PHYS 1441 – Section 002
Lecture #18

Wednesday, April 3, 2013
Dr. Jaehoon Yu

- Collisions
- Elastic Collisions
- Perfectly Inelastic Collisions
- Concept of the Center of Mass
- Fundamentals of the Rotational Motion
- Rotational Kinematics

Announcements

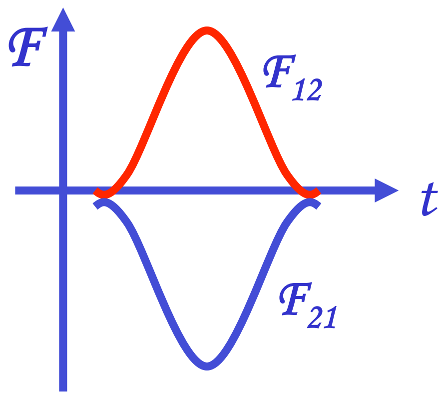
- Quiz #4
  - When: Beginning of class Monday, Apr. 8
  - Coverage: CH6.3 to what we cover this Wednesday
- Second non-comp term exam
  - Date and time: 4:00pm, Wednesday, April 17 in class
  - Coverage: CH6.3 through what we finish Monday, April 15
- Special colloquium for 15 point extra credit
  - Wednesday, April 24, University Hall RM116
  - Class will be substituted by this colloquium
  - Dr. Ketevi Assamagan from Brookhaven National Laboratory on Higgs Discovery in ATLAS
  - Please mark your calendars!!

Collisions

Generalized collisions must cover not only the physical contact but also the collisions without physical contact such as that of electromagnetic ones in a microscopic scale.

Consider a case of a collision between a proton on a helium ion.

The collisions of these ions never involve physical contact because the electromagnetic repulsive force between these two become great as they get closer causing a collision.



Assuming no external forces, the force exerted on particle 1 by particle 2, F_{21}, changes the momentum of particle 1 by

$$\Delta\vec{p}_1 = \vec{F}_{21}\Delta t$$

Likewise for particle 2 by particle 1

$$\Delta\vec{p}_2 = \vec{F}_{12}\Delta t$$

Using Newton's 3rd law we obtain

$$\Delta\vec{p}_2 = \vec{F}_{12}\Delta t = -\vec{F}_{21}\Delta t = -\Delta\vec{p}_1$$

So the momentum change of the system in the collision is 0, and the momentum is conserved

$$\Delta\vec{p} = \Delta\vec{p}_1 + \Delta\vec{p}_2 = 0$$

$$\vec{p}_{system} = \vec{p}_1 + \vec{p}_2 = \text{constant}$$

Elastic and Inelastic Collisions

Momentum is conserved in any collisions as long as external forces are negligible.

Collisions are classified as elastic or inelastic based on whether the ***kinetic energy is conserved, meaning whether the KE is the same*** *before and after the collision.*

Elastic Collision	*A collision in which the total kinetic energy and momentum are the same before and after the collision.*
Inelastic Collision	*A collision in which the total kinetic energy is not the same before and after the collision, but momentum is.*

Two types of inelastic collisions: Perfectly inelastic and inelastic

Perfectly Inelastic: *Two objects stick together after the collision, moving together at a certain velocity.*

Inelastic: *Colliding objects do not stick together after the collision but some kinetic energy is lost.*

Note: Momentum is constant in all collisions but kinetic energy is only in elastic collisions.

