PHYS 1441 – Section 002
Lecture #13

Monday, March 4, 2013

Dr. Jaehoon Yu

- Newton's Law of Universal Gravitation
- Motion in Resistive Force
- Work done by a constant force

Announcements

- Quiz 3 Results
  - Class average: 21/40
    - Equivalent to 52.5/100
    - Previous scores: 65/100 and 60/100
  - Top score: 39/40
- Midterm comprehensive exam
  - Wednesday, Mar. 20
  - In SH103
  - Covers CH1.1 through what we learn this Wednesday
  - Will prepare a 150 problem mid-term preparation set for you
    - Will distribute in class this Wednesday
- Spring break next week
  - No class during the week!

Special Project #4

- Using the fact that g=9.80m/s^2 on the Earth's surface, find the average density of the Earth.
  - Use the following information only but without computing the volume explicitly
    - The gravitational constant $G = 6.67 \times 10^{-11}\ N \cdot m^2 / kg^2$
    - The radius of the Earth $R_E = 6.37 \times 10^3\ km$
- 20 point extra credit
- Due: Monday, Mar. 25
- You must show your OWN, detailed work to obtain any credit!!

Newton's Law of Universal Gravitation

People have been very curious about the stars in the sky, making observations for a long~ time. The data people collected, however, have not been explained until Newton has discovered the law of gravitation.

Every object in the Universe attracts every other object with a force that is directly **proportional to the product of their masses** and **inversely proportional to the square of the distance** between them.

How would you write this law mathematically?

$$F_g \propto \frac{m_1 m_2}{r_{12}^2} \quad \xrightarrow{\text{With } G} \quad F_g = G\frac{m_1 m_2}{r_{12}^2}$$

G is the universal gravitational constant, and its value is

$$G = 6.673 \times 10^{-11}$$

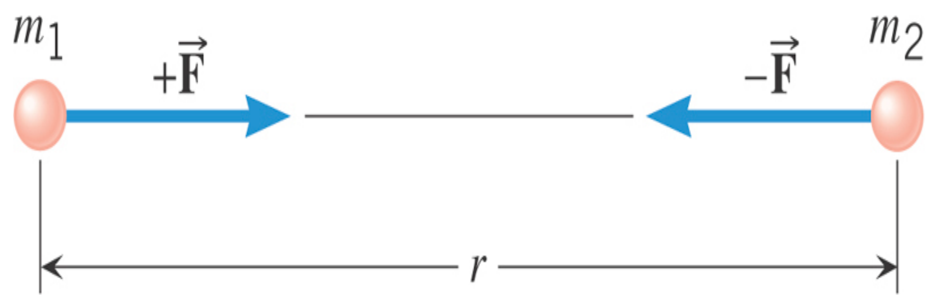
Unit? $N \cdot m^2 / kg^2$

This constant is not given by the theory but must be measured by experiments.

This form of forces is known as **the inverse-square law**, because the magnitude of the force is inversely proportional to the square of the distances between the objects.

Ex. Gravitational Attraction

What is the magnitude of the gravitational force that acts on each particle in the figure, assuming m_1=12kg, m_2=25kg, and r=1.2m?

