PHYS 1441 – Section 002
Lecture #6

Monday, Feb. 4, 2013

Dr. Jaehoon Yu

- Properties and operations of vectors
- Components of the 2D Vector
- Understanding the 2 Dimensional Motion
- 2D Kinematic Equations of Motion
- Projectile Motion

Announcements

- 1st term exam
  - In class, Wednesday, Feb. 13
  - Coverage: CH1.1 – what we finish Monday, Feb. 11, plus Appendix A1 – A8
  - Mixture of free response problems and multiple choice problems
  - Please do not miss the exam! You will get an F if you miss any exams!
- Quiz #2
  - Early in class this Wednesday, Feb. 6
  - Covers CH1.6 – what we finish today (CH3.4?)
- Homework
  - I see that Quest now is accepting payments through Feb.25. Please take an action quickly so that your homework is undisrupted..

Vector and Scalar

Vector quantities have both magnitudes (sizes) and directions

Velocity, acceleration, force, momentum, etc

Normally denoted in **BOLD** letters, $\mathbf{F}$, or a letter with arrow on top $\vec{F}$

Their sizes or magnitudes are denoted with normal letters, F, or absolute values: $\left|\vec{F}\right|$ or $\left|\mathbf{F}\right|$

Scalar quantities have magnitudes only

Can be completely specified with a value and its unit

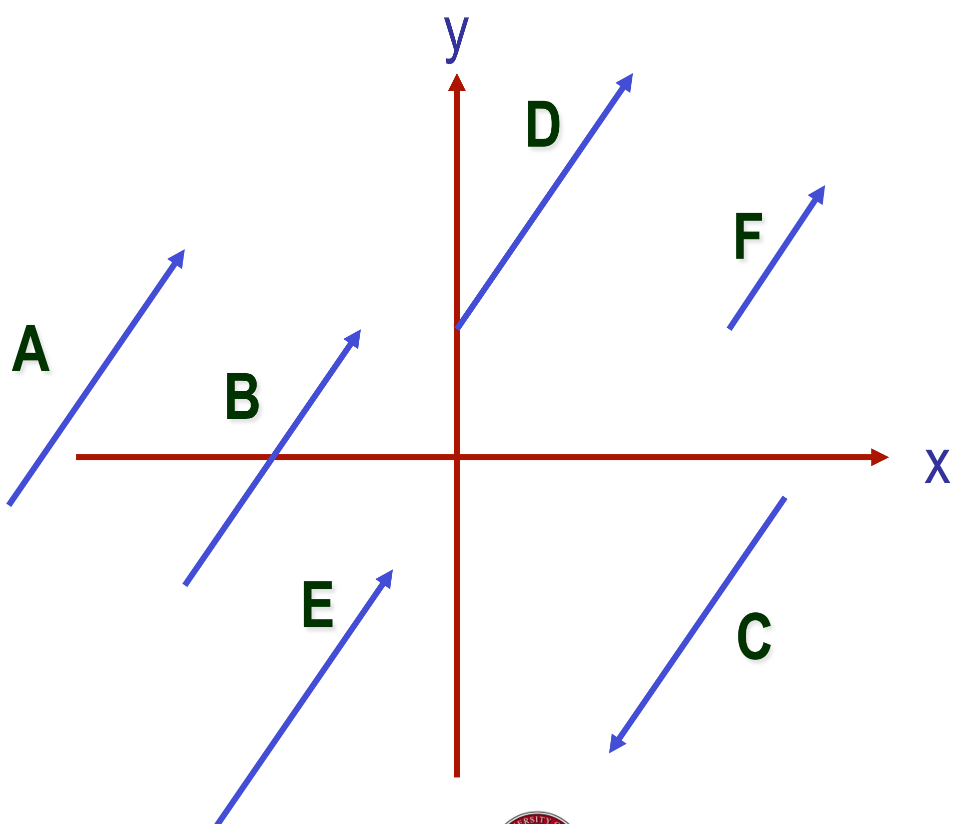
Speed, energy, heat, mass, time, etc

Normally denoted in normal letters, E

Both have units!!!

Properties of Vectors

- Two vectors are the same if their **sizes** and the **directions** are the same, no matter where they are on a coordinate system!! ➔ You can move them around as you wish as long as their directions and sizes are kept the same.



Which ones are the same vectors?

A=B=E=D

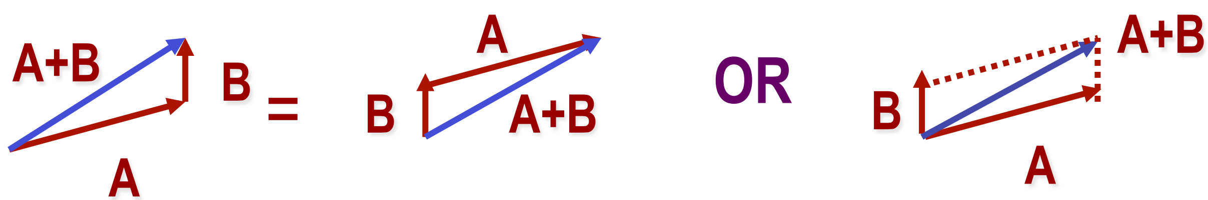
Why aren't the others?

C: The same magnitude but opposite direction: **C=-A:** A negative vector

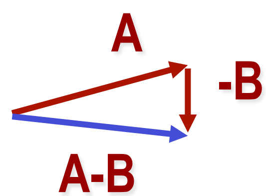
F: The same direction but different magnitude

Vector Operations

- Addition:
  - Triangular Method: One can add vectors by connecting the head of one vector to the tail of the other (head-to-tail)
  - Parallelogram method: Connect the tails of the two vectors and extend
  - Addition is commutative: Changing order of operation does not affect the results **A+B=B+A**, **A+B+C+D+E=E+C+A+B+D**



- Subtraction:
  - The same as adding a negative vector: **A** - **B** = **A** + (-**B**)



