PHYS 1444 – Section 004
Lecture #23

Monday, April 30, 2012

Dr. Jaehoon Yu

- EM Waves from Maxwell's equations
- Derivation of Speed of EM Waves
- Light as EM Wave
- Energy in EM Wave
- Energy Transport and the Poynting Vector

Today's homework is None!!

Announcements

- Final exam
  - Comprehensive
  - Date and time: 5:30 – 8:00pm, coming Monday, May 7
  - Location: SH103
  - Coverage: CH. 21 – 1 through CH31 – 10 plus Appendices A and B
  - Please do NOT miss the exam!!
- Reading Assignments
  - CH31 – 9 and CH31 – 10
- Please fill out the online teaching evaluation form
- No class Wednesday, May 2
- Colloquium 4pm, Wednesday, May 2
  - Dr. Haleh Hanavand

Physics Department
The University of Texas at Arlington
COLLOQUIUM

Diphoton Physics at the LHC with the ATLAS Experiment

Dr. Haleh Hadavand
Candidate: Faculty Research Associate
Southern Methodist University

4:00 pm Wednesday May 2, 2012 room 101 SH

Abstract:

The two photon final state provides a clear experimental signature for both the Standard Model Higgs and beyond Standard Model theories with extra spatial dimensions. Although the Standard Model has been validated with high precision measurements over the years, the Higgs Boson, predicted by the theory, has yet to be observed. The large difference between the strengths of electro-weak and gravity forces, known as the hierarchy problem, is also not explained by the theory. A class of theories that address the hierarchy problem evoke extra spatial dimensions. Examples of such models include the Randall-Sundrum (RS) model, ADD model, and Universal extra dimensions. I will describe some recent ATLAS results using the 2011 dataset with two photon final states in the context of the Higgs and extra dimension models.

Refreshments w[illegible] the Physics lounge

Maxwell's Equations w/ Q=I=0

- In the region of free space where Q=0 and I=0, the four Maxwell's equations become

$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_0} \quad \xrightarrow{Q_{encl}=0} \quad \oint \vec{E}\cdot d\vec{A} = 0$$

$$\oint \vec{B}\cdot d\vec{A} = 0 \quad \xrightarrow{\text{No Changes}} \quad \oint \vec{B}\cdot d\vec{A} = 0$$

$$\oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi_B}{dt} \quad \xrightarrow{\text{No Changes}} \quad \oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{encl} + \mu_0\varepsilon_0\frac{d\Phi_E}{dt} \quad \xrightarrow{I_{encl}=0} \quad \oint \vec{B}\cdot d\vec{l} = \mu_0\varepsilon_0\frac{d\Phi_E}{dt}$$

One can observe the symmetry between electricity and magnetism.

The last equation is the most important one for EM waves and their propagation!!

EM Waves from Maxwell's Equations

- If the wave is sinusoidal w/ wavelength λ and frequency *f*, such traveling wave can be written as

$$E = E_y = E_0 \sin(kx - \omega t)$$

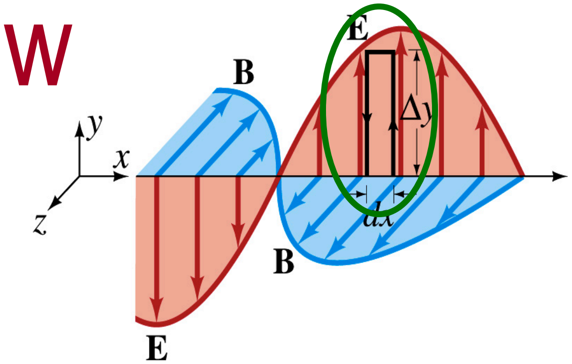
$$B = B_z = B_0 \sin(kx - \omega t)$$

  – Where

$$k = \frac{2\pi}{\lambda} \qquad \varpi = 2\pi f \qquad \text{Thus} \qquad f\lambda = \frac{\omega}{k} = \mathrm{v}$$

  – What is v?
    - It is the speed of the traveling wave
  – What are E_0 and B_0?
    - The amplitudes of the EM wave. Maximum values of **E** and **B** field strengths.