Elastic and Perfectly Inelastic Collisions

In perfectly inelastic collisions, the objects stick together after the collision, moving together. Momentum is conserved in this collision, so the final velocity of the stuck system is

$$m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = (m_1 + m_2)\vec{v}_f$$

$$\vec{v}_f = \frac{m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i}}{(m_1 + m_2)}$$

How about the elastic collision?

$$m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f}$$

In elastic collisions, both the momentum and the kinetic energy are conserved. Therefore, the final speeds in an elastic collision can be obtained in terms of initial speeds as

$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

$$m_1\left(v_{1i}^2 - v_{1f}^2\right) = m_2\left(v_{2i}^2 - v_{2f}^2\right)$$

$$m_1\left(v_{1i} - v_{1f}\right)\left(v_{1i} + v_{1f}\right) = m_2\left(v_{2i} - v_{2f}\right)\left(v_{2i} + v_{2f}\right)$$

From momentum conservation above

$$m_1\left(v_{1i} - v_{1f}\right) = m_2\left(v_{2i} - v_{2f}\right)$$

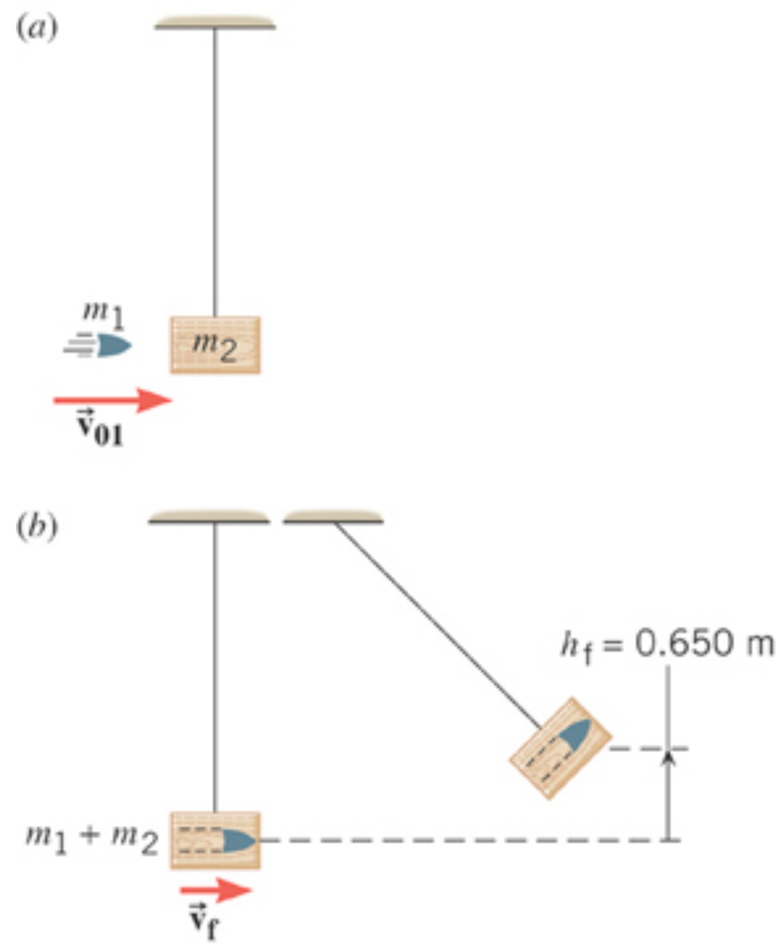
$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1i} + \left(\frac{2m_2}{m_1 + m_2}\right) v_{2i}$$

$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right) v_{1i} + \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{2i}$$

What happens when the two masses are the same?

Ex. A Ballistic Pendulum

The mass of the block of wood is 2.50-kg and the mass of the bullet is 0.0100-kg. The block swings to a maximum height of 0.650 m above the initial position. Find the initial speed of the bullet.



What kind of collision? Perfectly inelastic collision

No net external force ➔ momentum conserved

$$m_1 v_{f1} + m_2 v_{f2} = m_1 v_{01} + m_2 v_{02}$$

$$(m_1 + m_2)\, v_f = m_1 v_{01}$$

Solve for V_{01}

$$v_{01} = \frac{(m_1 + m_2) v_f}{m_1}$$

What do we not know? The final speed!!

How can we get it? Using the mechanical energy conservation!