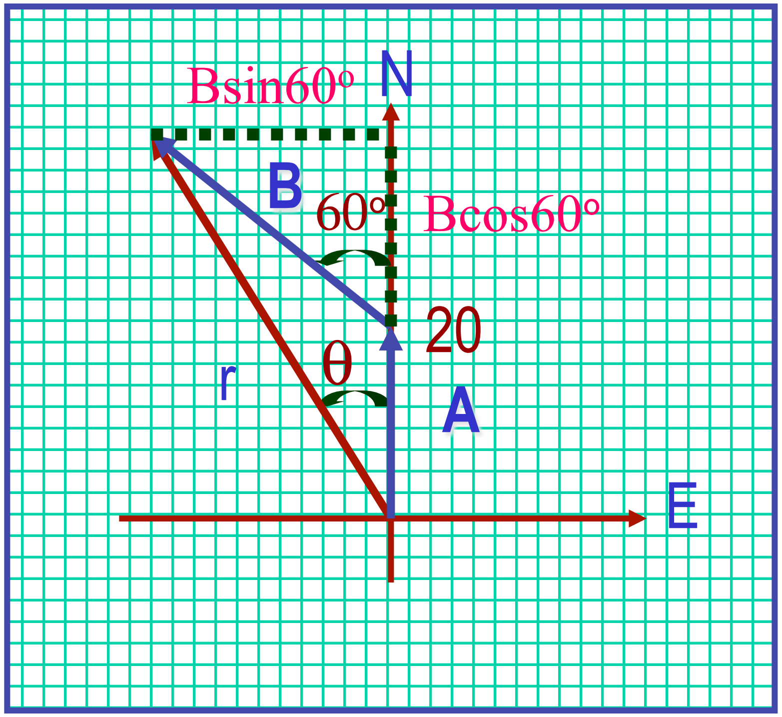
Since subtraction is the equivalent to adding a negative vector, subtraction is also commutative!!!

- Multiplication by a scalar is increasing the magnitude **A**, **B**=2**A**

$$\boxed{|\mathcal{B}| = 2|\mathcal{A}|}$$

Example for Vector Addition

A car travels 20.0km due north followed by 35.0km in a direction 60.0º West of North. Find the magnitude and direction of resultant displacement.



$$r = \sqrt{\left(A + B\cos\theta\right)^2 + \left(B\sin\theta\right)^2}$$

$$= \sqrt{A^2 + B^2\left(\cos^2\theta + \sin^2\theta\right) + 2AB\cos\theta}$$

$$= \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$$= \sqrt{(20.0)^2 + (35.0)^2 + 2\times 20.0\times 35.0\cos 60}$$

$$= \sqrt{2325} = 48.2(km)$$

$$\theta = \tan^{-1}\frac{\left|\vec{B}\right|\sin 60}{\left|\vec{A}\right| + \left|\vec{B}\right|\cos 60}$$

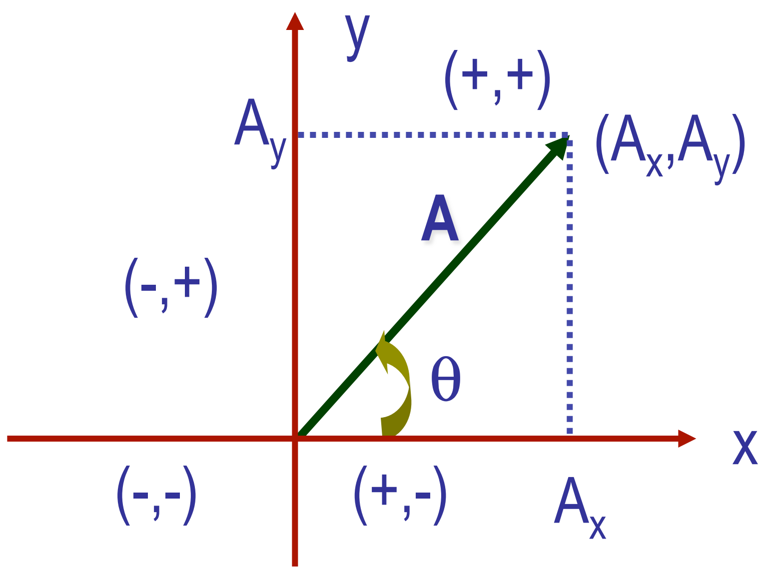
$$= \tan^{-1}\frac{35.0\sin 60}{20.0 + 35.0\cos 60}$$

$$= \tan^{-1}\frac{30.3}{37.5} = 38.9^\circ \text{ to W wrt N}$$

Do this using components!!

Components and Unit Vectors

Coordinate systems are useful in expressing vectors in their components



$$A_x = \left|\vec{A}\right|\cos\theta$$

$$A_y = \left|\vec{A}\right|\sin\theta$$

} Components

$$\left|\vec{A}\right| = \sqrt{A_x^{\,2} + A_y^{\,2}}$$

} Magnitude

$$\left|\vec{A}\right| = \sqrt{\left(\left|\vec{A}\right|\cos\theta\right)^2 + \left(\left|\vec{A}\right|\sin\theta\right)^2}$$

$$= \sqrt{\left|\vec{A}\right|^2\left(\cos^2\theta + \sin^2\theta\right)} = \left|\vec{A}\right|$$

Unit Vectors

- Unit vectors are the ones that tells us the directions of the components
  - Very powerful and makes vector notation and operations much easier!
- **Dimensionless**
- **Magnitudes these vectors are exactly 1**
- Unit vectors are usually expressed in **i, j, k** or $\vec{i},\ \vec{j},\ \vec{k}$

So a vector **A** can be expressed as

$$\vec{A} = A_x\vec{i} + A_y\vec{j} = \left|\vec{A}\right|\cos\theta\,\vec{i} + \left|\vec{A}\right|\sin\theta\,\vec{j}$$

Examples of Vector Operations

Find the resultant vector which is the sum of **A**=(2.0**i**+2.0**j**) and **B** =(2.0**i**-4.0**j**)

$$\vec{C} = \vec{A} + \vec{B} = \left(2.0\vec{i} + 2.0\vec{j}\right) + \left(2.0\vec{i} - 4.0\vec{j}\right)$$
$$= \left(2.0 + 2.0\right)\vec{i} + \left(2.0 - 4.0\right)\vec{j} = 4.0\vec{i} - 2.0\vec{j}\left(m\right)$$

$$\left|\vec{C}\right| = \sqrt{\left(4.0\right)^2 + \left(-2.0\right)^2}$$
$$= \sqrt{16 + 4.0} = \sqrt{20} = 4.5(m)$$

$$\theta = \tan^{-1}\frac{C_y}{C_x} = \tan^{-1}\frac{-2.0}{4.0} = -27^\circ$$

Find the resultant displacement of three consecutive displacements: $\mathbf{d_1}$=(15**i**+30**j** +12**k**)cm, $\mathbf{d_2}$=(23**i**+14**j** -5.0**k**)cm, and $\mathbf{d_3}$=(-13**i**+15**j**)cm

$$\vec{D} = \vec{d}_1 + \vec{d}_2 + \vec{d}_3 = \left(15\vec{i} + 30\vec{j} + 12\vec{k}\right) + \left(23\vec{i} + 14\vec{j} - 5.0\vec{k}\right) + \left(-13\vec{i} + 15\vec{j}\right)$$
$$= \left(15 + 23 - 13\right)\vec{i} + \left(30 + 14 + 15\right)\vec{j} + \left(12 - 5.0\right)\vec{k} = 25\vec{i} + 59\vec{j} + 7.0\vec{k}(cm)$$

