# PHYS 1441 – Section 002
# Lecture #14

*Wednesday, March 6, 2013*
*Dr. Jaehoon Yu*

- Work done by a constant force
- Scalar Product of the Vector
- Work with friction
- Work-Kinetic Energy Theorem
- Potential Energy

Today's homework is homework #8, due 11pm, Monday, Mar. 18!!

# Announcements

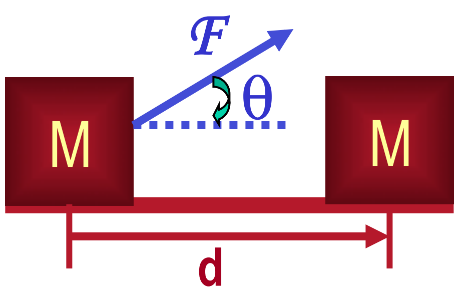
- Midterm comprehensive exam
  - Wednesday, Mar. 20
  - In SH103
  - Covers CH1.1 through what we learn today (CH6.3?)
- Spring break next week
  - No class during the week!

# Reminder: Special Project #4

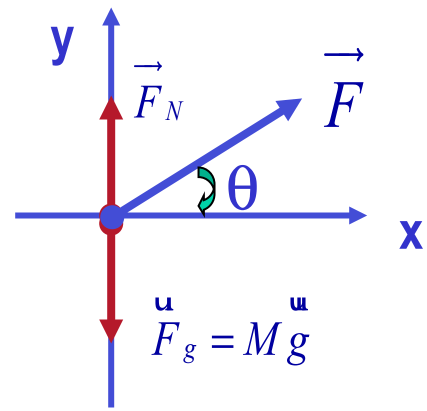
- Using the fact that g=9.80m/s$^2$ on the Earth's surface, find the average density of the Earth.
  - Use the following information only but without computing the volume explicitly
    - The gravitational constant $G = 6.67 \times 10^{-11}\, N \cdot m^2 / kg^2$
    - The radius of the Earth $R_E = 6.37 \times 10^3\, km$
- 20 point extra credit
- Due: Monday, Mar. 25
- You must show your OWN, detailed work to obtain any credit!!

# Work Done by a Constant Force

*A meaningful work in physics is done only when the net forces exerted on an object changes the energy of the object.*




**Free Body Diagram**




Which force did the work? Force $\vec{F}$ Why?

What kind? Scalar

How much work did it do? $W = \left(\sum \vec{F}\right) \cdot \vec{d} = Fd\cos\theta$

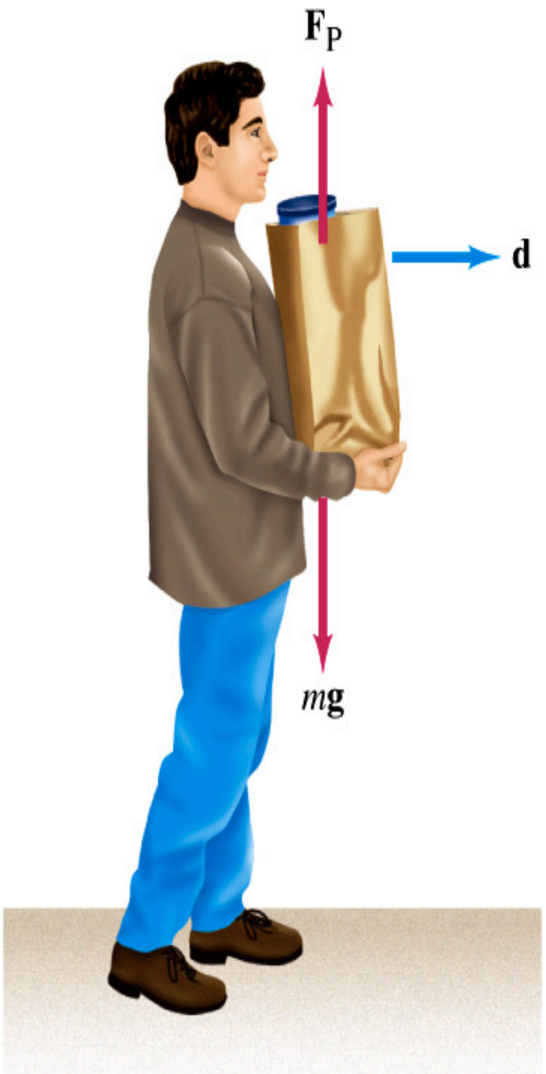
Unit? $N \cdot m$
$= J$ (for Joule)

What does this mean?