Ex. A Ballistic Pendulum, cnt'd

Now using the mechanical energy conservation

$$\tfrac{1}{2}mv^2 = mgh$$

$$(m_1+m_2)gh_f = \tfrac{1}{2}(m_1+m_2)v_f^2$$

$$gh_f = \tfrac{1}{2}v_f^2$$

Solve for V_f

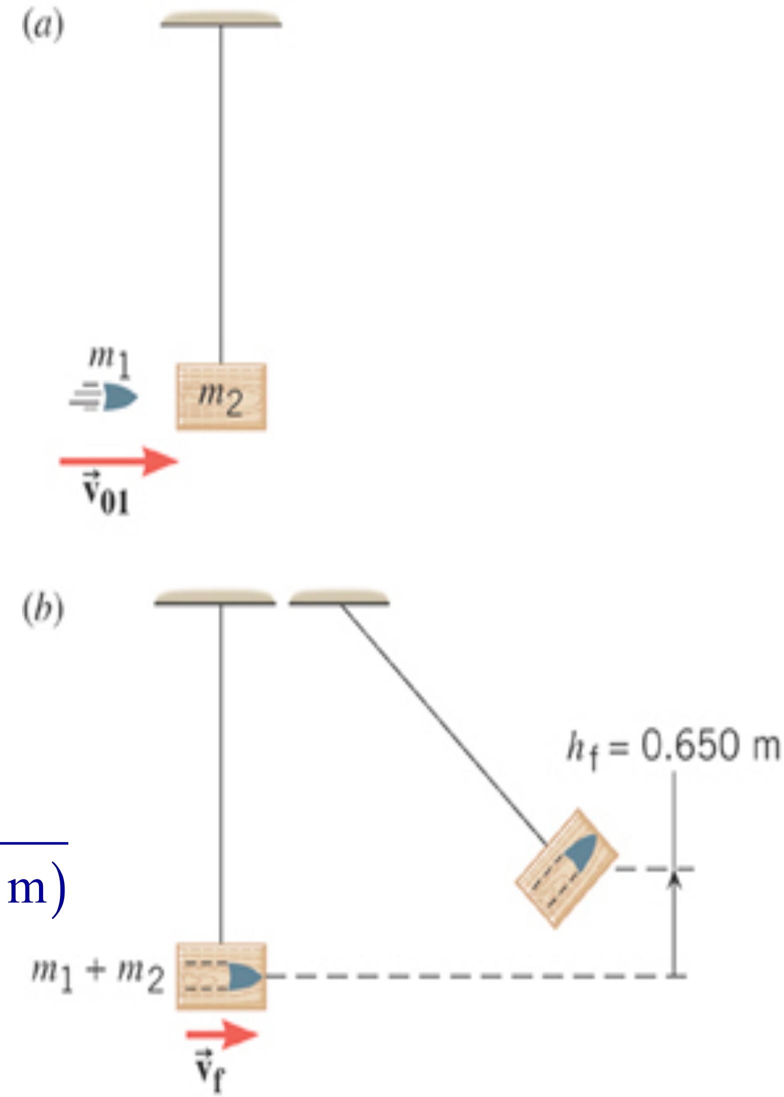
$$v_f = \sqrt{2gh_f} = \sqrt{2\left(9.80\,\mathrm{m/s^2}\right)\left(0.650\ \mathrm{m}\right)}$$