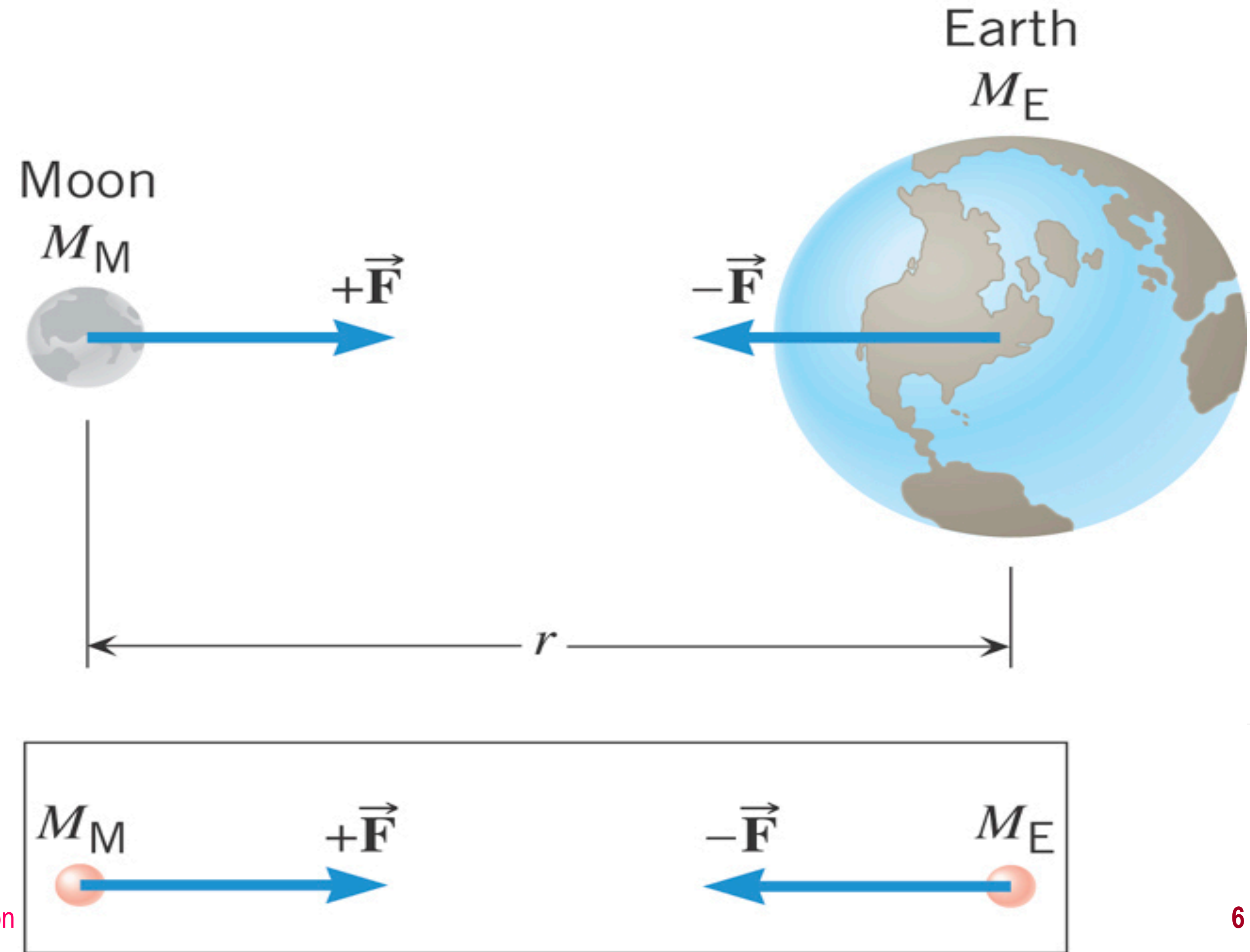


$$F = G\frac{m_1 m_2}{r^2}$$

$$= \left(6.67\times10^{-11}\ \mathrm{N\cdot m^2/kg^2}\right)\frac{(12\ \mathrm{kg})(25\ \mathrm{kg})}{(1.2\ \mathrm{m})^2}$$

$$= 1.4\times10^{-8}\ \mathrm{N}$$

Why does the Moon orbit the Earth?



Gravitational Force and Weight

Gravitational Force, $\mathcal{F}_g$

The attractive force exerted on an object by the Earth

$$\vec{F}_G = m\vec{a} = m\vec{g}$$

Weight of an object with mass M is

$$W = \left|\vec{F}_G\right| = M\left|\vec{g}\right| = Mg$$

What is the SI unit of weight? $\boxed{\mathrm{N}}$

*Since weight depends on the magnitude of gravitational acceleration, **g**, it varies depending on geographical location.*

By measuring the forces one can determine masses. This is why you can measure mass using the spring scale.