**Physically meaningful work is done only by the component of the force along the movement of the object.**

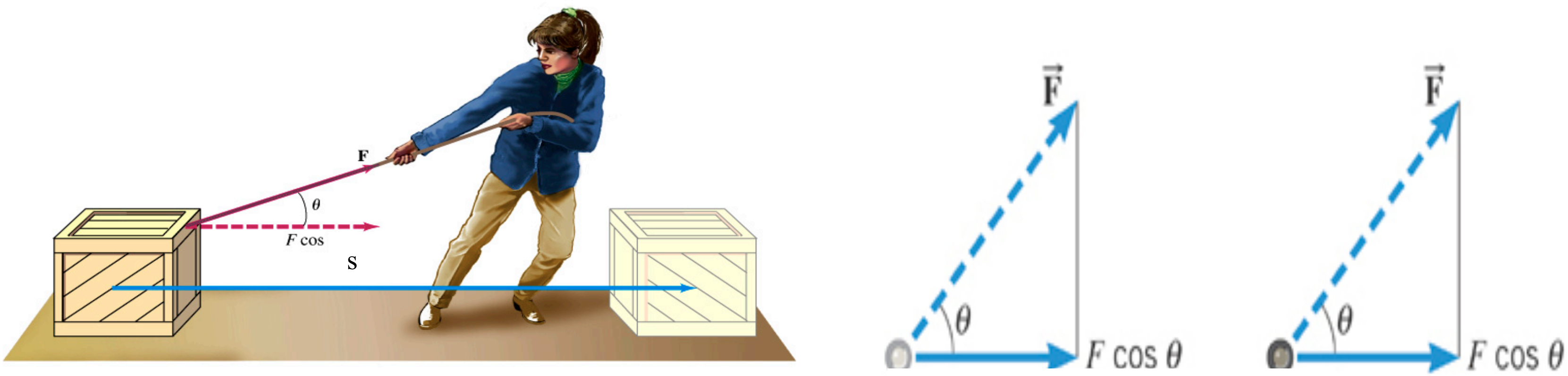
**Work is an <u>energy transfer</u>!!**

# Let's think about the meaning of work!




- A person is holding a grocery bag and walking at a constant velocity.
- Are his hands doing any work ON the bag?
  - No
  - Why not?
  - Because the force hands exert on the bag, $F_p$, is perpendicular to the displacement!!
  - This means that hands are not adding any energy to the bag.
- So what does this mean?
  - In order for a force to perform any meaningful work, the energy of the object the force exerts on must change due to that force!!
- What happened to the person?
  - He spends his energy just to keep the bag up but did not perform any work on the bag.

# Work done by a constant force




(a) (b)

$$W = \vec{F} \cdot \vec{s}$$
$$= \left(F\cos\theta\right)s$$

$$\cos 0^{o} = 1$$
$$\cos 90^{o} = 0$$
$$\cos 180^{o} = -1$$

# Scalar Product of Two Vectors

- Product of magnitude of the two vectors and the cosine of the angle between them $\vec{A}\cdot\vec{B} \equiv \left|\vec{A}\right|\left|\vec{B}\right|\cos\theta$
- Operation is commutative $\vec{A}\cdot\vec{B} = \left|\vec{A}\right|\left|\vec{B}\right|\cos\theta = \left|\vec{B}\right|\left|\vec{A}\right|\cos\theta = \vec{B}\cdot\vec{A}$
- Operation follows the distribution law of multiplication $\vec{A}\cdot\left(\vec{B}+\vec{C}\right) = \vec{A}\cdot\vec{B} + \vec{A}\cdot\vec{C}$
- Scalar products of Unit Vectors $\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1 \quad \hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0$
- How does scalar product look in terms of components?

