PHYS 1441 – Section 002
Lecture #5

Wednesday, Jan. 30, 2013

Dr. Jaehoon Yu

- One Dimensional Motion
  - One dimensional Kinematic Equations
  - How do we solve kinematic problems?
  - Falling motions

Today's homework is homework #4, due 11pm, Tuesday, Feb. 5!!

Announcements

- E-mail subscription
  - 84/99 subscribed! ➔ Please subscribe ASAP
  - I am still getting replies!   Thanks!
  - Please check your e-mail and reply to ME ASAP…
- Homework
  - If you haven't done yet, please get this done TODAY!!
  - Homework is 25% of your grade so without doing it well, it will be very hard for you to obtain good grade!
- 1st term exam
  - In class, Wednesday, Feb. 13
  - Coverage: CH1.1 – what we finish Monday, Feb. 11, plus Appendix A1 – A8
  - Mixture of free response problems and multiple choice problems
  - Please do not miss the exam!   You will get an F if you miss any exams!

Reminder: Special Project #1

- Derive the quadratic equation for $yx^2-zx+v=0$ ➔ 5 points
  - This means that you need to solve the above equation and find the solutions for x!
- Derive the kinematic equation $v^2 = v_0^2 + 2a\left(x - x_0\right)$ from first principles and the known kinematic equations ➔ 10 points
- You must **show your OWN work in detail** to obtain the full credit
  - Must be in much more detail than in this lecture note!!!
  - Please do not copy from the lecture note or from your friends. You will all get 0!
- Due Monday, Feb. 4

The Direction (sign) of the Acceleration

- If the velocity **INCREASES**, the acceleration must be in the **SAME** direction as the velocity!!
  - If the positive velocity increases, what sign is the acceleration?
    - Positive!!
  - If the negative velocity increases, what sign is the acceleration?
    - Negative
- If the velocity **DECREASES**, the acceleration must be in the **OPPOSITE** direction to the velocity!!
  - If the positive velocity decreases, what sign is the acceleration?
    - Negative
  - If the negative velocity decreases, what sign is the acceleration?
    - Positive

One Dimensional Motion

- Let's focus on the simplest case: acceleration is a constant ($a=a_0$)
- Using the definitions of average acceleration and velocity, we can derive equations of motion (description of motion, velocity and position as a function of time)

$$\bar{a}_x = a_x = \frac{v_{xf} - v_{xi}}{t_f - t_i} \quad \text{(If } t_f=t \text{ and } t_i=0\text{)} \quad a_x = \frac{v_{xf} - v_{xi}}{t} \Rightarrow v_{xf} = v_{xi} + a_x t$$

For constant acceleration, average velocity is a simple numeric average

$$\bar{v}_x = \frac{v_{xi} + v_{xf}}{2} = \frac{2v_{xi} + a_x t}{2} = v_{xi} + \frac{1}{2} a_x t$$

$$\bar{v}_x = \frac{x_f - x_i}{t_f - t_i} \quad \text{(If } t_f=t \text{ and } t_i=0\text{)} \quad \bar{v}_x = \frac{x_f - x_i}{t} \Rightarrow x_f = x_i + \bar{v}_x t$$

Resulting Equation of Motion becomes

$$x_f = x_i + \bar{v}_x t = x_i + v_{xi} t + \frac{1}{2} a_x t^2$$

One Dimensional Motion cont'd

Average velocity $\bar{v}_x = \frac{v_{xi} + v_{xf}}{2}$ $\Rightarrow$ $x_f = x_i + \bar{v}_x t = x_i + \left(\frac{v_{xi} + v_{xf}}{2}\right)t$

Since $a_x = \frac{v_{xf} - v_{xi}}{t}$ **Solving for t** $\Rightarrow$ $t = \frac{v_{xf} - x_{xi}}{a_x}$

Substituting t in the above equation,

$$x_f = x_i + \left(\frac{v_{xf} + v_{xi}}{2}\right)\left(\frac{v_{xf} - v_{xi}}{a_x}\right) = x_i + \frac{v_{xf}^2 - v_{xi}^2}{2a_x}$$

Resulting in

$$v_{xf}^2 = v_{xi}^2 + 2a_x\left(x_f - x_i\right)$$

Kinematic Equations of Motion on a Straight Line Under Constant Acceleration

$$v_{xf}(t) = v_{xi} + a_x t$$

Velocity as a function of time

$$x_f - x_i = \bar{v}_x t = \frac{1}{2}\left(v_{xf} + v_{xi}\right) t$$

Displacement as a function of velocities and time

$$x_f = x_i + v_{xi} t + \frac{1}{2} a_x t^2$$

Displacement as a function of time, velocity, and acceleration

$$v_{xf}^2 = v_{xi}^2 + 2a_x\left(x_f - x_i\right)$$

Velocity as a function of Displacement and acceleration

You may use different forms of Kinetic equations, depending on the information given to you for specific physical problems!!

How do we solve a problem using the kinematic formula for constant acceleration?

- Identify what information is given in the problem.
  - Initial and final velocity?
  - Acceleration?
  - Distance?
  - Time?
- Identify what the problem wants you to find out.
- Identify which kinematic formula is most appropriate and easiest to solve for what the problem wants.
  - Often multiple formulae can give you the answer for the quantity you are looking for. ➔ Do not just use any formula but use the one that makes the problem easiest to solve.
- Solve the equation for the quantity want!