Magnitude

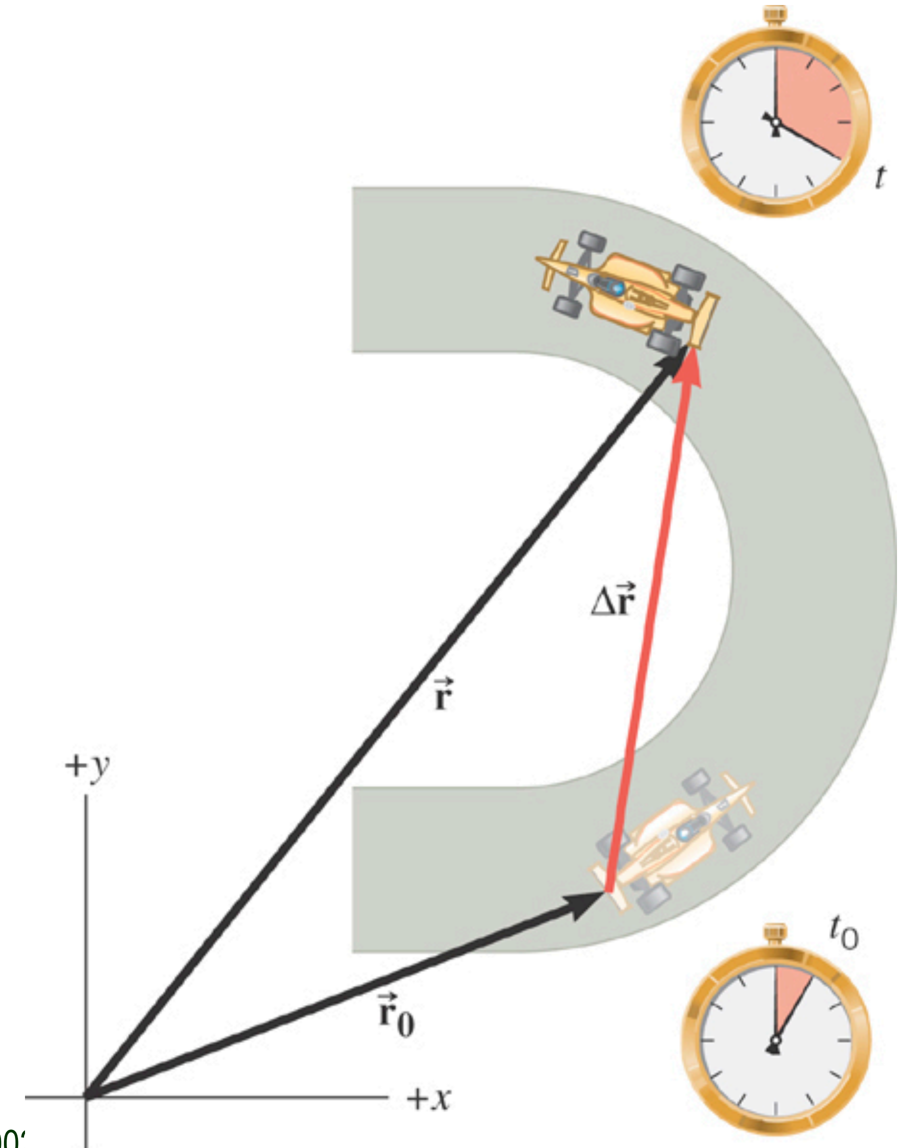
$$\left|\vec{D}\right| = \sqrt{\left(25\right)^2 + \left(59\right)^2 + \left(7.0\right)^2} = 65(cm)$$

2D Displacement

$\vec{\mathbf{r}}_o =$ initial position

$\vec{\mathbf{r}} =$ final position

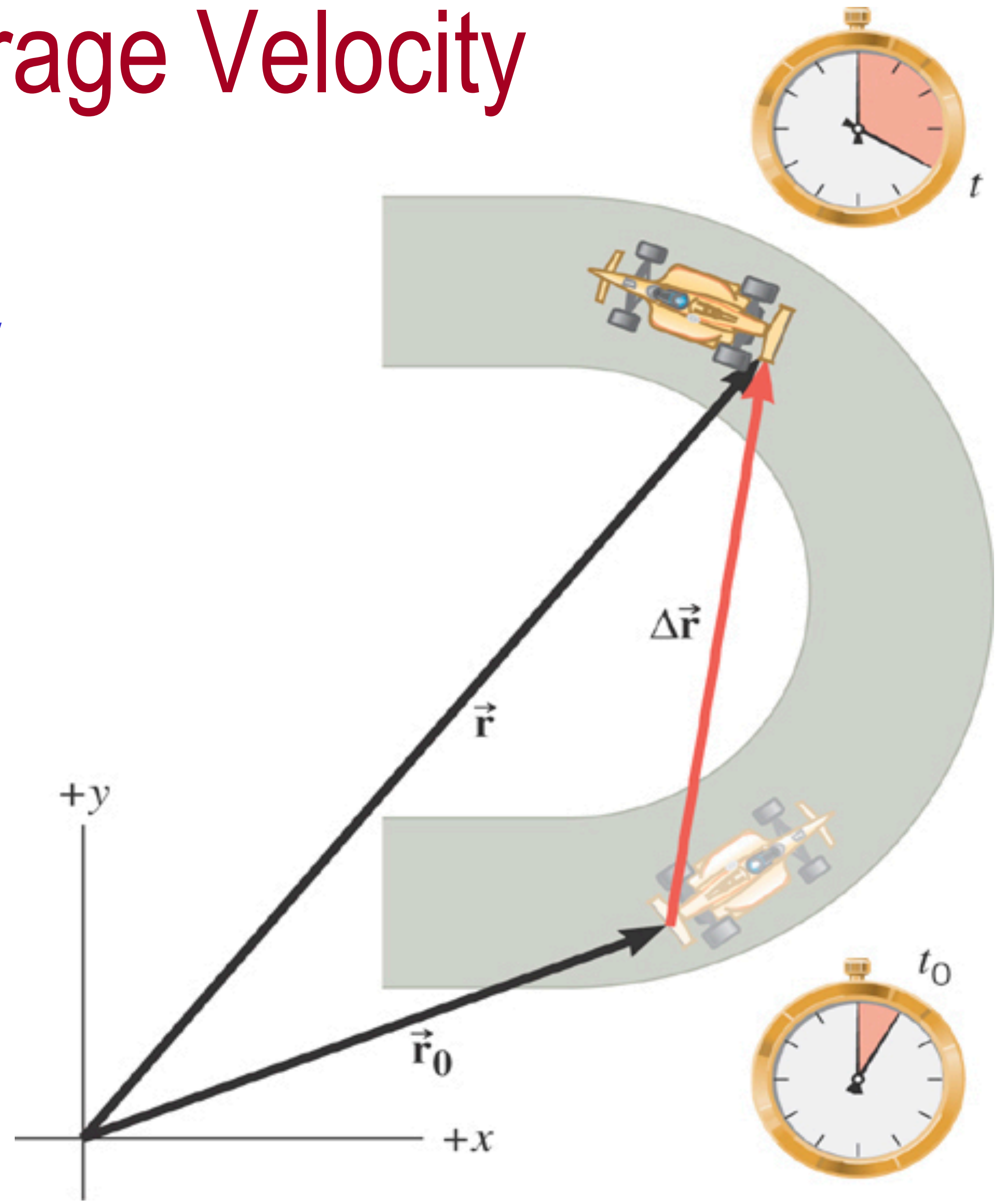
Displacement: $\Delta\vec{\mathbf{r}} = \vec{\mathbf{r}} - \vec{\mathbf{r}}_o$