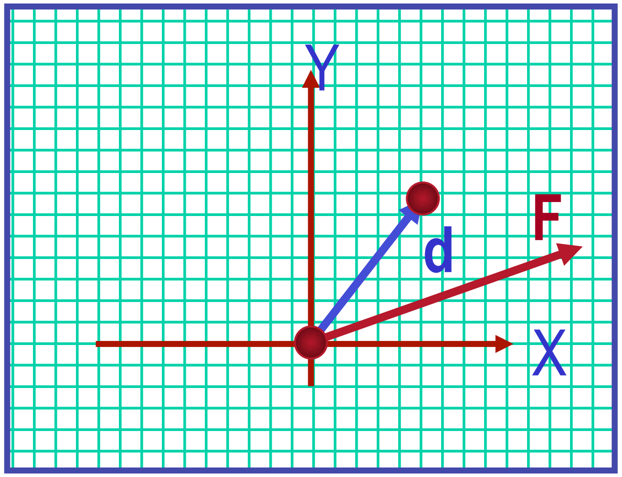
$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k} \qquad \vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$$

$$\vec{A}\cdot\vec{B} = \left(A_x\hat{i} + A_y\hat{j} + A_z\hat{k}\right)\cdot\left(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}\right) = \left(A_xB_x\,\hat{i}\cdot\hat{i} + A_yB_y\,\hat{j}\cdot\hat{j} + A_zB_z\,\hat{k}\cdot\hat{k}\right) + \underbrace{cross\ terms}_{=0}$$

$$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$$

# Example of Work by Scalar Product

A particle moving on the xy plane undergoes a displacement **d**=(2.0**i**+3.0**j**)m as a constant force **F**=(5.0**i**+2.0**j**) N acts on the particle.




a) Calculate the magnitude of the displacement and that of the force.

$$\left|\vec{d}\right| = \sqrt{d_x^2 + d_y^2} = \sqrt{(2.0)^2 + (3.0)^2} = 3.6m$$

$$\left|\vec{F}\right| = \sqrt{F_x^2 + F_y^2} = \sqrt{(5.0)^2 + (2.0)^2} = 5.4N$$

b) Calculate the work done by the force F.

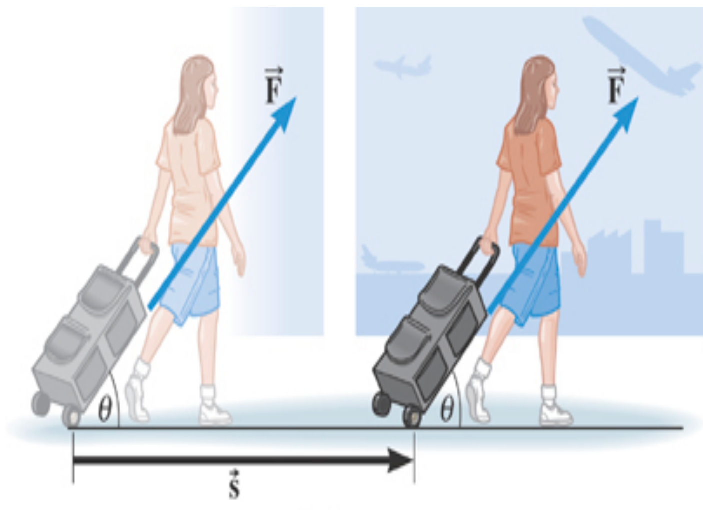
$$W = \vec{F} \cdot \vec{d} = \left(2.0\hat{i} + 3.0\hat{j}\right)\cdot\left(5.0\hat{i} + 2.0\hat{j}\right) = 2.0\times 5.0\,\hat{i}\cdot\hat{i} + 3.0\times 2.0\,\hat{j}\cdot\hat{j} = 10 + 6 = 16(J)$$

Can you do this using the magnitudes and the angle between **d** and **F**?

$$W = \vec{F} \cdot \vec{d} = \left|\vec{F}\right|\left|\vec{d}\right|\cos\theta$$

# Ex. Pulling A Suitcase-on-Wheel

Find the work done by a 45.0N force in pulling the suitcase in the figure at an angle 50.0° for a distance s=75.0m.



(a)

$$W = \left(\sum \vec{F}\right)\cdot\vec{d} = \left|\left(\sum \vec{F}\right)\cos\theta\right|\left|\vec{d}\right|$$

$$= \left(45.0\cdot\cos 50^{\circ}\right)\cdot 75.0 = 2170J$$

Does work depend on mass of the object being worked on?