Example 2.8

Suppose you want to design an air-bag system that can protect the driver in a head-on collision at a speed 100km/hr (~60miles/hr). Estimate how fast the air-bag must inflate to effectively protect the driver. Assume the car crumples upon impact over a distance of about 1m. How does the use of a seat belt help the driver?

How long does it take for the car to come to a full stop? ⇨ As long as it takes for it to crumple.

The initial speed of the car is $v_{xi} = 100km/h = \dfrac{100000m}{3600s} = 28m/s$

We also know that $v_{xf} = 0m/s$ and $x_f - x_i = 1m$

Using the kinematic formula $v_{xf}^2 = v_{xi}^2 + 2a_x\left(x_f - x_i\right)$

The acceleration is $a_x = \dfrac{v_{xf}^2 - v_{xi}^2}{2\left(x_f - x_i\right)} = \dfrac{0-\left(28m/s\right)^2}{2\times 1m} = -390m/s^2$

Thus the time for air-bag to deploy is $t = \dfrac{v_{xf} - v_{xi}}{a} = \dfrac{0-28m/s}{-390m/s^2} = 0.07s$

Falling Motion

- Falling motion is a motion under the influence of the gravitational pull (gravity) only; Which direction is a freely falling object moving? **Yes, down to the center of the earth!!**
  - A motion under constant acceleration
  - All kinematic formula we learned can be used to solve for falling motions.
- Gravitational acceleration is inversely proportional to the square of the distance between the object and the center of the earth
- The magnitude of the gravitational acceleration is $g=9.80m/s^2$ on the surface of the earth.
- The direction of gravitational acceleration is **ALWAYS** toward the center of the earth, which we normally call (-y); where up and down direction are indicated as the variable "y"
- Thus the correct denotation of gravitational acceleration on the surface of the earth is $g=-9.80m/s^2$ when +y points upward
- The difference is that the object initially moving upward will turn around and come down!

Example for Using 1D Kinematic Equations on a Falling object

A stone was thrown straight upward at t=0 with +20.0m/s initial velocity on the roof of a 50.0m high building,

What is the acceleration in this motion? $\boxed{a=g=-9.80 m/s^2}$

(a) Find the time the stone reaches at the maximum height.

What happens at the maximum height? The stone stops; V=0

$$v_f = v_{yi} + a_y t = +20.0 - 9.80t = 0.00 m/s \xrightarrow{\text{Solve for t}} t = \frac{20.0}{9.80} = 2.04s$$

(b) Find the maximum height.

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2 = 50.0 + 20 \times 2.04 + \frac{1}{2} \times (-9.80) \times (2.04)^2$$

$$= 50.0 + 20.4 = 70.4(m)$$

Example of a Falling Object cnt'd

(c) Find the time the stone reaches back to its original height.

$$t = 2.04 \times 2 = 4.08s$$

(d) Find the velocity of the stone when it reaches its original height.

$$v_{yf} = v_{yi} + a_y t = 20.0 + (-9.80) \times 4.08 = -20.0(m/s)$$

(e) Find the velocity and position of the stone at t=5.00s.

Velocity

$$v_{yf} = v_{yi} + a_y t = 20.0 + (-9.80) \times 5.00 = -29.0(m/s)$$

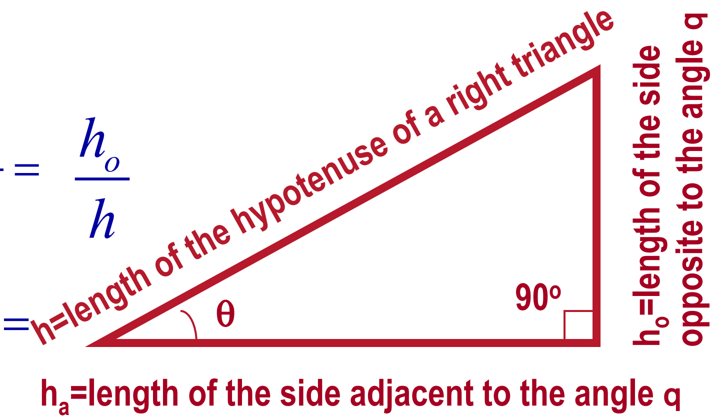
Position

$$y_f = y_i + v_{yi} t + \frac{1}{2} a_y t^2$$

$$= 50.0 + 20.0 \times 5.00 + \frac{1}{2} \times (-9.80) \times (5.00)^2 = +27.5(m)$$

Trigonometry Refresher

- Definitions of sinθ, cosθ and tanθ



$$\sin\theta = \frac{\text{Length of the opposite side to }\theta}{\text{Length of the hypotenuse of the right triangle}} = \frac{h_o}{h}$$

$$\cos\theta = \frac{\text{Length of the adjacent side to }\theta}{\text{Length of the hypotenuse of the right triangle}} = \frac{h_a}{h}$$

Solve for θ

$$\theta = \cos^{-1}\left(\frac{h_a}{h}\right)$$

$$\tan\theta = \frac{\text{Length of the opposite side to }\theta}{\text{Length of the adjacent side to }\theta} = \frac{h_o}{h_a}$$

Solve for θ

$$\theta = \tan^{-1}\left(\frac{h_o}{h_a}\right)$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{h_o}{\cancel{h}}}{\frac{h_a}{\cancel{h}}} = \frac{h_o}{h_a}$$

Pythagorian theorem: For right angle triangles

$$h^2 = h_o^2 + h_a^2 \Rightarrow h = \sqrt{h_o^2 + h_a^2}$$