Using the solution obtained previously, we obtain

$$v_{01} = \frac{(m_1+m_2)v_f}{m_1} = \frac{(m_1+m_2)\sqrt{2gh_f}}{m_1}$$

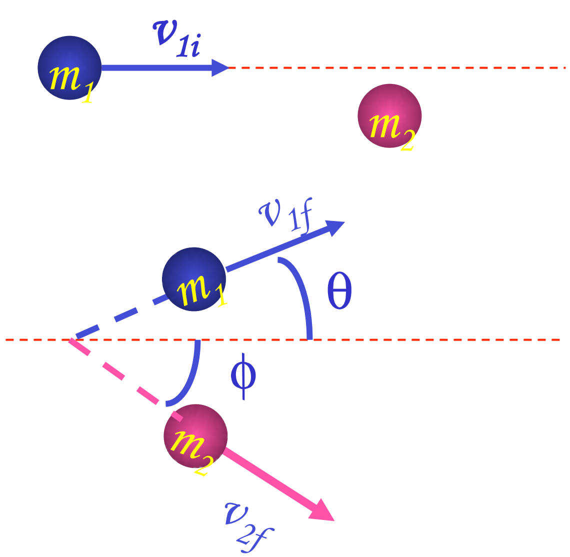
$$= \left(\frac{0.0100\ \mathrm{kg} + 2.50\ \mathrm{kg}}{0.0100\ \mathrm{kg}}\right)\sqrt{2\left(9.80\,\mathrm{m/s^2}\right)\left(0.650\ \mathrm{m}\right)}$$

$$= +896\,\mathrm{m/s}$$



Two dimensional Collisions

In two dimension, one needs to use components of momentum and apply momentum conservation to solve physical problems.



$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$

x-comp. $$m_1 v_{1ix} + m_2 v_{2ix} = m_1 v_{1fx} + m_2 v_{2fx}$$

y-comp. $$m_1 v_{1iy} + m_2 v_{2iy} = m_1 v_{1fy} + m_2 v_{2fy}$$

Consider a system of two particle collisions and scatters in two dimension as shown in the picture. (This is the case at fixed target accelerator experiments.) The momentum conservation tells us:

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1i}$$

$$m_1 v_{1ix} = m_1 v_{1fx} + m_2 v_{2fx} = m_1 v_{1f}\cos\theta + m_2 v_{2f}\cos\varphi$$

$$m_1 v_{1iy} = 0 = m_1 v_{1fy} + m_2 v_{2fy} = m_1 v_{1f}\sin\theta - m_2 v_{2f}\sin\varphi$$

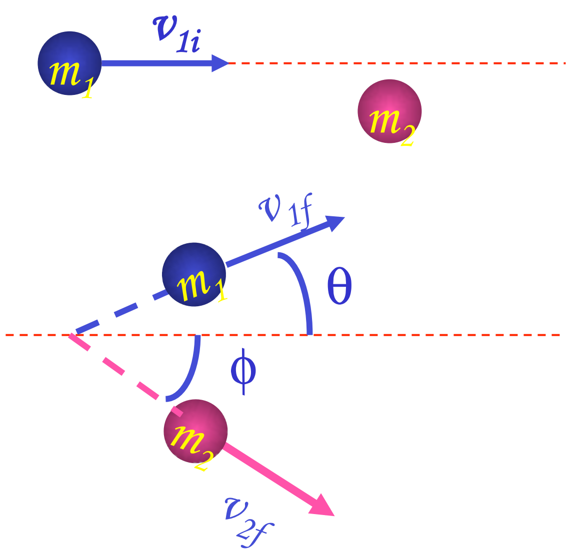
And for the elastic collisions, the kinetic energy is conserved:

$$\frac{1}{2}m_1 v_{1i}^2 = \frac{1}{2}m_1 v_{1f}^2 + \frac{1}{2}m_2 v_{2f}^2$$

What do you think we can learn from these relationships?

Example for Two Dimensional Collisions

Proton #1 with a speed 3.50×10^5 m/s collides elastically with proton #2 initially at rest. After the collision, proton #1 moves at an angle of 37^o to the horizontal axis and proton #2 deflects at an angle f to the same axis. Find the final speeds of the two protons and the scattering angle of proton #2, ϕ.



Since both the particles are protons $m_1=m_2=m_p$.