2D Average Velocity

Average velocity is the displacement divided by the elapsed time.

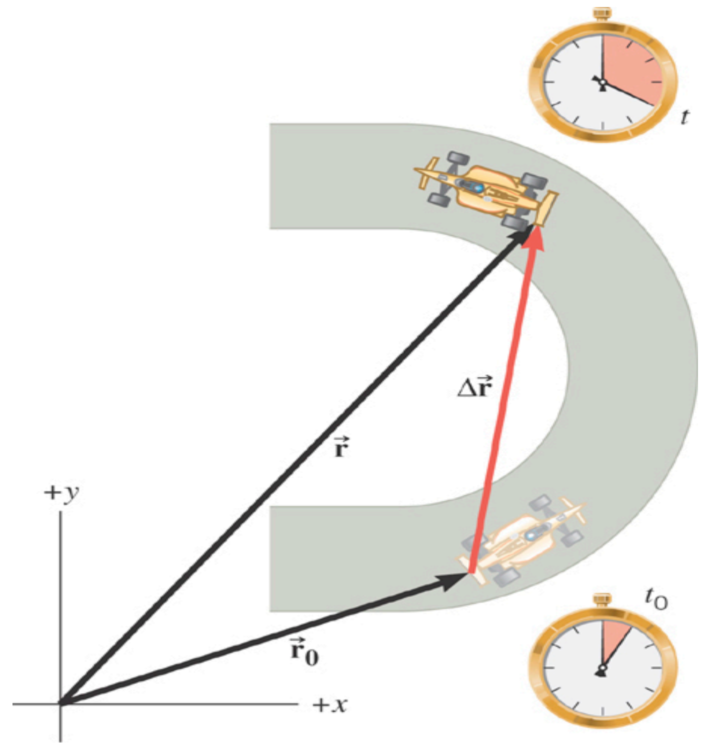
$$\overline{\vec{\mathbf{v}}} = \frac{\vec{\mathbf{r}} - \vec{\mathbf{r}}_o}{t - t_o} = \frac{\Delta\vec{\mathbf{r}}}{\Delta t}$$

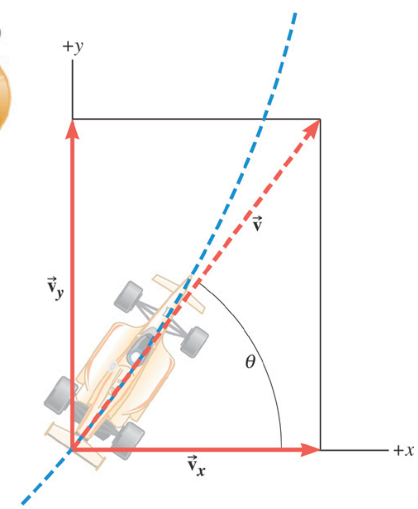


2D Instantaneous Velocity

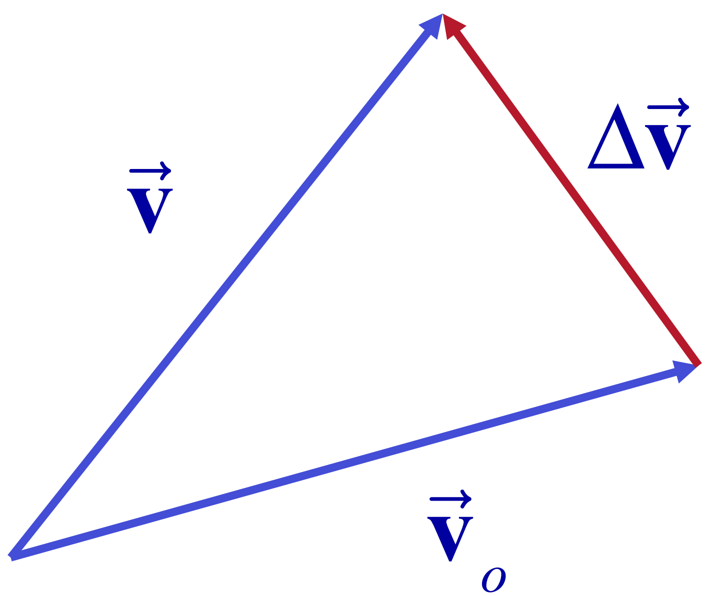
The ***instantaneous velocity*** indicates how fast the car moves and the direction of motion at each instant of time.

$$\vec{\mathbf{v}} = \lim_{\Delta t \to 0} \frac{\Delta \vec{\mathbf{r}}}{\Delta t}$$