Gravitational Acceleration

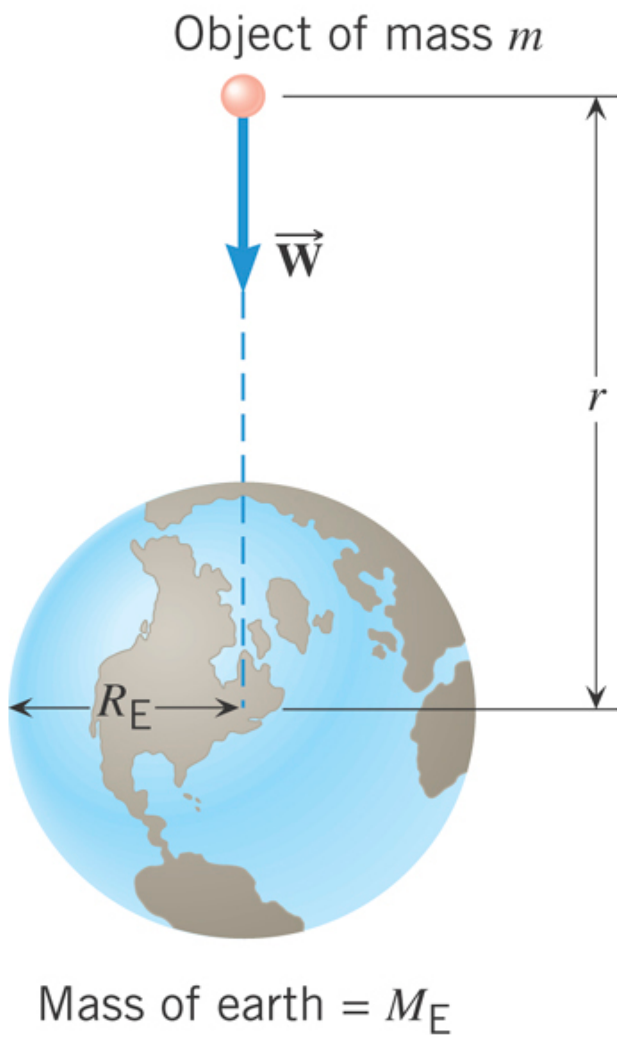
$$W = G\frac{M_E m}{r^2}$$

$$W = mg$$

$$\cancel{m}g = G\frac{M_E \cancel{m}}{r^2}$$

$$g = G\frac{M_E}{r^2}$$

Gravitational acceleration at distance r from the center of the earth!



What is the SI unit of g? $\boxed{m/s^2}$

Magnitude of the gravitational acceleration on the surface of the Earth

Gravitational force on the surface of the earth:

$$F_G = G\frac{M_E m}{r^2} = G\frac{M_E \cancel{m}}{R_E^2} = \cancel{m}g$$

$$g = G\frac{M_E}{R_E^2}$$

$G = 6.67\times10^{-11}\,\mathrm{N\cdot m^2/kg^2}$

$M_E = 5.98\times10^{24}\ \mathrm{kg};\quad R_E = 6.38\times10^{6}\ m$

$$= \left(6.67\times10^{-11}\,\mathrm{N\cdot m^2/kg^2}\right)\frac{\left(5.98\times10^{24}\ \mathrm{kg}\right)}{\left(6.38\times10^{6}\ \mathrm{m}\right)^2}$$

$$= 9.80\ \mathrm{m/s^2}$$

Example for Universal Gravitation

Using the fact that $g=9.80m/s^2$ on the Earth's surface, find the average density of the Earth.