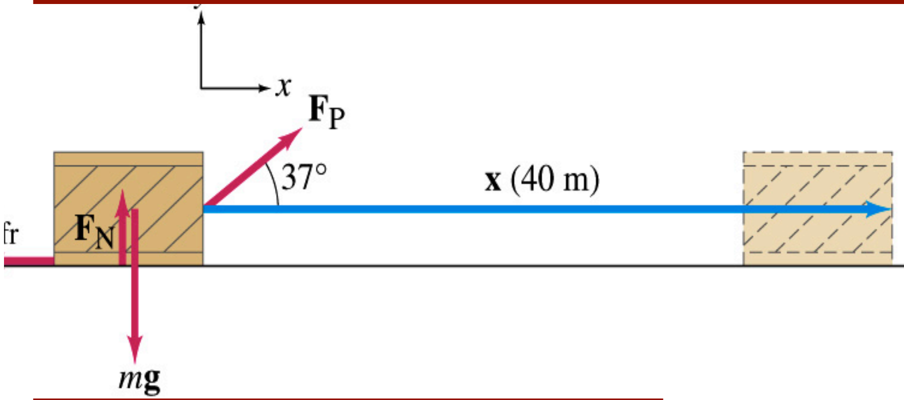
Yes

Why don't I see the mass term in the work at all then?

It is reflected in the force. If an object has smaller mass, it would take less force to move it at the same acceleration than a heavier object. So it would take less work. Which makes perfect sense, doesn't it?

# Ex. 6.1 Work done on a crate

A person pulls a 50kg crate 40m along a horizontal floor by a constant force $F_p$=100N, which acts at a 37° angle as shown in the figure. The floor is rough and exerts a friction force $F_{fr}$=50N. Determine (a) the work done by each force and (b) the net work done on the crate.




What are the forces exerting on the crate?

$F_p$ | $F_{fr}$ | $F_G$=-mg | $F_N$=+mg

Which force performs the work on the crate?

$F_p$ | $F_{fr}$

Work done on the crate by $F_G$

$$W_G = \vec{F}_G \cdot \vec{x} = -mg\cos\left(-90^\circ\right) \cdot \left|\vec{x}\right| = 0J$$

Work done on the crate by $F_N$

$$W_N = \vec{F}_N \cdot \vec{x} = mg\cos 90^\circ \cdot \left|\vec{x}\right| = 100 \cdot \cos 90^\circ \cdot 40 = 0J$$

Work done on the crate by $F_p$:

$$W_p = \vec{F}_p \cdot \vec{x} = \left|\vec{F}_p\right| \cos 37^\circ \cdot \left|\vec{x}\right| = 100 \cdot \cos 37^\circ \cdot 40 = 3200J$$

Work done on the crate by $F_{fr}$:

$$W_{fr} = \vec{F}_{fr} \cdot \vec{x} = \left|\vec{F}_{fr}\right| \cos 180^\circ \cdot \left|\vec{x}\right| = 50 \cdot \cos 180^\circ \cdot 40 = -2000J$$

So the net work on the crate

$$W_{net} = W_N + W_G + W_p + W_{fr} = 0 + 0 + 3200 - 2000 = 1200\left(J\right)$$

This is the same as

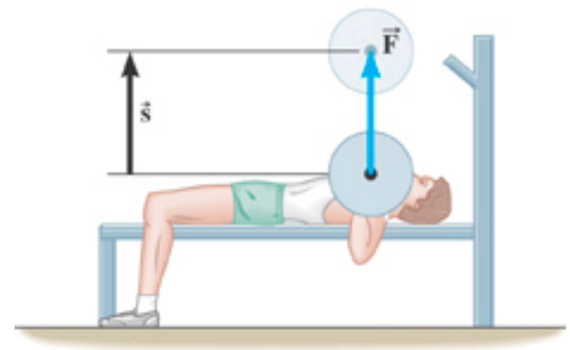
$$W_{net} = \sum\left(\vec{F} \cdot \vec{x}\right) = \left(\vec{F}_N \cdot \vec{x} + \vec{F}_G \cdot \vec{x} + \vec{F}_p \cdot \vec{x} + \vec{F}_{fr} \cdot \vec{x}\right)$$

# Ex. Bench Pressing and The Concept of Negative Work

(a)

A weight lifter is bench-pressing a barbell whose weight is 710N a distance of 0.65m above his chest. Then he lowers it the same distance. The weight is raised and lowered at a constant velocity. Determine the work in the two cases.