2D Average Acceleration



$$\bar{\vec{a}} = \frac{\vec{v} - \vec{v}_o}{t - t_o} = \frac{\Delta\vec{v}}{\Delta t}$$

Displacement, Velocity, and Acceleration in 2-dim

- Displacement: $$\Delta\vec{r} = \vec{r}_f - \vec{r}_i$$
- Average Velocity: $$\vec{v} \equiv \frac{\Delta\vec{r}}{\Delta t} = \frac{\vec{r}_f - \vec{r}_i}{t_f - t_i}$$
- Instantaneous Velocity: $$\vec{v} \equiv \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t}$$
- Average Acceleration $$\vec{a} \equiv \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i}$$
- Instantaneous Acceleration: $$\vec{a} \equiv \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t}$$

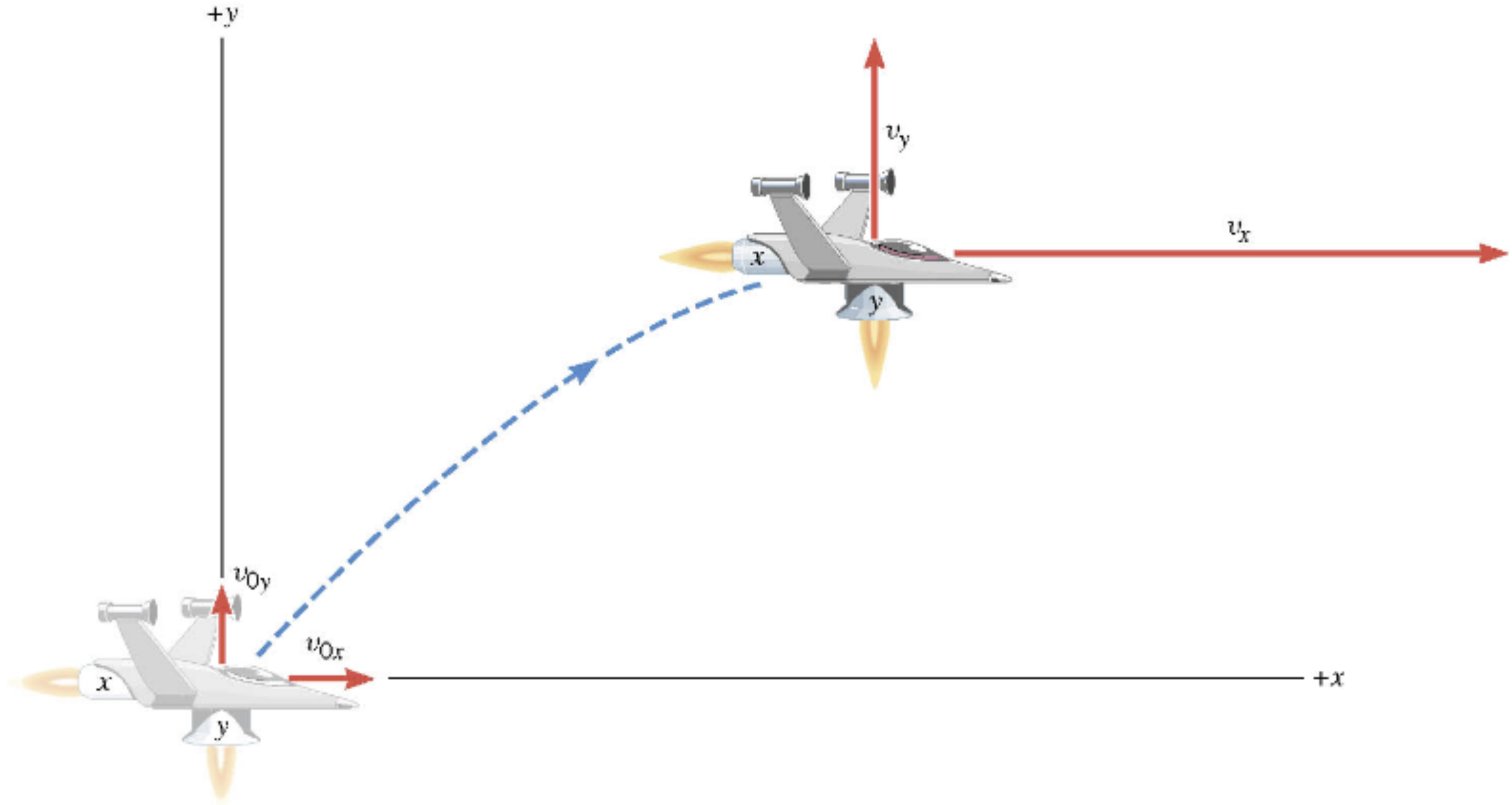
How is each of these quantities defined in 1-D?

Kinematic Quantities in 1D and 2D

Quantities	1 Dimension	2 Dimension
Displacement	$\Delta x = x_f - x_i$	$\Delta \vec{r} = \vec{r}_f - \vec{r}_i$
Average Velocity	$v_x = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$	$\vec{v} \equiv \frac{\Delta \vec{r}}{\Delta t} = \frac{\vec{r}_f - \vec{r}_i}{t_f - t_i}$
Inst. Velocity	$v_x \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$	$\vec{v} \equiv \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t}$
Average Acc.	$a_x \equiv \frac{\Delta v_x}{\Delta t} = \frac{v_{xf} - v_{xi}}{t_f - t_i}$	$\vec{a} \equiv \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i}$
Inst. Acc.	$a_x \equiv \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t}$	$\vec{a} \equiv \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t}$