From Faraday's Law



- Let's apply Faraday's law

$$\oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

  – to the rectangular loop of height Δy and width dx

- $\vec{E}\cdot d\vec{l}$ along the top and bottom of the loop is 0. Why?
  – Since **E** is perpendicular to dℓ.
  – So the result of the integral through the loop counterclockwise becomes $\oint \vec{E}\cdot d\vec{l} = \vec{E}\cdot d\vec{x} + \left(\vec{E}+d\vec{E}\right)\cdot\Delta\vec{y} + \vec{E}\cdot d\vec{x}' + \vec{E}\cdot\Delta\vec{y}' =$

$$= 0 + \left(E+dE\right)\Delta y - 0 - E\Delta y = dE\Delta y$$

  – For the right-hand side of Faraday's law, the magnetic flux through the loop changes as

$$-\frac{d\Phi_B}{dt} = \frac{dB}{dt}dx\Delta y$$

Thus

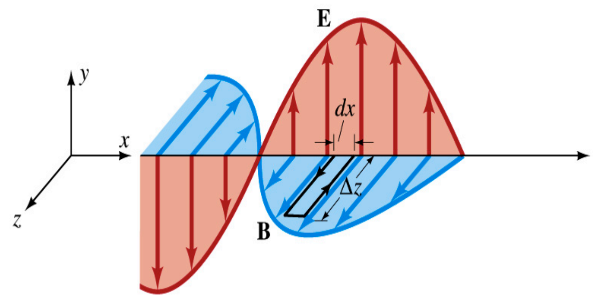
$$dE\Delta y = -\frac{dB}{dt}dx\Delta y$$

$$\frac{dE}{dx} = -\frac{dB}{dt}$$

Since E and B depend on x and t

$$\boxed{\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}}$$

From Modified Ampére's Law



- Let's apply Maxwell's 4th equation

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

  – to the rectangular loop of length Δz and width dx

- $\vec{B} \cdot d\vec{l}$ along the x-axis of the loop is 0
  – Since **B** is perpendicular to d*l*.
  – So the result of the integral through the loop counterclockwise becomes $\oint \vec{B} \cdot d\vec{l} = B\Delta z - \left(B + dB\right)\Delta z = -dB\Delta z$
  – For the right-hand side of the equation is

$$\mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} = \mu_0 \varepsilon_0 \frac{dE}{dt} dx\Delta z$$

Thus

$$-dB\Delta z = \mu_0 \varepsilon_0 \frac{dE}{dt} dx\Delta z$$

  – $\frac{dB}{dx} = -\mu_0 \varepsilon_0 \frac{dE}{dt}$

Since E and B depend on x and t

$$\boxed{\frac{\partial B}{\partial x} = -\mu_0 \varepsilon_0 \frac{\partial E}{\partial t}}$$

Relationship between **E**, **B** and **v**

- Let's now use the relationship from Faraday's law $\frac{\partial E}{\partial x}=-\frac{\partial B}{\partial t}$
- Taking the derivatives of E and B as given their traveling wave form, we obtain

$$\frac{\partial E}{\partial x}=\frac{\partial}{\partial x}\left(E_0\sin\left(kx-\omega t\right)\right)=kE_0\cos\left(kx-\omega t\right)$$

$$\frac{\partial B}{\partial t}=\frac{\partial}{\partial t}\left(B_0\sin\left(kx-\omega t\right)\right)=-\omega B_0\cos\left(kx-\omega t\right)$$

Since $\frac{\partial E}{\partial x}=-\frac{\partial B}{\partial t}$ **We obtain** $kE_0\cancel{\cos\left(kx-\omega t\right)}=\omega B_0\cancel{\cos\left(kx-\omega t\right)}$