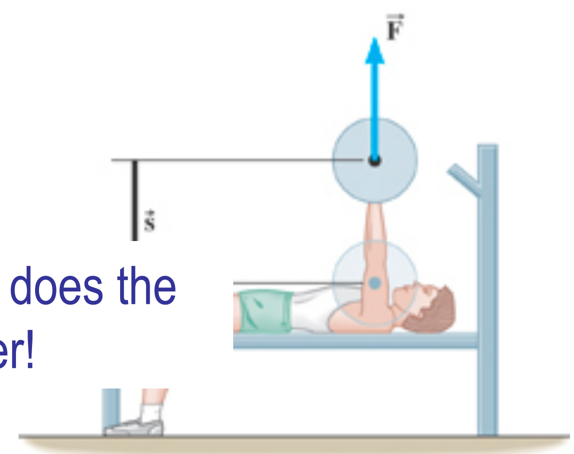
What is the angle between the force and the displacement?




(b)

$$W = \left(F\cos 0\right)s = Fs$$
$$= 710 \cdot 0.65 = +460\left(J\right)$$

$$W = \left(F\cos 180\right)s = -Fs$$
$$= -710 \cdot 0.65 = -460\left(J\right)$$

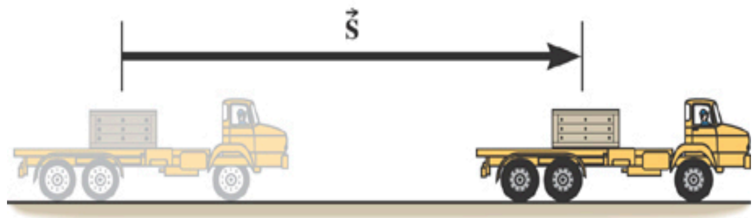



(c)

What does the negative work mean? The gravitational force does the work on the weight lifter!

# Ex. Accelerating a Crate

The truck is accelerating at a rate of +1.50 $m/s^2$. The mass of the crate is 120-kg and it does not slip. The magnitude of the displacement is 65 m. What is the total work done on the crate by all of the forces acting on it?



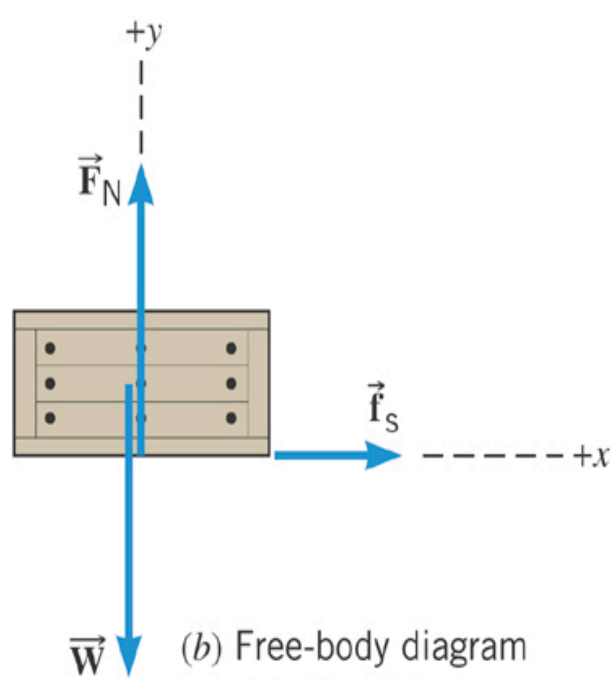
(a)

What are the forces acting in this motion?

Gravitational force on the crate, weight, **W** or $\mathbf{F_g}$

Normal force force on the crate, $\mathbf{F_N}$

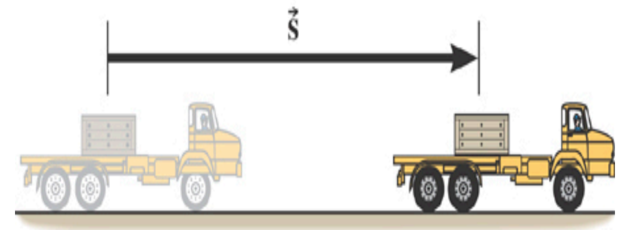
Static frictional force on the crate, $\mathbf{f_s}$



(b) Free-body diagram for the crate

# Ex. Continued…

Lets figure out what the work done by each of the forces in this motion is.



