Though, Faraday was given the credit since he published his work before Henry did
      - He also did a lot of detailed studies on magnetic induction

Electromagnetic Induction

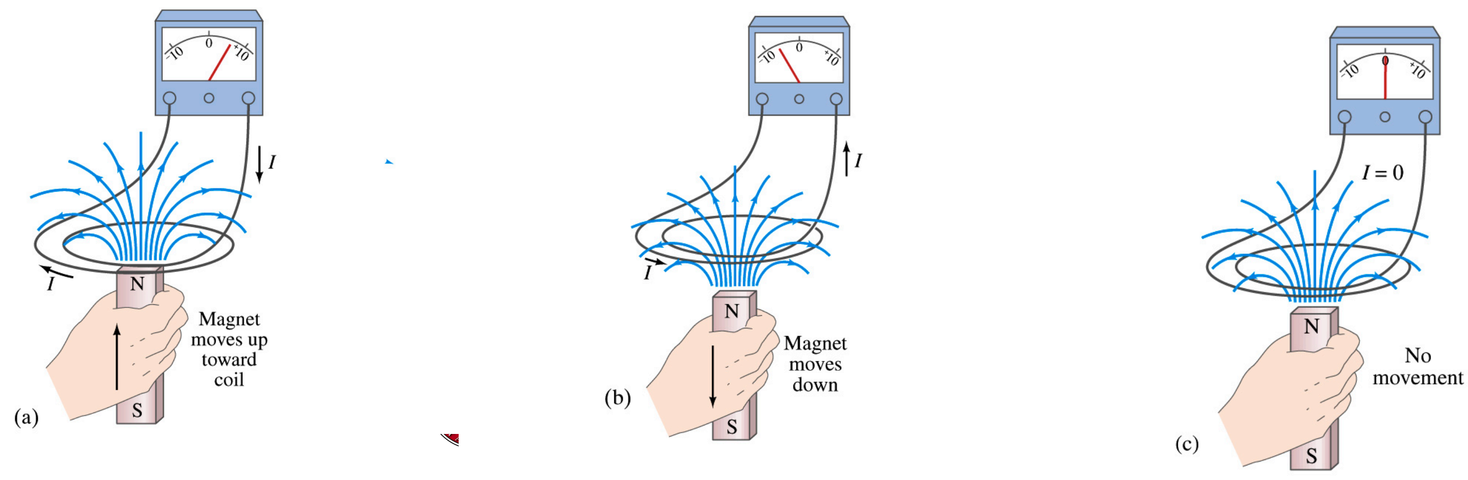
- Faraday used an apparatus below to show that magnetic field can induce current



- Despite his hope he did not see steady current induced on the other side when the switch is thrown
- But he did see that the needle on the Galvanometer turns strongly when the switch is initially thrown and is opened
  - When the magnetic field through coil Y changes, a current flows as if there were a source of emf
- Thus he concluded that **an induced emf is produced by a changing magnetic field** → **Electromagnetic Induction**

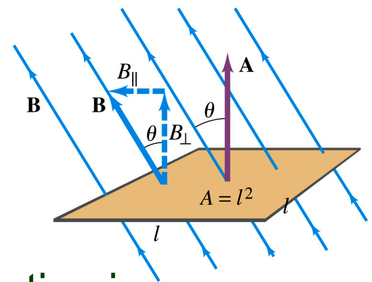
Electromagnetic Induction

- Further studies on electromagnetic induction taught
  - If a magnet is moved quickly into a coil of wire, a current is induced in the wire.
  - If a magnet is removed from the coil, a current is induced in the wire in the opposite direction
  - By the same token, the current can also be induced if the magnet stays put but the coil moves toward or away from the magnet
  - Current is also induced if the coil rotates.
- In other words, it does not matter whether the magnet or the coil moves. It is the relative motion that counts.



Magnetic Flux

- So what do you think is the induced emf proportional to?
  - The rate of changes of the magnetic field?
    - the higher the changes the higher the induction
  - Not really, it rather depends on the rate of change of the **magnetic flux**, Φ_B.
  - Magnetic flux is defined as (just like the electric flux)
  - $$\Phi_B = B_\perp A = BA\cos\theta = \vec{B}\cdot\vec{A}$$
    - θ is the angle between **B** and the area vector **A** whose direction is perpendicular to the face of the loop based on the right-hand rule
  - What kind of quantity is the magnetic flux?
    - Scalar. Unit?
    - $T\cdot m^2$ or weber $1Wb = 1T\cdot m^2$



- If the area of the loop is not simple or B is not uniform, the magnetic flux can be written as $$\Phi_B = \int \vec{B}\cdot d\vec{A}$$

Faraday's Law of Induction

- In terms of magnetic flux, we can formulate Faraday's findings
  - The emf induced in a circuit is equal to the rate of change of magnetic flux through the circuit

$$\varepsilon = -\frac{d\Phi_B}{dt}$$

Faraday's Law of Induction

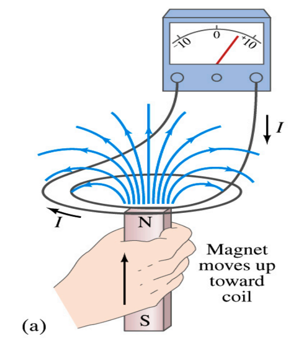
- If the circuit contains N closely wrapped loops, the total induced emf is the sum of emf induced in each loop

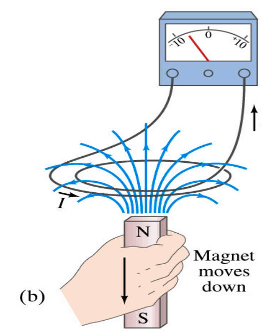
$$\varepsilon = -N\frac{d\Phi_B}{dt}$$

  - Why negative?
    - Has got a lot to do with the direction of induced emf…

Lenz's Law

- It is experimentally found that
  - An induced emf gives rise to a current whose magnetic field opposes the original change in flux ➔ This is known as **Lenz's Law**
  - In other words, an induced emf is always in a direction that opposes the original change in flux that caused it.
  - We can use Lenz's law to explain the following cases in the figures
    - When the magnet is moving into the coil
      - Since the external flux increases, the field inside the coil takes the opposite direction to minimize the change and causes the current to flow clockwise
    - When the magnet is moving out
      - Since the external flux decreases, the field inside the coil takes the opposite direction to compensate the loss, causing the current to flow counter-clockwise





- Which law is Lenz's law result of?
  - Energy conservation. Why?