Thus $\frac{E_0}{B_0}=\frac{\omega}{k}=\mathrm{v}$

  - Since E and B are in phase, we can write $E/B=\mathrm{v}$
    - This is valid at any point and time in space. What is v?
      - The velocity of the wave

Speed of EM Waves

- Let's now use the relationship from Apmere's law $\frac{\partial B}{\partial x}=-\varepsilon_0\mu_0\frac{\partial E}{\partial t}$
- Taking the derivatives of E and B as given their traveling wave form, we obtain

$$\frac{\partial B}{\partial x}=\frac{\partial}{\partial x}\left(B_0\sin\left(kx-\omega t\right)\right)=kB_0\cos\left(kx-\omega t\right)$$

$$\frac{\partial E}{\partial t}=\frac{\partial}{\partial t}\left(E_0\sin\left(kx-\omega t\right)\right)=-\omega E_0\cos\left(kx-\omega t\right)$$

Since $\frac{\partial B}{\partial x}=-\varepsilon_0\mu_0\frac{\partial E}{\partial t}$ **We obtain** $kB_0\cos\cancel{\left(kx-\omega t\right)}=\varepsilon_0\mu_0\omega E_0\cos\cancel{\left(kx-\omega t\right)}$

Thus $\frac{B_0}{E_0}=\frac{\varepsilon_0\mu_0\omega}{k}=\varepsilon_0\mu_0 v$

- However, from the previous page we obtain $E_0/B_0=\mathrm{v}=\frac{1}{\varepsilon_0\mu_0\mathrm{v}}$
- Thus $\mathrm{v}^2=\frac{1}{\varepsilon_0\mu_0}$

$$\boxed{\mathrm{v}=\frac{1}{\sqrt{\varepsilon_0\mu_0}}=\frac{1}{\sqrt{\left(8.85\times10^{-12}\,C^2/N\cdot m^2\right)\cdot\left(4\pi\times10^{-7}\,T\cdot m/A\right)}}=3.00\times10^{8}\,m/s}$$

The speed of EM waves is the same as the speed of light. EM waves behaves like the light.

Speed of Light w/o Sinusoidal Wave Forms

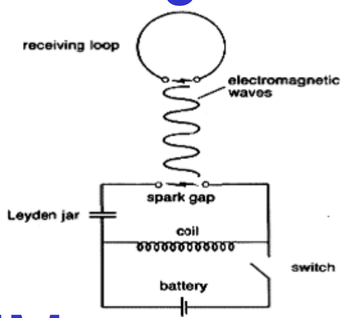
- Taking the time derivative on the relationship from Ampere's laws, we obtain $\frac{\partial^2 B}{\partial x \partial t} = -\varepsilon_0 \mu_0 \frac{\partial^2 E}{\partial t^2}$
- By the same token, we take position derivative on the relationship from Faraday's law $\frac{\partial^2 E}{\partial x^2} = -\frac{\partial^2 B}{\partial x \partial t}$
- From these, we obtain

$$\frac{\partial^2 E}{\partial t^2} = \frac{1}{\varepsilon_0 \mu_0} \frac{\partial^2 E}{\partial x^2} \quad \textbf{and} \quad \frac{\partial^2 B}{\partial t^2} = \frac{1}{\varepsilon_0 \mu_0} \frac{\partial^2 B}{\partial x^2}$$

- Since the equation for traveling wave is $\frac{\partial^2 x}{\partial t^2} = \mathrm{v}^2 \frac{\partial^2 x}{\partial x^2}$
- By correspondence, we obtain $\mathrm{v}^2 = \frac{1}{\varepsilon_0 \mu_0}$
- A natural outcome of Maxwell's equations is that E and B obey the wave equation for waves traveling w/ speed $\mathrm{v} = 1 / \sqrt{\varepsilon_0 \mu_0}$
  - Maxwell predicted the existence of EM waves based on this

Light as EM Wave

- People knew some 60 years before Maxwell that light behaves like a wave, but …
  - They did not know what kind of waves they are.
    - Most importantly what is it that oscillates in light?