(a)

Work done by the gravitational force on the crate, **W** or $\mathbf{F_g}$

$$W = \left(F_g \cos\left(-90^o\right)\right)s = 0$$

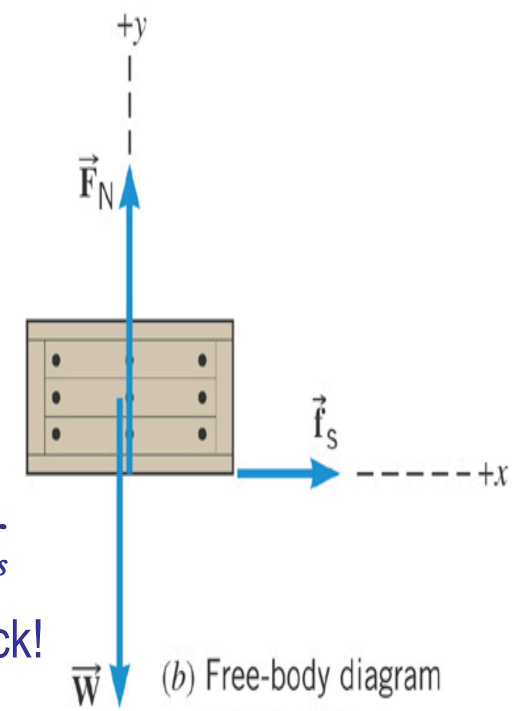
Work done by Normal force force on the crate, $\mathbf{F_N}$

$$W = \left(F_N \cos\left(+90^o\right)\right)s = 0$$

Work done by the static frictional force on the crate, $f_s$

$$f_s = ma = (120\text{ kg})\left(1.5\,\text{m/s}^2\right) = 180\text{N}$$

$$W = f_s \cdot s = \left[(180\text{N})\cos 0\right](65\text{ m}) = 1.2\times 10^4\,\text{J}$$



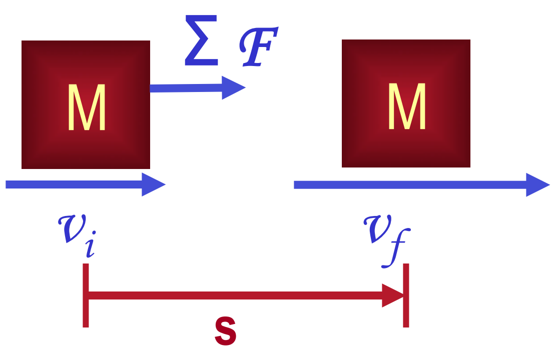
(b) Free-body diagram for the crate

Which force did the work? Static frictional force on the crate, $f_s$

How? By holding on to the crate so that it moves with the truck!

# Kinetic Energy and Work-Kinetic Energy Theorem

- Some problems are hard to solve using Newton's second law
  - If forces exerting on an object during the motion are complicated
  - Relate the work done on the object by the net force to the change of the speed of the object




Suppose net force $\Sigma F$ was exerted on an object for displacement d to increase its speed from $v_i$ to $v_f$.

The work on the object by the net force $\Sigma F$ is

$$W = \left(\sum \vec{F}\right)\cdot\vec{s} = (ma\cos 0)s = (ma)s$$

Using the kinematic equation of motion $2as = v_f^2 - v_0^2$ ⇒ $as = \frac{v_f^2 - v_0^2}{2}$

Work $W = (ma)s = \frac{1}{2}m\left(v_f^2 - v_0^2\right) = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2$

