# PHYS 1441 – Section 002
# Lecture #7

*Wednesday, Feb. 6, 2013*
*Dr. Jaehoon Yu*

- What is the Projectile Motion?
- How do we solve projectile motion problems?
- Maximum Range and Height

Today's homework is homework #5, due 11pm, Tuesday, Feb. 12!!

# Announcements

- 1st term exam
  - In class, Wednesday, Feb. 13
  - Coverage: CH1.1 – what we finish Monday, Feb. 11, plus Appendix A1 – A8
  - Mixture of free response problems and multiple choice problems
  - Please do not miss the exam!   You will get an F if you miss any exams!

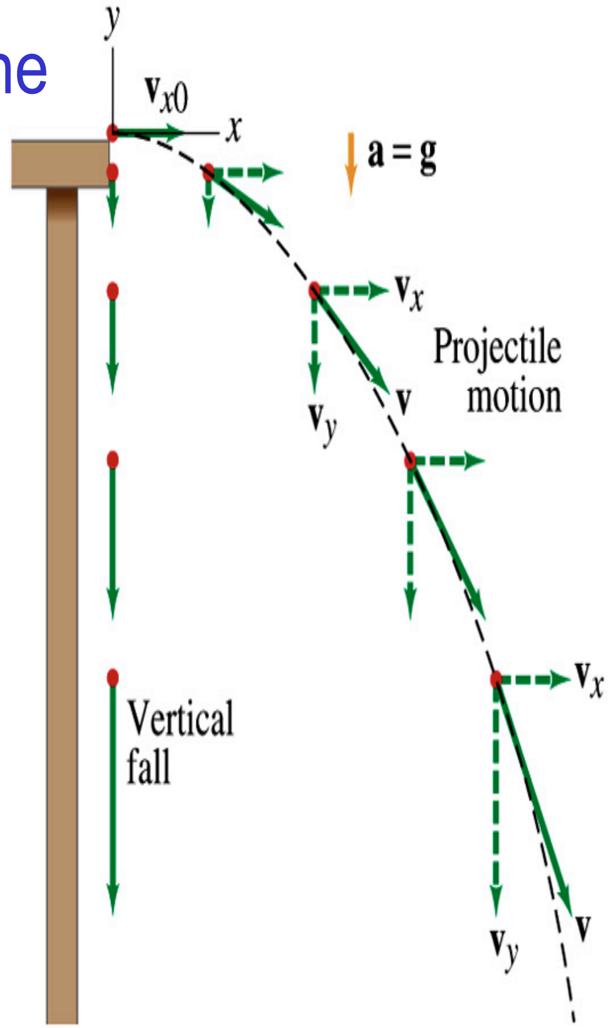
# Special Project #2 for Extra Credit

- Show that the trajectory of a projectile motion is a parabola!!
  - 20 points
  - Due: Monday, Feb. 18
  - You MUST show full details of your OWN computations to obtain any credit
    - Beyond what was covered in page 7 of this lecture note!!