Using momentum conservation, one obtains

x-comp. $m_p v_{1i} = m_p v_{1f}\cos\theta + m_p v_{2f}\cos\phi$

y-comp. $m_p v_{1f}\sin\theta - m_p v_{2f}\sin\phi = 0$

Canceling m_p and putting in all known quantities, one obtains

$$v_{1f}\cos 37^\circ + v_{2f}\cos\phi = 3.50\times10^5 \quad (1)$$

$$v_{1f}\sin 37^\circ = v_{2f}\sin\phi \quad (2)$$

From kinetic energy conservation:

$$\left(3.50\times10^5\right)^2 = v_{1f}^2 + v_{2f}^2 \quad (3)$$

Solving Eqs. 1-3 equations, one gets

$$v_{1f} = 2.80\times10^5\, m/s$$

$$v_{2f} = 2.11\times10^5\, m/s$$

$$\phi = 53.0^\circ$$

Do this at home☺

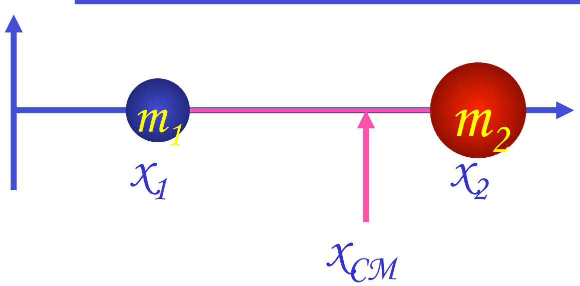
Center of Mass

We've been solving physical problems treating objects as sizeless points with masses, but in realistic situations objects have shapes with masses distributed throughout the body.

Center of mass of a system is the average position of the system's mass and represents the motion of the system as if all the mass is on that point.

What does above statement tell you concerning the forces being exerted on the system?

The total external force exerted on the system of total mass M causes the center of mass to move at an acceleration given by $\vec{a} = \sum \vec{F} / M$ *as if the entire mass of the system is on the center of mass.*



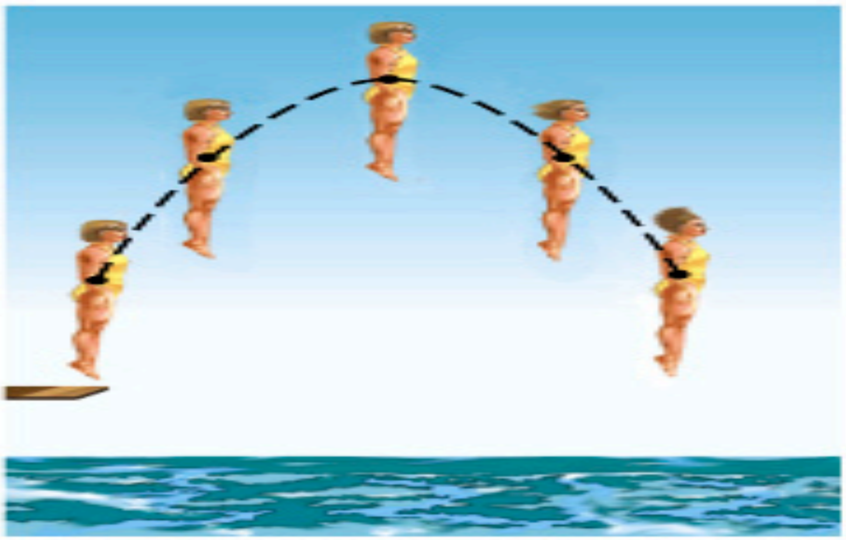
Consider a massless rod with two balls attached at either end.

The position of the center of mass of this system is the mass averaged position of the system

$$x_{CM} \equiv \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

CM is closer to the heavier object

Motion of a Diver and the Center of Mass



(a)

Diver performs a simple dive.
The motion of the center of mass follows a parabola since it is a projectile motion.