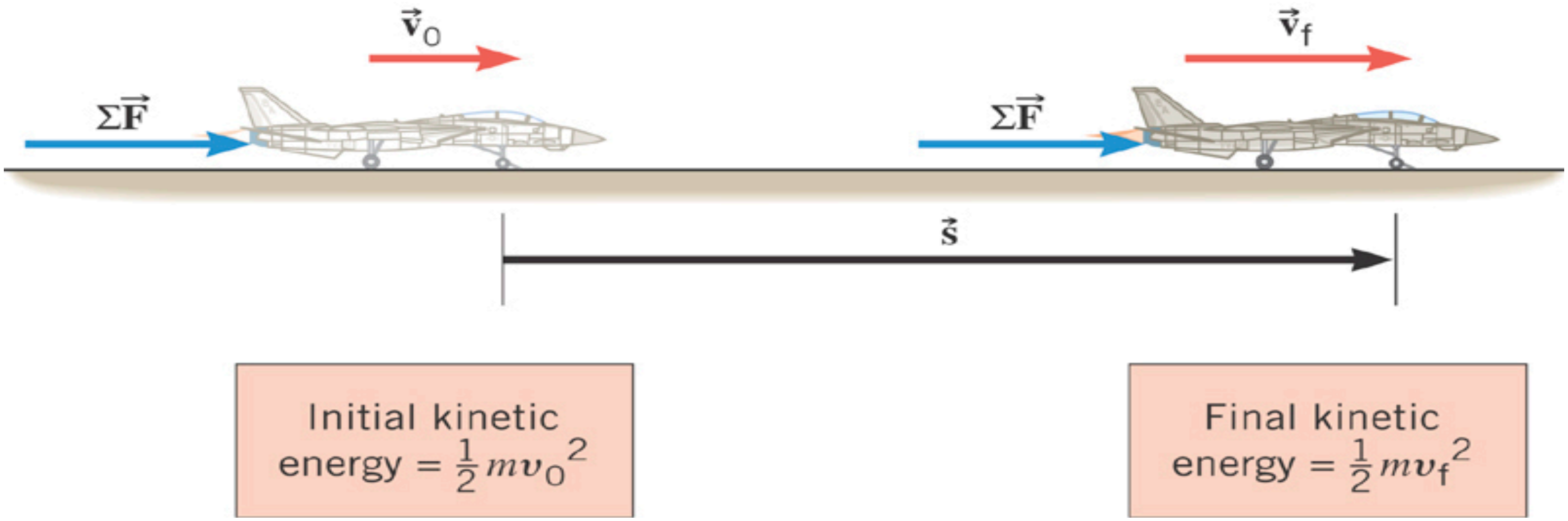
Kinetic Energy $\boxed{KE \equiv \frac{1}{2}mv^2}$

Work $W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = KE_f - KE_i = \Delta KE$

**Work done by the net force causes change in the object's kinetic energy.**

**Work-Kinetic Energy Theorem**

# Work-Kinetic Energy Theorem

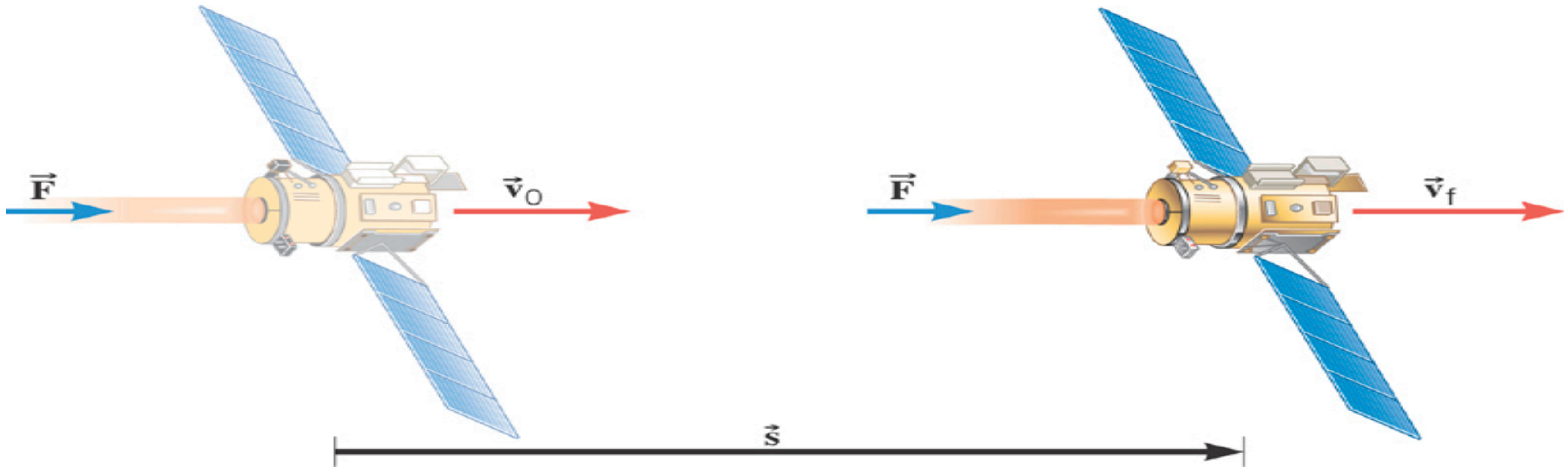



When a net external force by the jet engine does work on an object, the kinetic energy of the object changes according to