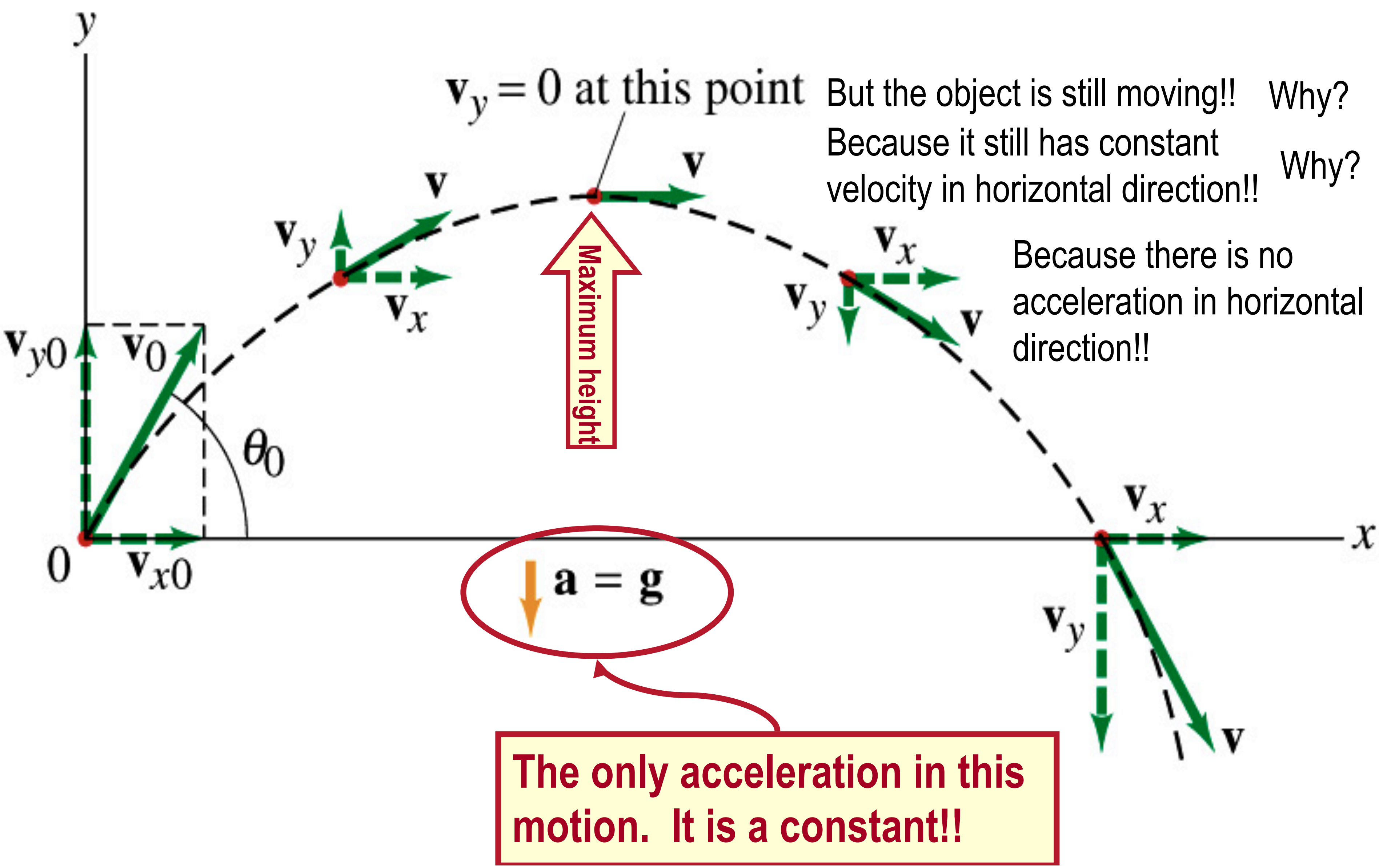
# What is the Projectile Motion?

- A 2-dim motion of an object under the gravitational acceleration with the following assumptions
  - Free fall acceleration, $g$, is **constant** over the range of the motion
    - $\vec{g} = -9.8\vec{j}\ (m/s^2)$
    - $a_x = 0\,m/s^2$ and $a_y = -9.8\,m/s^2$
  - Air resistance and other effects are negligible
- A motion under constant acceleration!!!! ➔ Superposition of two motions
  - Horizontal motion with constant velocity ( **no acceleration** ) $v_{xf} = v_{x0}$
  - Vertical motion under constant acceleration ( $g$ )

$$v_{yf} = v_{y0} + a_y t = v_{y0} + (-9.8)t$$



# Projectile Motion



# Kinematic Equations for a projectile motion

**x-component**

$$a_x = 0$$

$$v_x = v_{xo}$$

$$\Delta x = v_{xo} t$$

$$v_{x0}^2 = v_{xo}^2$$

$$\Delta x = v_{xo} t$$

**y-component**

$$a_y = -\left|\vec{g}\right| = -9.8\,m/s^2$$

$$v_y = v_{yo} - gt$$

$$\Delta y = \tfrac{1}{2}\left(v_{yo} + v_y\right) t$$

$$v_y^2 = v_{yo}^2 - 2gy$$

$$\Delta y = v_{yo} t - \tfrac{1}{2} g t^2$$

## *Show that a projectile motion is a parabola!!!*

x-component $v_{xi} = v_i \cos\theta_i$

y-component $v_{yi} = v_i \sin\theta_i$

$$\vec{a} = a_x \vec{i} + a_y \vec{j} = -g\vec{j}$$

In a projectile motion, the only acceleration is gravitational one whose direction is always toward the center of the earth (downward).