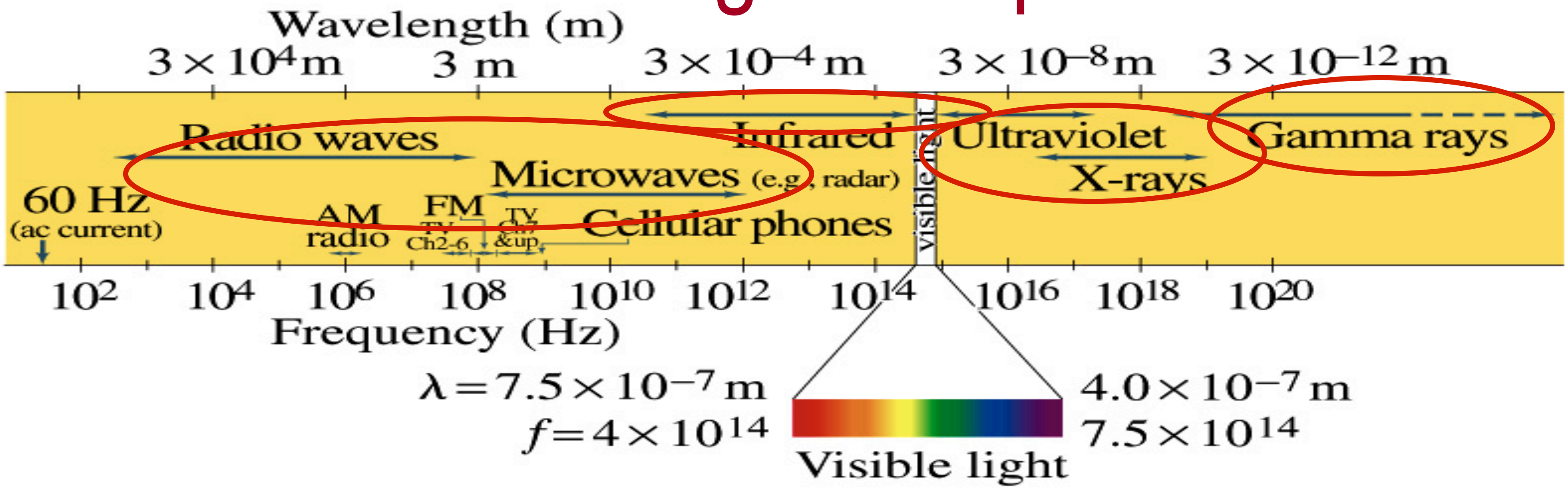


- Heinrich Hertz first generated and detected EM waves experimentally in 1887 using a spark gap apparatus
  - Charge was rushed back and forth in a short period of time, generating waves with frequency about 10^9Hz (these are called radio waves)
  - He detected using a loop of wire in which an emf was produced when a changing magnetic field passed through
  - These waves were later shown to travel at the speed of light and behave exactly like the light just not visible

Light as EM Wave

- The wavelengths of visible light were measured in the first decade of the 19th century
  - The visible light wave length were found to be between 4.0×10^{-7}m (400nm) and 7.5×10^{-7}m (750nm)
  - The frequency of visible light is fl=c
    - Where f and l are the frequency and the wavelength of the wave
      - What is the range of visible light frequency?
      - 4.0×10^{14}Hz to 7.5×10^{14}Hz
    - c is 3×10^{8}m/s, the speed of light
- EM Waves, or EM radiation, are categorized using EM spectrum

Electromagnetic Spectrum



- Low frequency waves, such as radio waves or microwaves can be easily produced using electronic devices
- Higher frequency waves are produced natural processes, such as emission from atoms, molecules or nuclei
- Or they can be produced from acceleration of charged particles
- Infrared radiation (IR) is mainly responsible for the heating effect of the Sun
  - The Sun emits visible lights, IR and UV
    - The molecules of our skin resonate at infrared frequencies so IR is preferentially absorbed and thus warm up

Example 31 – 3

Wavelength of EM waves. Calculate the wavelength (a) of a 60-Hz EM wave, (b) of a 93.3-MHz FM radio wave and (c) of a beam of visible red light from a laser at frequency 4.74×10^{14}Hz.