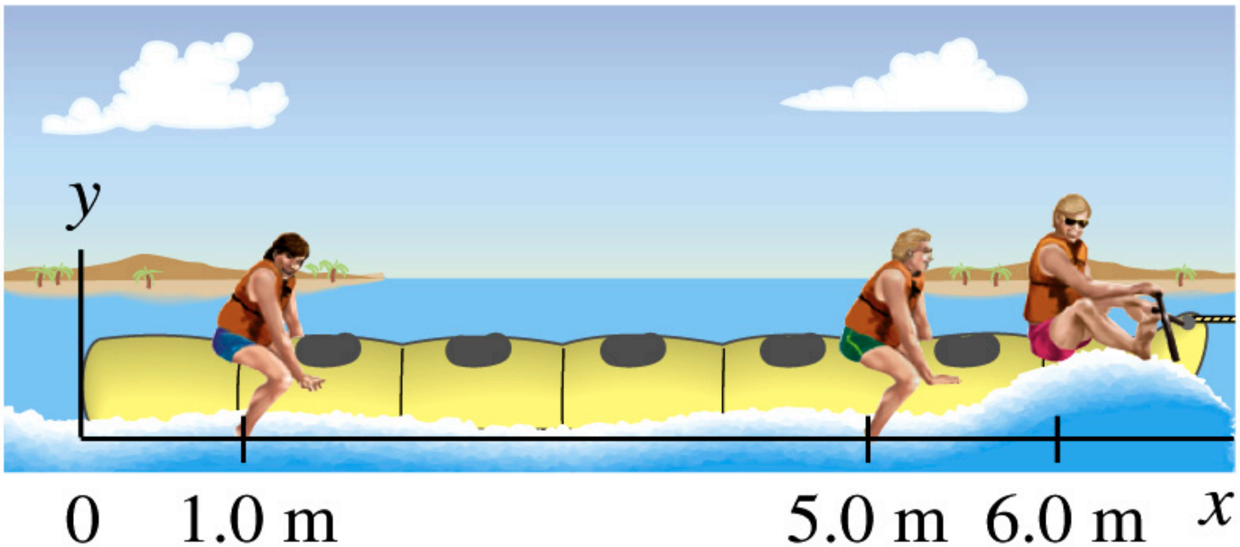
(b)

Diver performs a complicated dive.
The motion of the center of mass still follows the same parabola since it still is a projectile motion.

The motion of the center of mass of the diver is always the same.

Ex. 7 – 12 Center of Mass

Thee people of roughly equivalent mass M on a lightweight (air-filled) banana boat sit along the x axis at positions x_1=1.0m, x_2=5.0m, and x_3=6.0m. Find the position of CM.