$a_x=0$

$$x_f = v_{xi}t = v_i \cos\theta_i t$$

$$\boxed{t = \frac{x_f}{v_i \cos\theta_i}}$$

$$y_f = v_{yi}t + \frac{1}{2}(-g)\, t^2 = v_i \sin\theta_i t - \frac{1}{2} g t^2$$

Plug t into the above

$$y_f = v_i \sin\theta_i \left(\frac{x_f}{v_i \cos\theta_i}\right) - \frac{1}{2} g \left(\frac{x_f}{v_i \cos\theta_i}\right)^2$$

$$\boxed{y_f = x_f \tan\theta_i - \left(\frac{g}{2v_i^2 \cos^2\theta_i}\right) x_f^2}$$

What kind of parabola is this?

# Example for a Projectile Motion

A ball is thrown with an initial velocity **v**=(20**i**+40**j**)m/s. Estimate the time of flight and the distance from the original position when the ball lands.

Which component determines the flight time and the distance?

Flight time is determined by the $y$ component, because the ball stops moving when it is on the ground after the flight.

$$y_f = 40t + \frac{1}{2}(-g)t^2 = 0m$$

$$t(80 - gt) = 0$$

So the possible solutions are…

$$\therefore t = 0 \text{ or } t = \frac{80}{g} \approx 8\sec$$

$$\therefore t \approx 8\sec$$

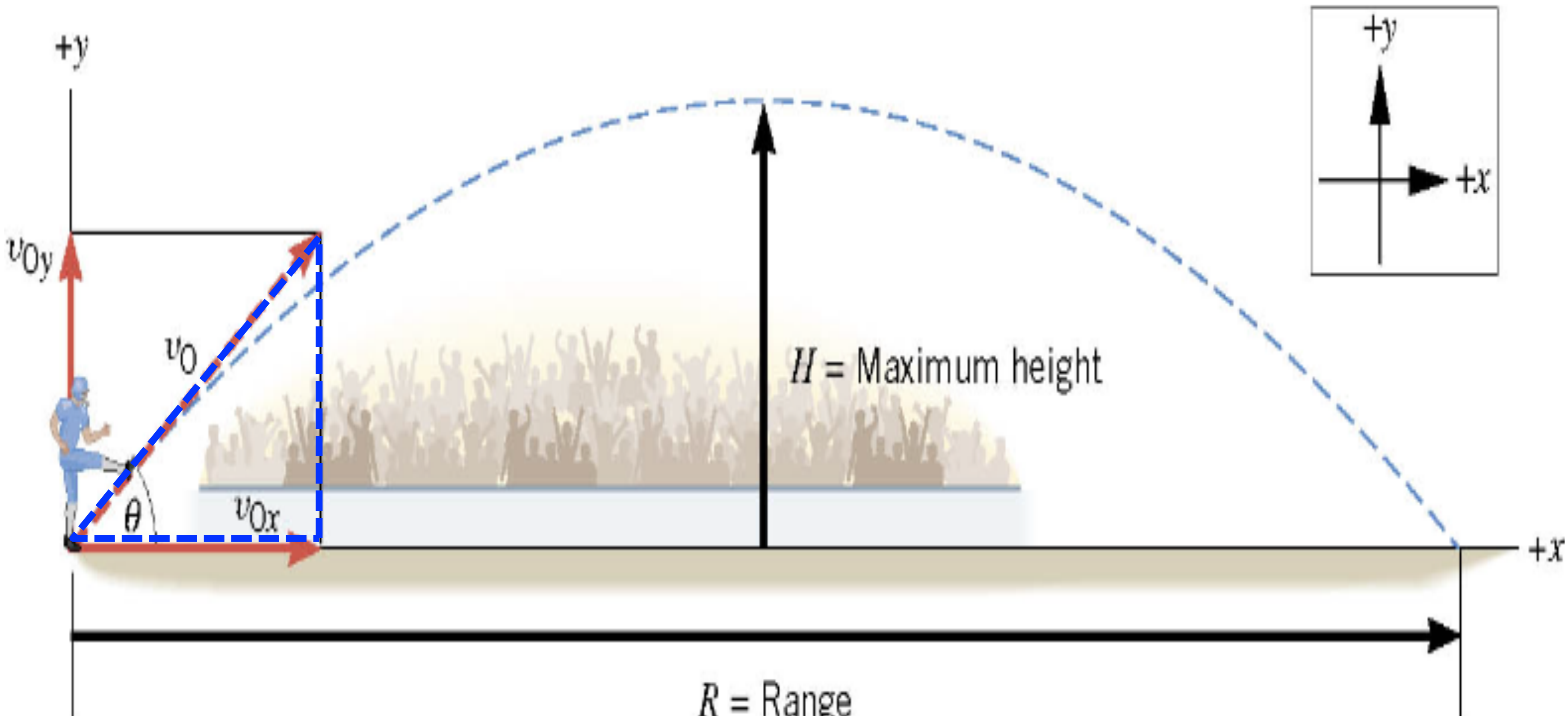
Why isn't t 0 the solution?