$$W = \mathrm{KE_f} - \mathrm{KE_o} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_o^2$$

# Ex. Deep Space 1

The mass of the space probe is 474-kg and its initial velocity is 275 m/s. If a 56.0-mN force acts on the probe parallel through a displacement of $2.42\times10^9$m, what is its final speed?




$$\left[\left(\sum \mathrm{F}\right)\cos\theta\right]s = \tfrac{1}{2}mv_{\mathrm{f}}^2 - \tfrac{1}{2}mv_o^2$$

Solve for $v_f$

$$v_f = \sqrt{v_o^2 + 2\left(\sum F\cos\theta\right)s/m} \qquad = \sqrt{\left(275\,\mathrm{m/s}\right)^2 + 2\left(5.60\times10^{-2}\mathrm{N}\right)\cos 0^\circ\left(2.42\times10^9\,\mathrm{m}\right)/474}$$

$$v_f = 805\,\mathrm{m/s}$$

# Ex. Satellite Motion and Work By the Gravity

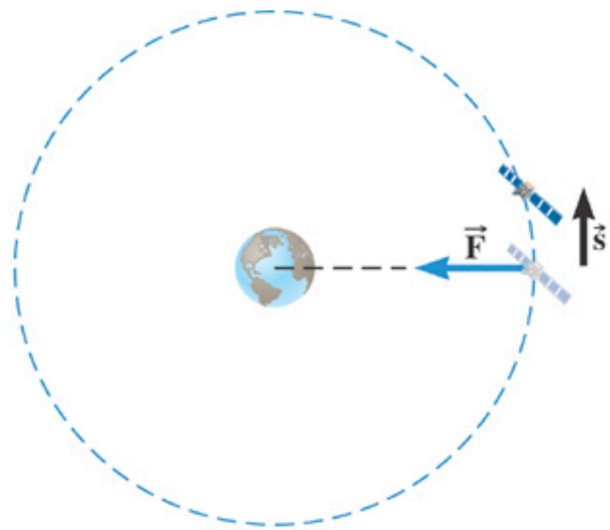
A satellite is moving about the earth in a circular orbit and an elliptical orbit. For these two orbits, determine whether the kinetic energy of the satellite changes during the motion.

For a circular orbit No change! Why not?

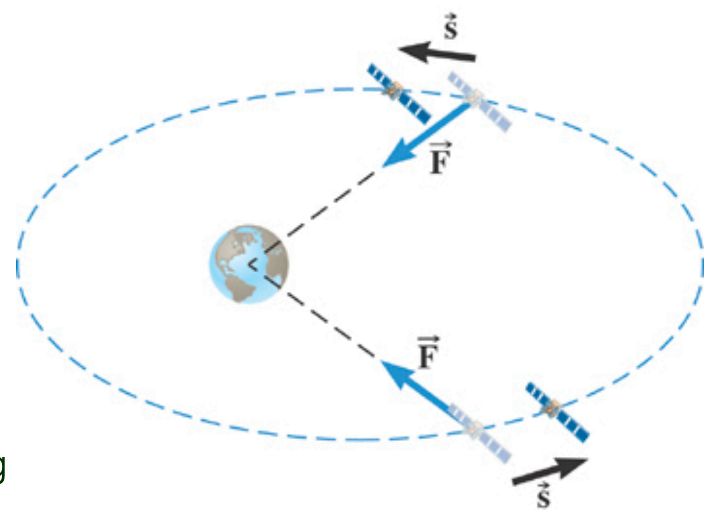
Gravitational force is the only external force but it is perpendicular to the displacement. So no work.

For an elliptical orbit Changes! Why?

Gravitational force is the only external force but its angle with respect to the displacement varies. So it performs work.




(a)




(b)