What is the relationship between speed of light, frequency and the wavelength? $c = f\lambda$

Thus, we obtain $\lambda = \frac{c}{f}$

For f=60Hz $$\lambda = \frac{3\times10^8\ m/s}{60s^{-1}} = 5\times10^6\,m$$

For f=93.3MHz $$\lambda = \frac{3\times10^8\ m/s}{93.3\times10^6\,s^{-1}} = 3.22m$$

For f=4.74×10^{14}Hz $$\lambda = \frac{3\times10^8\ m/s}{4.74\times10^{14}\,s^{-1}} = 6.33\times10^{-7}\,m$$

EM Wave in the Transmission Lines

- Can EM waves travel through a wire?
  - Can it not just travel through the empty space?
  - Nope. It sure can travel through a wire.
- When a source of emf is connected to a transmission line, the electric field within the wire does not set up immediately at all points along the line
  - When two wires are separated via air, the EM wave travel through the air at the speed of light, c.
  - However, through medium w/ permittivity ε and permeability μ, the speed of the EM wave is given $\mathrm{v} = 1/\sqrt{\varepsilon\mu} < c$
    - Is this faster than c? Nope! It is slower.

Example 31 – 5

Phone call time lag. You make a telephone call from New York to London. Estimate the time the electrical signal to travel to reach London (a) carried on a 5000km telephone cable under the Atlantic Ocean and (b) sent via satellite 36,000km above the ocean. Would this cause a noticeable delay in either case?

Time delay via the cable: $t = \frac{d}{c} = \frac{5 \times 10^{6}}{3 \times 10^{8}} = 0.017s$

Delay via satellite $t = \frac{2d_{s}}{c} = \frac{2 \times 3.6 \times 10^{7}}{3.0 \times 10^{8}} = 0.24s$

So in case of satellite, the delay is likely noticeable!!

Energy in EM Waves

- Since B=E/c and $c = 1/\sqrt{\varepsilon_0 \mu_0}$, we can rewrite the energy density

$$u = u_E + u_B = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2}\frac{B^2}{\mu_0} = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2}\frac{\varepsilon_0 \mu_0 E^2}{\mu_0} = \varepsilon_0 E^2 \qquad \boxed{u = \varepsilon_0 E^2}$$

  - Note that the energy density associate with B field is the same as that associate with E
  - So each field contribute half to the total energy
- By rewriting in B field only, we obtain

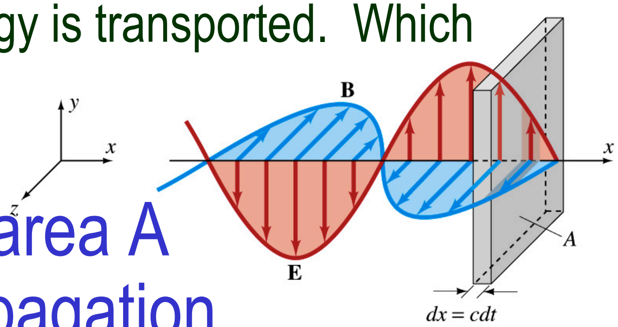
$$u = \frac{1}{2}\varepsilon_0 \frac{B^2}{\varepsilon_0 \mu_0} + \frac{1}{2}\frac{B^2}{\mu_0} = \frac{B^2}{\mu_0} \qquad \boxed{u = \frac{B^2}{\mu_0}}$$