Distance is determined by the $x$ component in 2-dim, because the ball is at $y=0$ position when it completed it's flight.

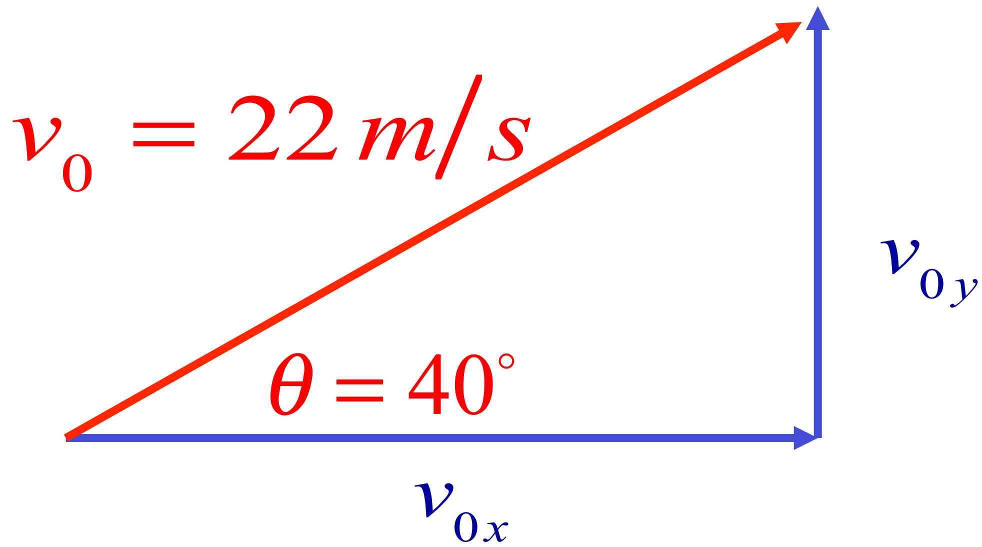
$$x_f = v_{xi}t = 20 \times 8 = 160(m)$$

# *Ex.3.5* The Height of a Kickoff

A placekicker kicks a football at an angle of 40.0 degrees and the initial speed of the ball is 22 m/s. Ignoring air resistance, determine the maximum height that the ball attains.



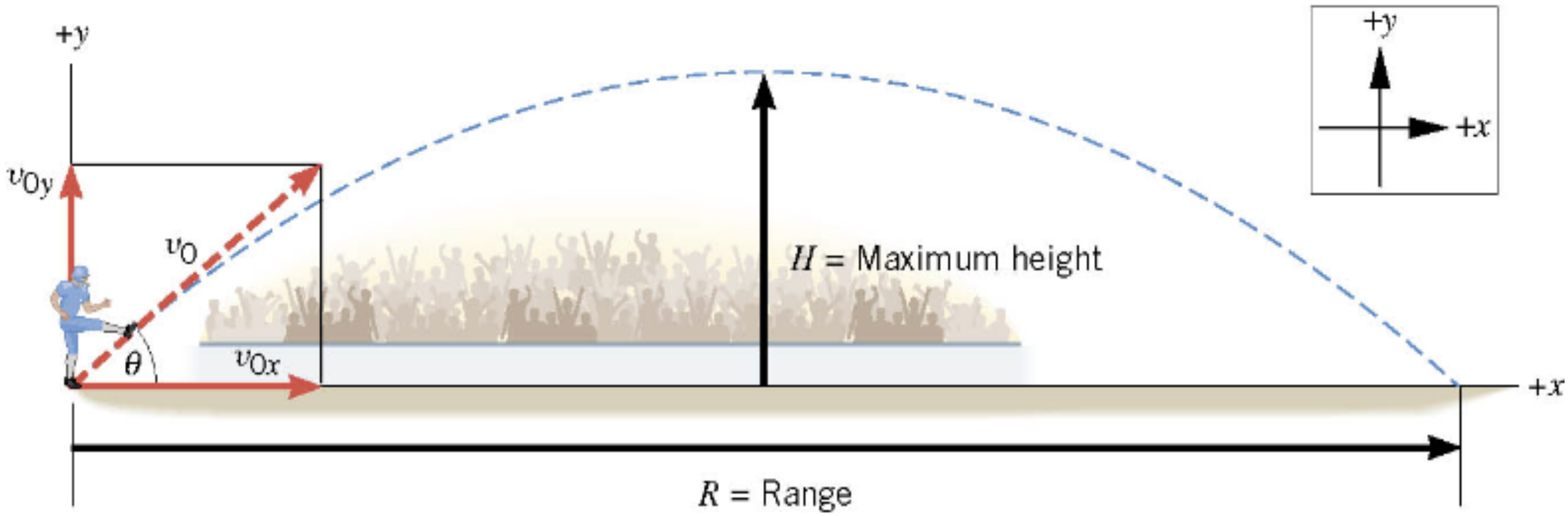
# First, the initial velocity components