Since the gravitational acceleration is

$$F_g = G\frac{M_E m}{R_E^2} = mg$$

Solving for g

$$g = G\frac{M_E}{R_E^2} = 6.67\times 10^{-11}\frac{M_E}{R_E^2}$$

Solving for M_E

$$M_E = \frac{R_E^2 g}{G}$$

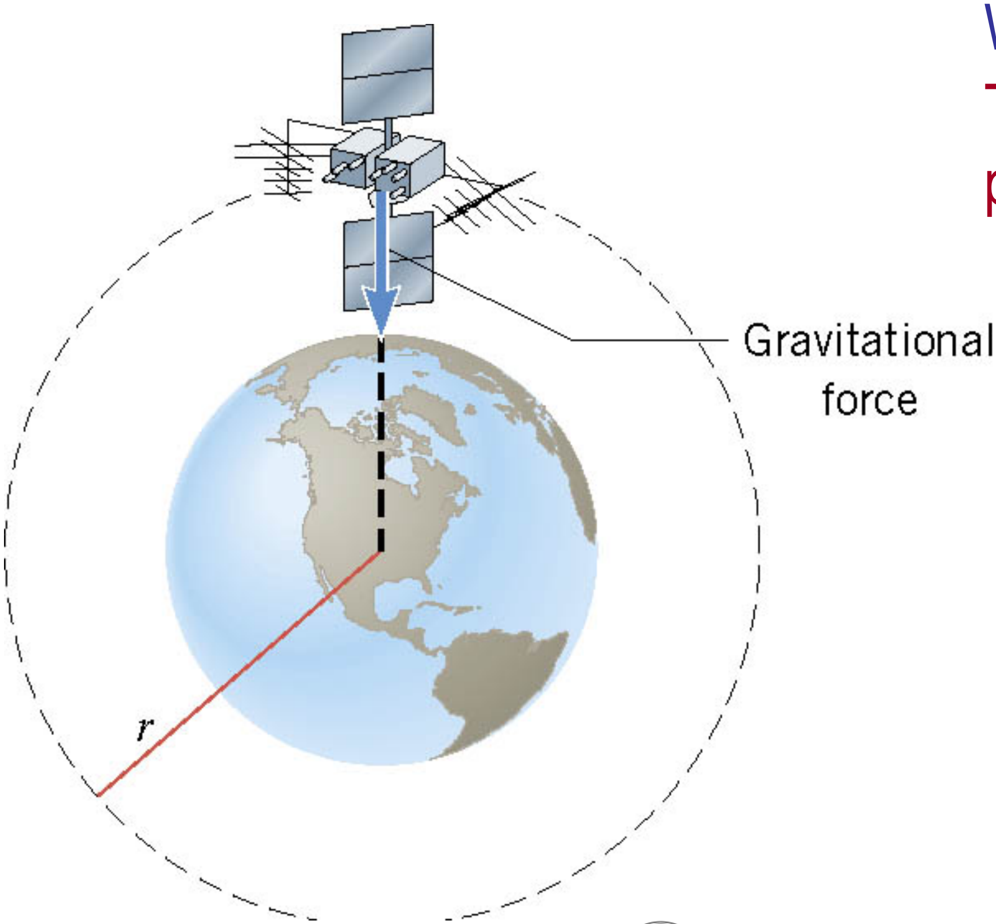
Therefore the density of the Earth is

$$\rho = \frac{M_E}{V_E} = \frac{\frac{R_E^2 g}{G}}{\frac{4\pi}{3}R_E^3} = \frac{3g}{4\pi G R_E}$$

$$= \frac{3\times 9.80}{4\pi\times 6.67\times 10^{-11}\times 6.37\times 10^6} = 5.50\times 10^3 kg/m^3$$

Satellite in Circular Orbits

There is only one speed that a satellite can have if the satellite is to remain in an orbit with a fixed radius.



What acts as the centripetal force?

The gravitational force of the earth pulling the satellite!

$$F_c = G\frac{mM_E}{r^2} = m\frac{v^2}{r}$$

$$v^2 = \frac{GM_E}{r}$$

$$\boxed{v = \sqrt{\frac{GM_E}{r}}}$$

Ex. Orbital Speed of the Hubble Space Telescope

Determine the speed of the Hubble Space Telescope orbiting at a height of 598 km above the earth's surface.