- We can also rewrite to contain both E and B

$$u = \varepsilon_0 E^2 = \varepsilon_0 EcB = \frac{\varepsilon_0 EB}{\sqrt{\varepsilon_0 \mu_0}} = \sqrt{\frac{\varepsilon_0}{\mu_0}} EB \qquad \boxed{u = \sqrt{\frac{\varepsilon_0}{\mu_0}} EB}$$

Energy Transport

- What is the energy the wave transport per unit time per unit area?
  - This is given by the vector **S**, the Poynting vector
    - The unit of **S** is **W/m²**.
    - The direction of **S** is the direction in which the energy is transported. Which direction is this?
      - The direction the wave is moving



- Let's consider a wave passing through an area A perpendicular to the x-axis, the axis of propagation
  - How much does the wave move in time dt?
    - dx=cdt
  - The energy that passes through A in time dt is the energy that occupies the volume dV, $dV = Adx = Acdt$
  - Since the energy density is $u=\varepsilon_0 E^2$, the total energy, dU, contained in the volume V is $dU = udV = \varepsilon_0 E^2 Acdt$

Energy Transport

- Thus, the energy crossing the area A per time dt is

$$\boxed{S = \frac{1}{A}\frac{dU}{dt} = \varepsilon_0 c E^2}$$

- Since E=cB and $c = 1/\sqrt{\varepsilon_0 \mu_0}$, we can also rewrite

$$\boxed{S = \varepsilon_0 c E^2 = \frac{cB^2}{\mu_0} = \frac{EB}{\mu_0}}$$

- Since the direction of S is along **v**, perpendicular to **E** and **B**, the Poynting vector **S** can be written

$$\boxed{\vec{S} = \frac{1}{\mu_0}\left(\vec{E} \times \vec{B}\right)}$$

What is the unit? W/m^2

  - This gives the energy transported per unit area per unit time at any instant

Average Energy Transport

- The average energy transport in an extended period of time is useful since sometimes we do not detect the rapid variation with respect to time because the frequency is so high.
- If E and B are sinusoidal, $\overline{E^2} = E_0^2/2$
- Thus we can write the magnitude of the average Poynting vector as

$$\boxed{\bar{S} = \frac{1}{2}\varepsilon_0 c E_0^2 = \frac{1}{2}\frac{c}{\mu_0}B_0^2 = \frac{E_0 B_0}{2\mu_0}}$$

  - This time averaged value of S is the intensity, defined as the average power transferred across unit area. E_0 and B_0 are maximum values.
- We can also write

$$\boxed{\bar{S} = \frac{E_{rms}B_{rms}}{\mu_0}}$$

  - Where E_{rms} and B_{rms} are the rms values ($E_{rms} = \sqrt{\overline{E^2}}$, $\mathrm{B}_{rms} = \sqrt{\overline{B^2}}$)

Example 31 – 6

E and B from the Sun. Radiation from the Sun reaches the Earth (above the atmosphere) at a rate of about 1350W/m^2. Assume that this is a single EM wave and calculate the maximum values of E and B.

What is given in the problem? The average S!!

$$\bar{S} = \frac{1}{2}\varepsilon_0 c E_0^2 = \frac{1}{2}\frac{c}{\mu_0}B_0^2 = \frac{E_0 B_0}{2\mu_0}$$

For E$_0$, $E_0 = \sqrt{\frac{2\bar{S}}{\varepsilon_0 c}} = \sqrt{\frac{2\cdot 1350\,W/m^2}{\left(8.85\times10^{-12}\,C^2/N\cdot m^2\right)\cdot\left(3.00\times10^8\,m/s\right)}} = 1.01\times10^3\,V/m$

For B$_0$ $B_0 = \frac{E_0}{c} = \frac{1.01\times10^3\,V/m}{3\times10^8\,m/s} = 3.37\times10^{-6}\,T$

You have worked very hard and well !!

This was one of my best semesters!!

Good luck with your final exams!!

Have a safe summer!