$$v_{0x} = v_o \cos\theta = (22\,\mathrm{m/s})\cos 40^\circ = 17\,\mathrm{m/s}$$

$$v_{0y} = v_o \sin\theta = (22\,\mathrm{m/s})\sin 40^\circ = 14\,\mathrm{m/s}$$

# Motion in y-direction is of the interest..




| $y$ | $a_y$ | $v_y$ | $v_{0y}$ | $t$ |
|---|---|---|---|---|
| ? | -9.8 m/s$^2$ | 0 m/s | +14 m/s | |

# Now the nitty, gritty calculations…

| $y$ | $a_y$ | $v_y$ | $v_{0y}$ | $t$ |
|---|---|---|---|---|
| ? | -9.80 m/s$^2$ | 0 | 14 m/s | |

What happens at the maximum height?

The ball's velocity in y-direction becomes 0!!

And the ball's velocity in x-direction? Stays the same!! Why?

Because there is no acceleration in x-direction!!

Which kinematic formula would you like to use?

$$v_y^2 = v_{oy}^2 + 2a_y y$$

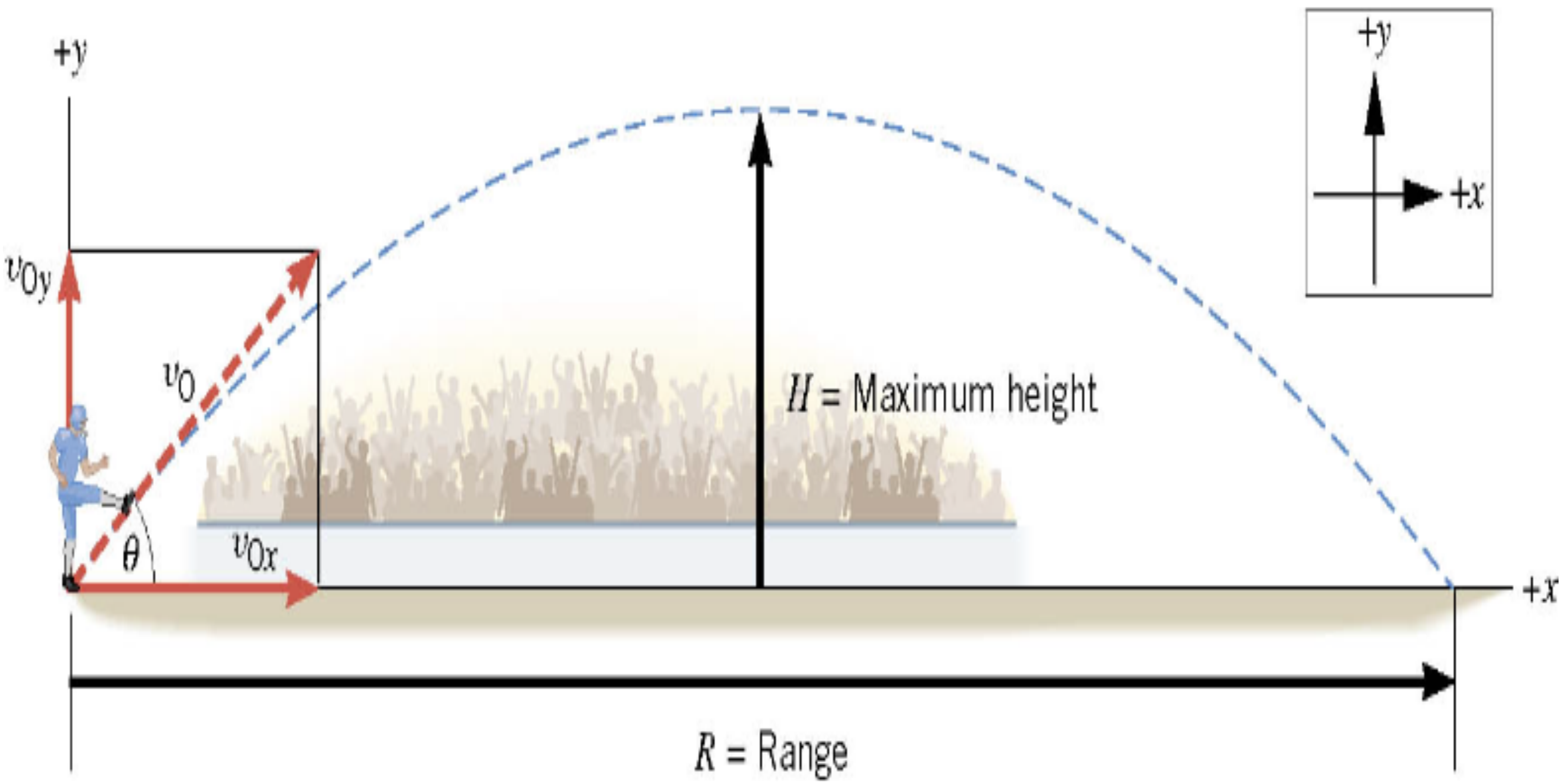
Solve for y

$$y = \frac{v_y^2 - v_{oy}^2}{2a_y}$$

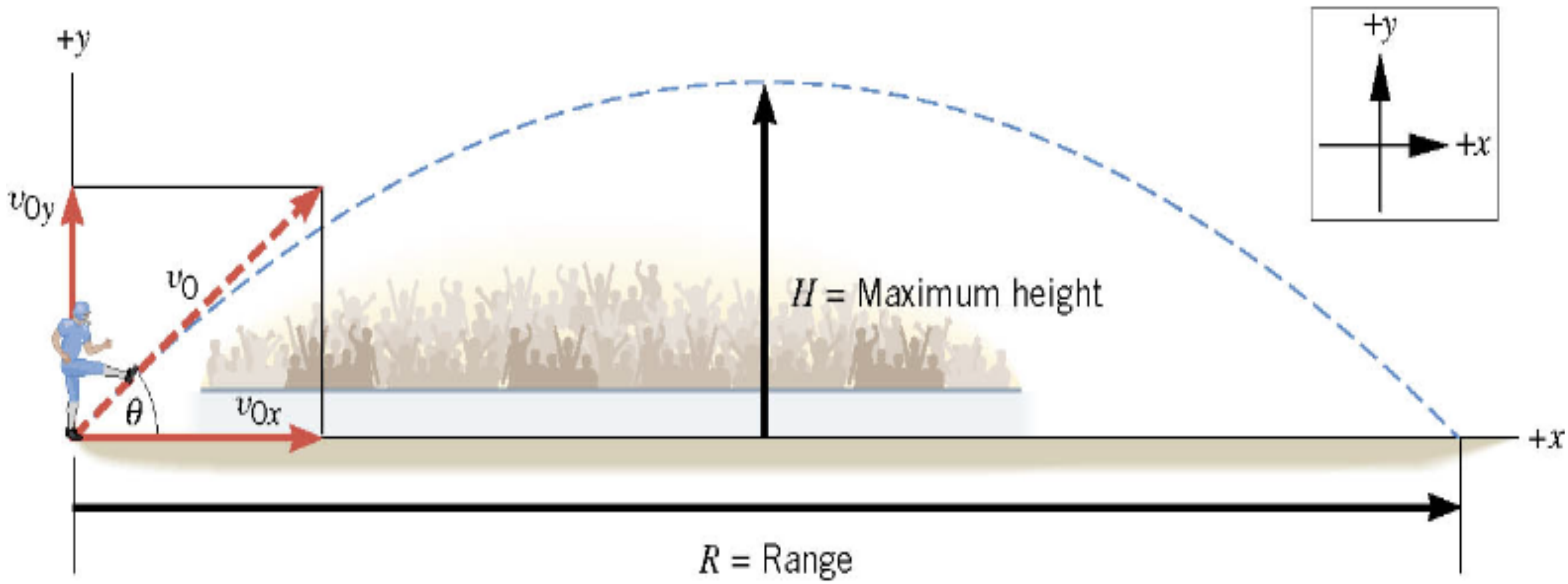
$$y = \frac{0 - (14\,\mathrm{m/s})^2}{2(-9.8\,\mathrm{m/s^2})} = +10\ \mathrm{m}$$

# *Ex.3.9 extended:* The Time of Flight of a Kickoff

What is the time of flight between kickoff and landing?