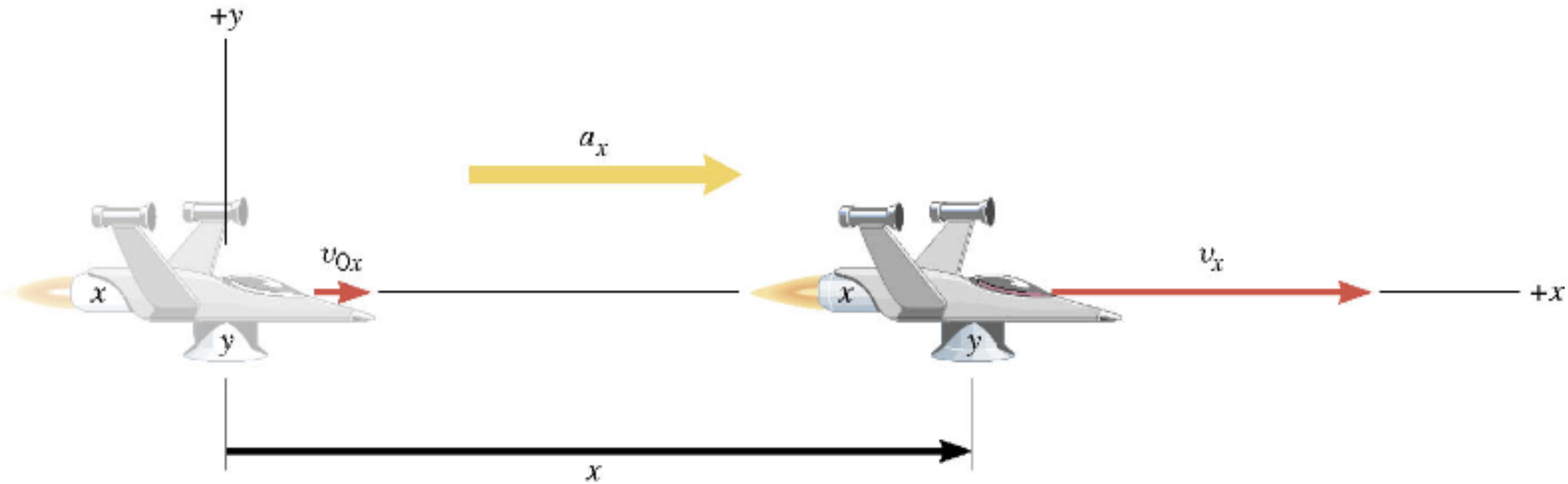
What is the difference between 1D and 2D quantities?

A Motion in 2 Dimension



This is a motion that could be viewed as two motions combined into one. (superposition…)

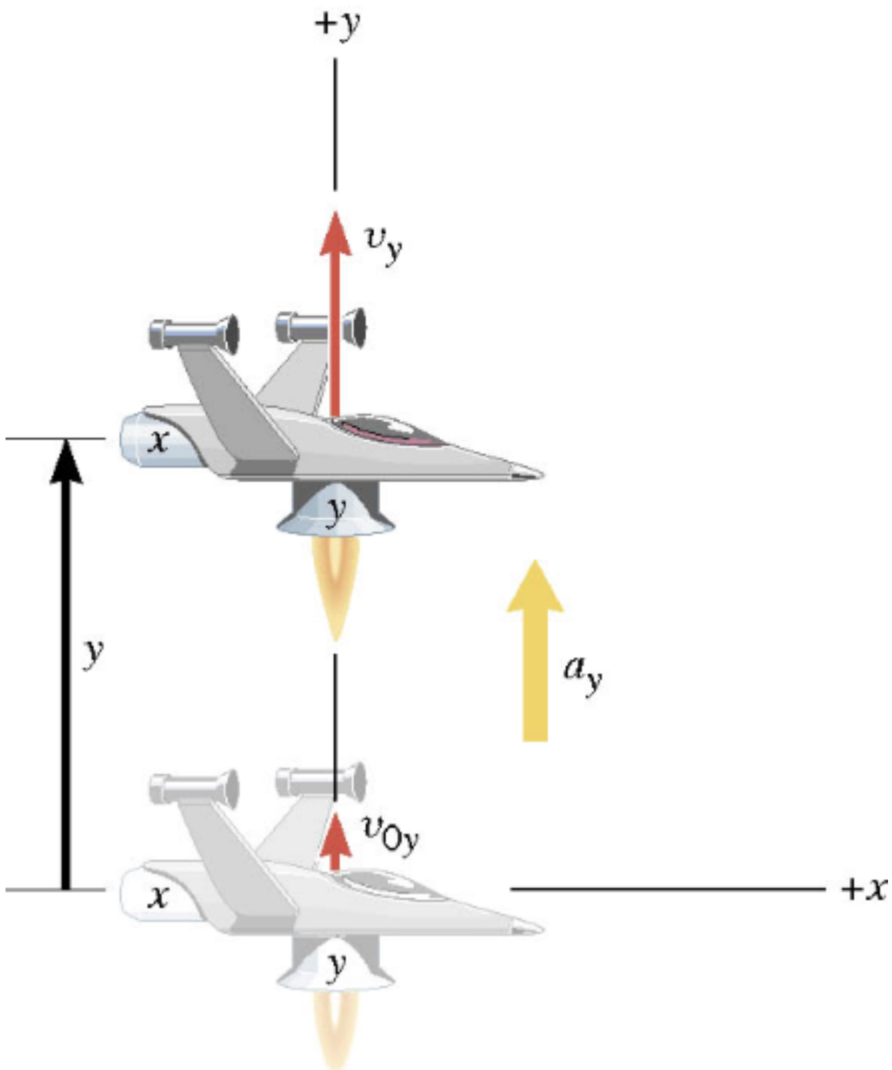
Motion in horizontal direction (x)



$$v_x = v_{xo} + a_x t \qquad x = \frac{1}{2}\left(v_{xo} + v_x\right)t$$

$$v_x^2 = v_{xo}^2 + 2a_x x \qquad x = v_{xo}t + \frac{1}{2}a_x t^2$$

Motion in vertical direction (y)