$$v = \sqrt{\frac{GM_E}{r}}$$

$$= \sqrt{\frac{\left(6.67\times10^{-11}\,\mathrm{N\cdot m^2/kg^2}\right)\left(5.98\times10^{24}\,\mathrm{kg}\right)}{6.38\times10^{6}\,\mathrm{m}+598\times10^{3}\,\mathrm{m}}}$$

$$= 7.56\times10^{3}\,\mathrm{m/s} \quad \left(16900\,\mathrm{mi/h}\right)$$

Period of a Satellite in an Orbit

Speed of a satellite $$v = \sqrt{\frac{GM_E}{r}} = \frac{2\pi r}{T}$$

$$\frac{GM_E}{r} = \left(\frac{2\pi r}{T}\right)^2$$

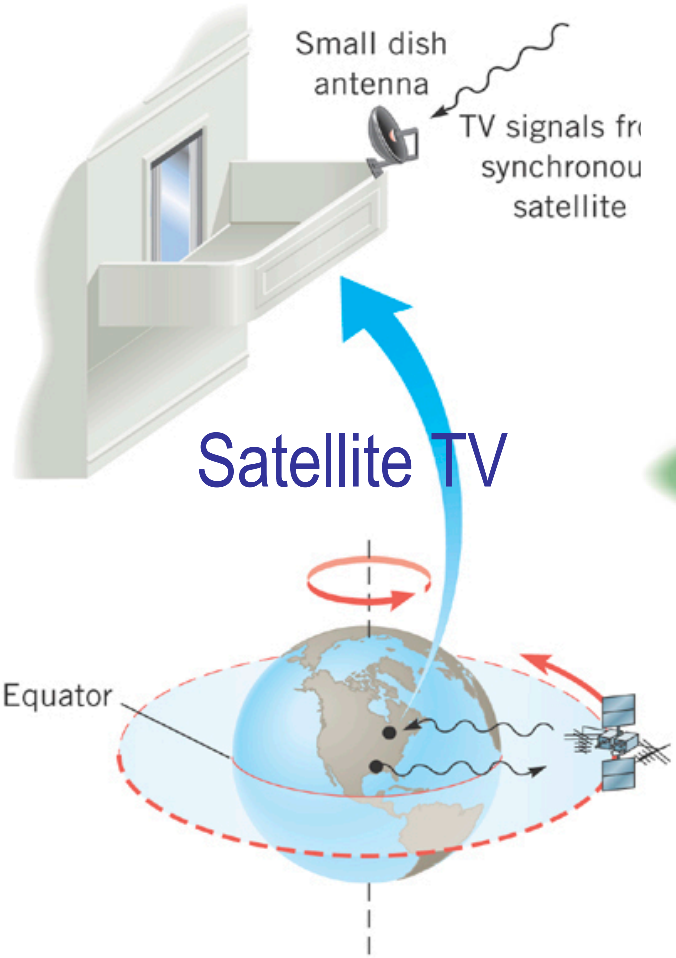
Square either side and solve for T2

$$T^2 = \frac{(2\pi)^2 r^3}{GM_E}$$

Period of a satellite $$T = \frac{2\pi r^{3/2}}{\sqrt{GM_E}}$$ Kepler's 3rd Law

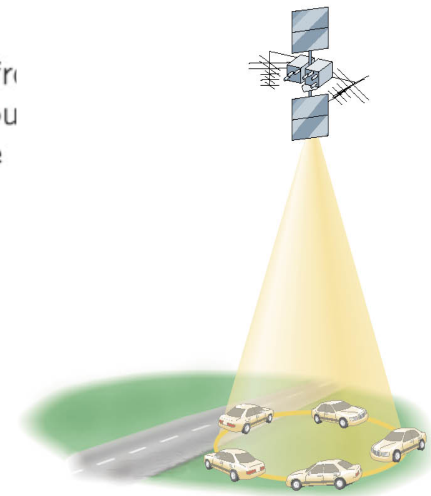
This is applicable to any satellite or even for planets and moons.

Geo-synchronous Satellites



Satellite TV

Global Positioning System (GPS)

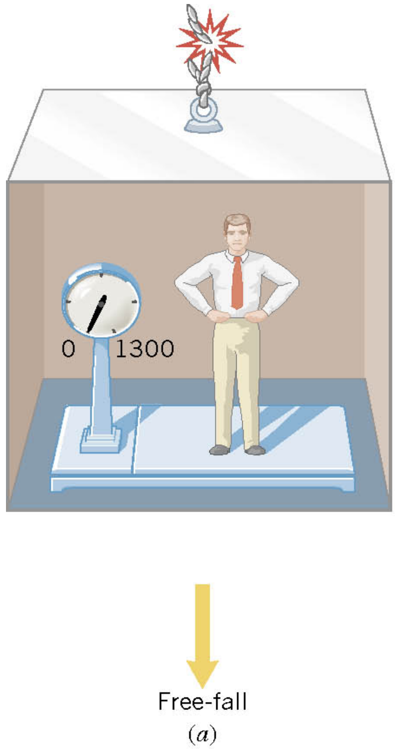


(a)

What period should these satellites have?