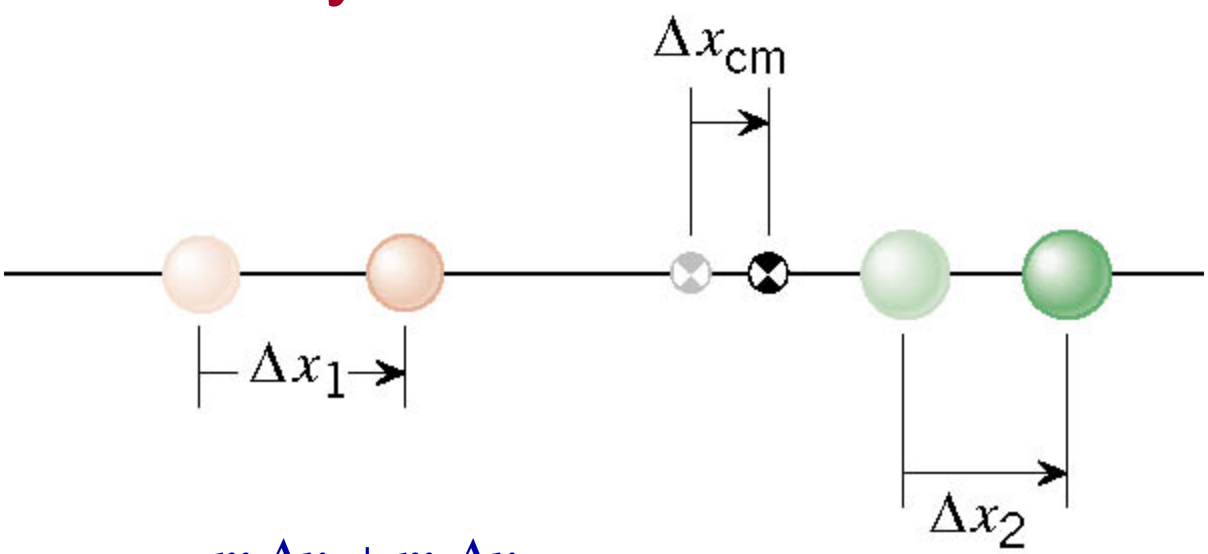


Using the formula for CM

$$x_{CM} = \frac{\sum_i m_i x_i}{\sum_i m_i}$$

$$= \frac{M \cdot 1.0 + M \cdot 5.0 + M \cdot 6.0}{M + M + M} = \frac{12.0M}{3M} = 4.0(m)$$

Velocity of the Center of Mass



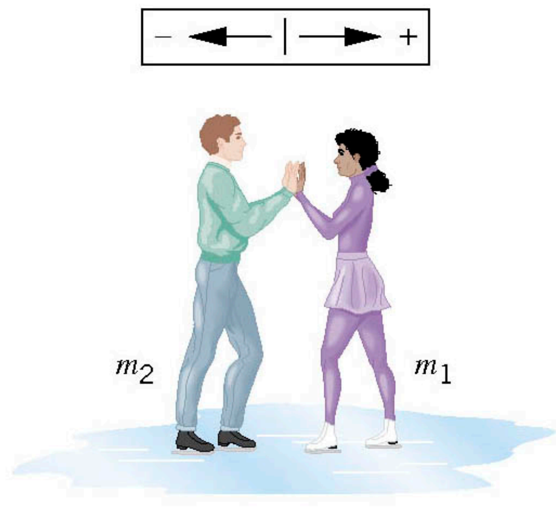
$$\Delta x_{cm} = \frac{m_1 \Delta x_1 + m_2 \Delta x_2}{m_1 + m_2}$$

$$v_{cm} = \frac{\Delta x_{cm}}{\Delta t} = \frac{m_1 \Delta x_1/\Delta t + m_2 \Delta x_2/\Delta t}{m_1 + m_2} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

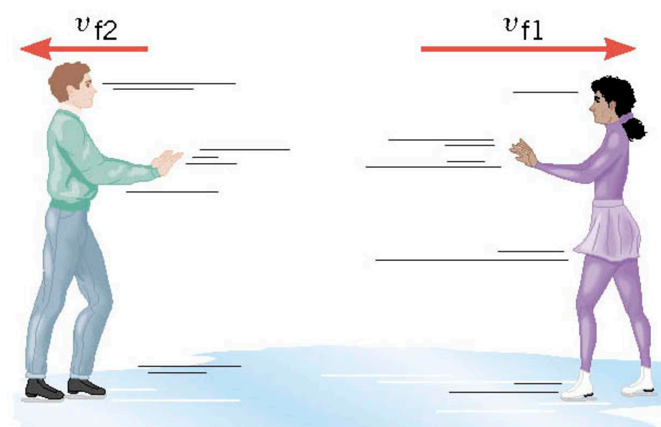
In an isolated system, the total linear momentum does not change, therefore the velocity of the center of mass does not change.

Another Look at the Ice Skater Problem

Starting from rest, two skaters push off against each other on ice where friction is negligible. One is a 54-kg woman and one is a 88-kg man. The woman moves away with a speed of +2.5 m/s.



(a) Before



(b) After

$$v_{10} = 0\,m/s \qquad v_{20} = 0\,m/s$$

$$v_{cm0} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = 0$$

$$v_{1f} = +2.5\,m/s \qquad v_{2f} = -1.5\,m/s$$

$$v_{cmf} = \frac{m_1 v_{1f} + m_2 v_{2f}}{m_1 + m_2}$$

$$= \frac{54 \cdot (+2.5) + 88 \cdot (-1.5)}{54 + 88} = \frac{3}{142} = 0.02 \approx 0\,m/s$$