# What is y when it reached the max range?




| $y$ | $a_y$ | $v_y$ | $v_{oy}$ | $t$ |
|---|---|---|---|---|
| 0 m | -9.80 m/s$^2$ | | 14 m/s | ? |

# Now solve the kinematic equations in y direction!!

| $y$ | $a_y$ | $v_y$ | $v_{oy}$ | $t$ |
|---|---|---|---|---|
| 0 | -9.80 m/s$^2$ | | 14 m/s | ? |

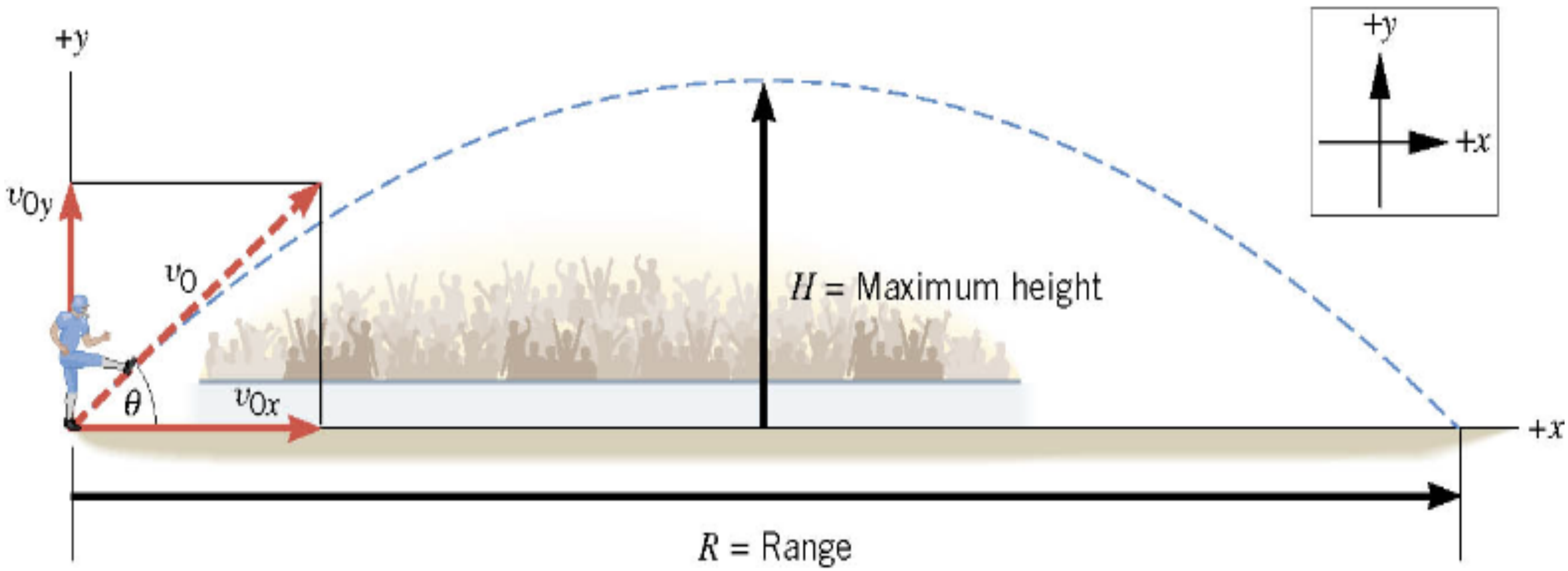
$$y = v_{oy}t + \tfrac{1}{2}a_y t^2 \xrightarrow{\text{Since } y=0} 0 = v_{oy}t + \tfrac{1}{2}a_y t^2 = t\left(v_{oy} + \tfrac{1}{2}a_y t\right)$$

Two soultions $t = 0$ or

$$v_{oy} + \tfrac{1}{2}a_y t = 0 \xrightarrow{\text{Solve for t}} t = \frac{-v_{oy}}{\frac{1}{2}a_y} = \frac{-2v_{oy}}{a_y} = \frac{-2\cdot 14}{-9.8} = 2.9s$$