The same as the earth!! 24 hours

Ex. Apparent Weightlessness and Free Fall



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In each case, what is the weight recorded by the scale?

Ex. Artificial Gravity

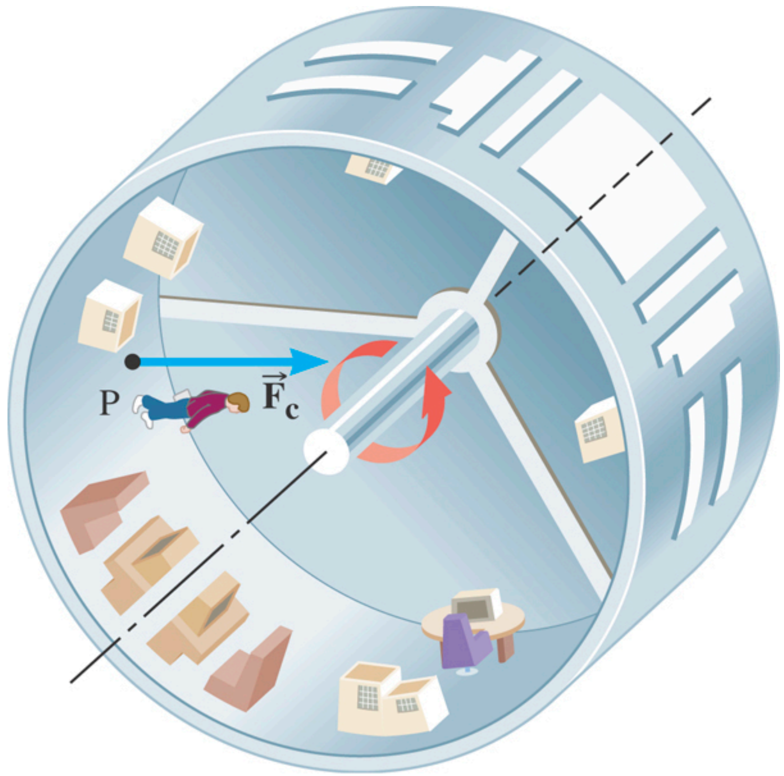
At what speed must the surface of the space station move so that the astronaut experiences a push on his feet equal to his weight on earth? The radius is 1700 m.

$$F_c = m\frac{v^2}{r} = mg$$

$$v = \sqrt{rg}$$

$$= \sqrt{(1700\ \mathrm{m})(9.80\,\mathrm{m/s^2})}$$

$$= 130\, m/s$$



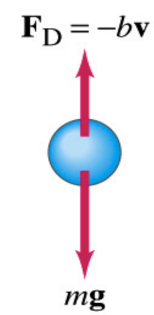
Motion in Resistive Forces

Medium can exert resistive forces on an object moving through it due to viscosity or other types frictional properties of the medium.

Some examples? | Air resistance, viscous force of liquid, etc

These forces are exerted on moving objects in opposite direction of the movement.

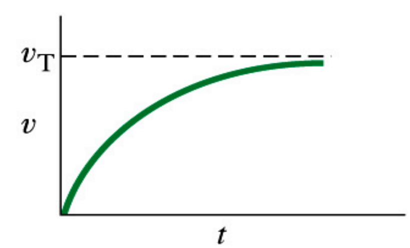
These forces are proportional to such factors as speed. They almost always increase with increasing speed.



(a)

Two different cases of proportionality:

1. Forces linearly proportional to speed:
   Slowly moving or very small objects
2. Forces proportional to square of speed:
   Large objects w/ reasonable speed



(b)