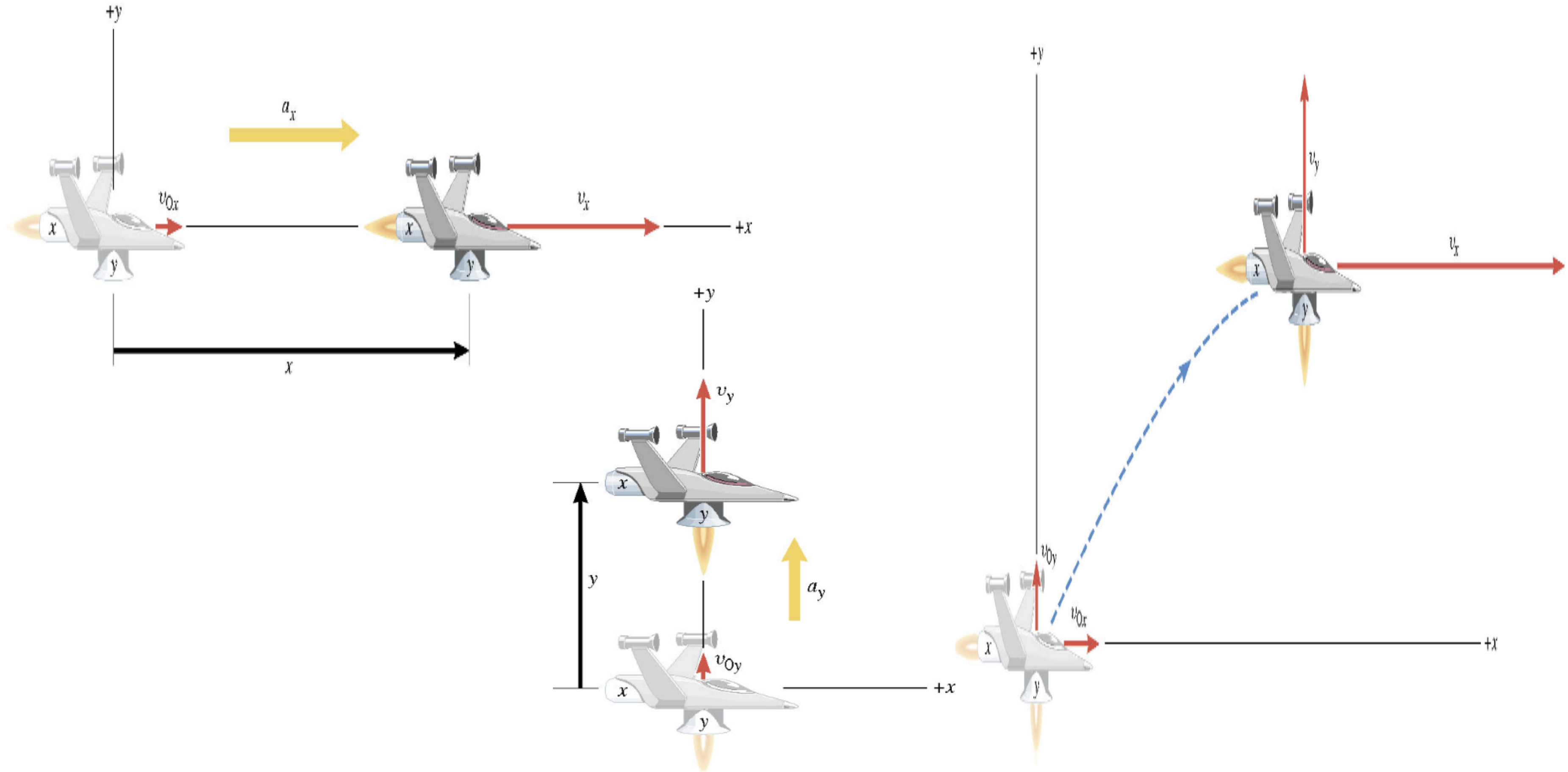
$$v_y = v_{yo} + a_y t$$

$$y = \frac{1}{2}\left(v_{yo} + v_y\right)t$$

$$v_y^2 = v_{yo}^2 + 2a_y y$$

$$y = v_{yo}t + \frac{1}{2}a_y t^2$$

A Motion in 2 Dimension



Imagine you add the two 1 dimensional motions on the left.
It would make up a one 2 dimensional motion on the right.

Kinematic Equations in 2-Dim

x-component

$$v_x = v_{xo} + a_x t$$

$$x = \frac{1}{2}\left(v_{xo} + v_x\right)t$$

$$v_x^2 = v_{xo}^2 + 2a_x x$$

$$\Delta x = v_{xo}t + \frac{1}{2}a_x t^2$$

y-component

$$v_y = v_{yo} + a_y t$$

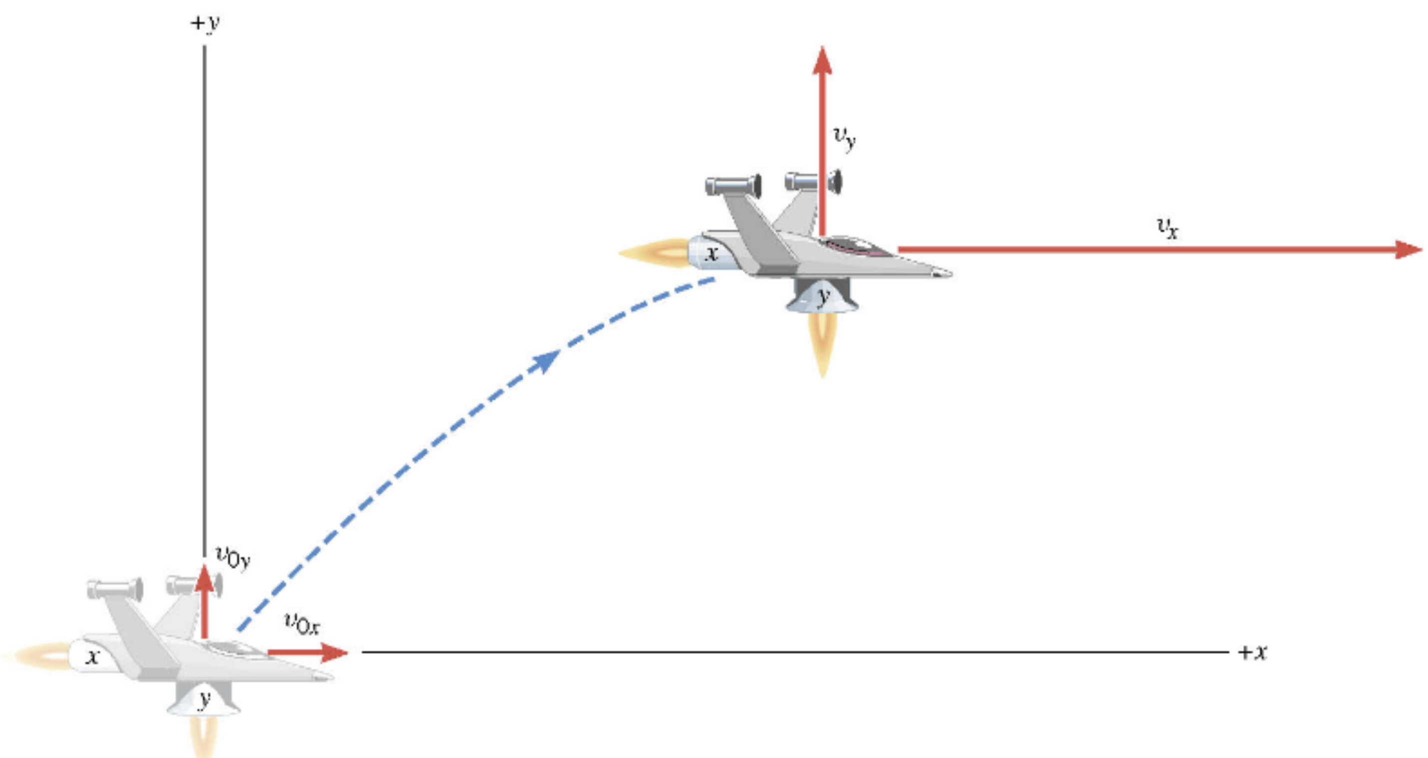
$$y = \frac{1}{2}\left(v_{yo} + v_y\right)t$$