# Ex.3.9 The Range of a Kickoff

## Calculate the range R of the projectile.




$$x = v_{ox}t + \tfrac{1}{2}a_x t^2 = v_{ox}t = (17\,\mathrm{m/s})(2.9\ \mathrm{s}) = +49\ \mathrm{m}$$

# Example for a Projectile Motion

- A stone was thrown upward from the top of a cliff at an angle of 37° to horizontal with initial speed of 65.0m/s. If the height of the cliff is 125.0m, how long is it before the stone hits the ground?

$$v_{xi} = v_i \cos\theta_i = 65.0 \times \cos 37^\circ = 51.9 m/s$$

$$v_{yi} = v_i \sin\theta_i = 65.0 \times \sin 37^\circ = 39.1 m/s$$

$$y_f = -125.0 = v_{yi}t - \frac{1}{2}gt^2$$

**Becomes**

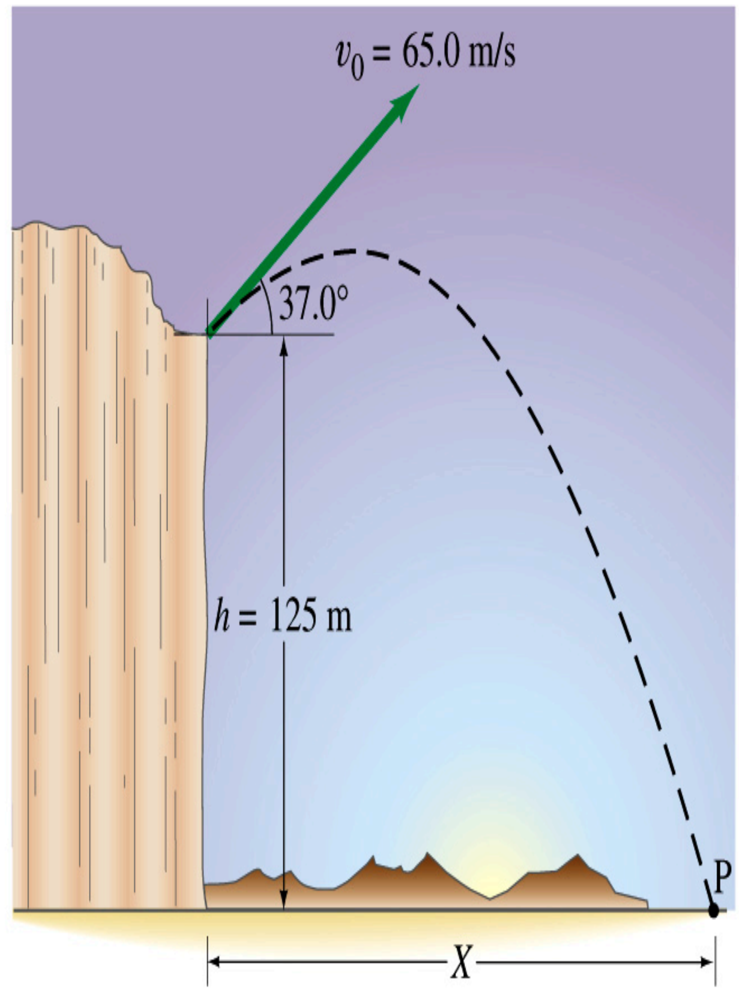
$$gt^2 - 78.2t - 250 = 9.80t^2 - 78.2t - 250 = 0$$

$$t = \frac{78.2 \pm \sqrt{(-78.2)^2 - 4 \times 9.80 \times (-250)}}{2 \times 9.80}$$

$$t = -2.43s \quad \text{or} \quad t = 10.4s$$

$$\boxed{t = 10.4s}$$

Since negative time does not exist.



# Example cont'd

- What is the speed of the stone just before it hits the ground?

$$v_{xf} = v_{xi} = v_i \cos\theta_i = 65.0 \times \cos 37^\circ = 51.9 m/s$$

$$v_{yf} = v_{yi} - gt = v_i \sin\theta_i - gt = 39.1 - 9.80 \times 10.4 = -62.8 m/s$$

$$|v| = \sqrt{v_{xf}^2 + v_{yf}^2} = \sqrt{51.9^2 + (-62.8)^2} = 81.5 m/s$$

- What are the maximum height and the maximum range of the stone?

**Do these yourselves at home for fun!!!**