$$v_y^2 = v_{yo}^2 + 2a_y y$$

$$\Delta y = v_{yo}t + \frac{1}{2}a_y t^2$$

Ex. A Moving Spacecraft

In the *x* direction, the spacecraft in zero-gravity zone has an initial velocity component of +22 m/s and an acceleration of +24 m/s^2. In the *y* direction, the analogous quantities are +14 m/s and an acceleration of +12 m/s^2. Find (a) *x* and v_x, (b) *y* and v_y, and (c) the final velocity of the spacecraft at time 7.0 s.



How do we solve this problem?

1. Visualize the problem → Draw a picture!
2. Decide which directions are to be called positive (+) and negative (-).
3. Write down the values that are given for any of the five kinematic variables associated with each direction.
4. Verify that the information contains values for at least three of the kinematic variables. Do this for *x* and *y* *separately.* Select the appropriate equation.
5. When the motion is divided into segments, remember that the final velocity of one segment is the initial velocity for the next.
6. Keep in mind that there may be two possible answers to a kinematics problem.

Ex. continued

In the x direction, the spacecraft in a zero gravity zone has an initial velocity component of +22 m/s and an acceleration of +24 m/s^2. In the y direction, the analogous quantities are +14 m/s and an acceleration of +12 m/s^2. Find (a) x and v_x, (b) y and v_y, and (c) the final velocity of the spacecraft at time 7.0 s.

x	a_x	v_x	v_{ox}	t
?	+24.0 m/s^2	?	+22.0 m/s	7.0 s

y	a_y	v_y	v_{oy}	t
?	+12.0 m/s^2	?	+14.0 m/s	7.0 s

First, the motion in x-direciton…

x	a_x	v_x	v_{ox}	t
?	+24.0 m/s^2	?	+22 m/s	7.0 s

$$\Delta x = v_{ox}t + \tfrac{1}{2}a_x t^2$$

$$= (22\,\mathrm{m/s})(7.0\ \mathrm{s}) + \tfrac{1}{2}(24\,\mathrm{m/s^2})(7.0\ \mathrm{s})^2 = +740\ \mathrm{m}$$

$$v_x = v_{ox} + a_x t$$

$$= (22\,\mathrm{m/s}) + (24\,\mathrm{m/s^2})(7.0\ \mathrm{s}) = +190\,\mathrm{m/s}$$

Now, the motion in y-direction…

y	a_y	v_y	v_{oy}	t
?	+12.0 m/s^2	?	+14 m/s	7.0 s

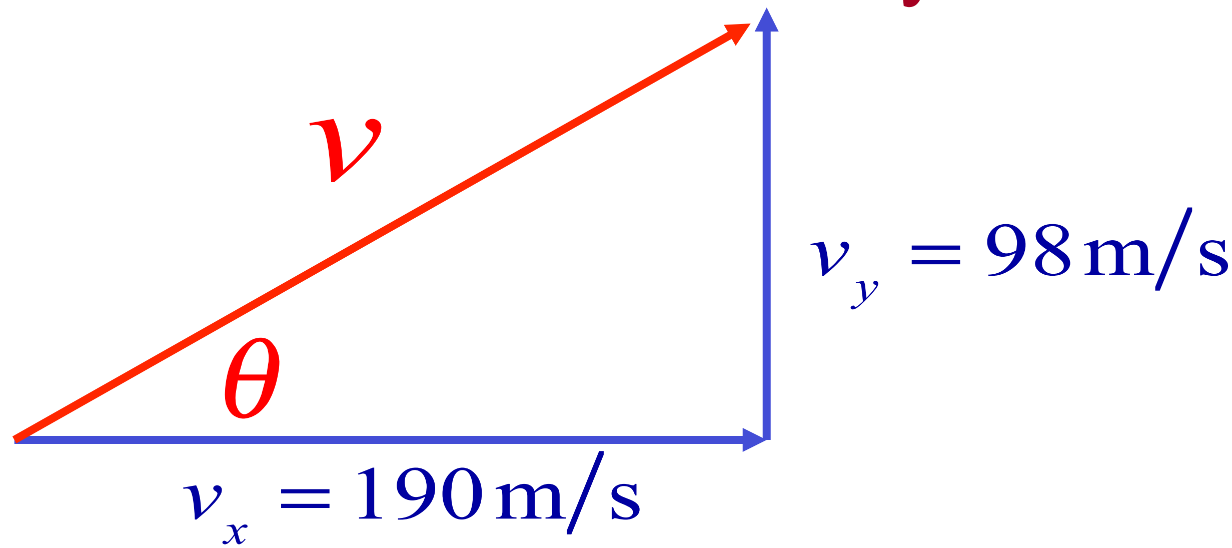
$$\Delta y = v_{oy}t + \tfrac{1}{2}a_y t^2$$

$$= (14\,\mathrm{m/s})(7.0\ \mathrm{s}) + \tfrac{1}{2}(12\,\mathrm{m/s^2})(7.0\ \mathrm{s})^2 = +390\ \mathrm{m}$$

$$v_y = v_{oy} + a_y t$$

$$= (14\,\mathrm{m/s}) + (12\,\mathrm{m/s^2})(7.0\ \mathrm{s}) = +98\,\mathrm{m/s}$$

The final velocity…



$$v = \sqrt{(190\,\mathrm{m/s})^2 + (98\,\mathrm{m/s})^2} = 210\,\mathrm{m/s}$$

$$\theta = \tan^{-1}(98/190) = 27^\circ$$

A vector can be fully described when the magnitude and the direction are given. Any other way to describe it?

Yes, you are right! Using components and unit vectors!!

$$\vec{v} = v_x\vec{i} + v_y\vec{j} = (190\vec{i} + 98\vec{j})\,m/s$$

If we visualize the motion…