

PHYS 1441 – Section 002

Lecture #19

Monday, April 8, 2013

Dr. Jaehoon Yu

- Fundamentals of the Rotational Motion
- Rotational Kinematics
- Equations of Rotational Kinematics
- Relationship Between Angular and Linear Quantities
- Rolling Motion of a Rigid Body

Today's homework is homework #10, due 11pm, Monday, Apr. 15!!



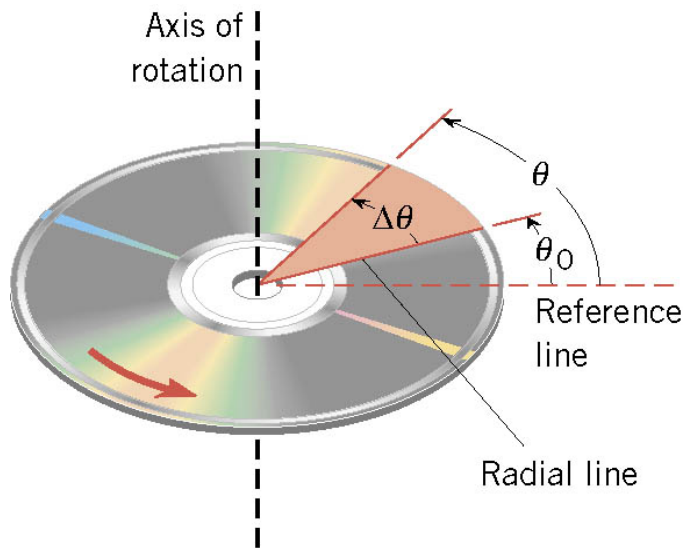
Announcements

- Second non-comp term exam
 - Date and time: 4:00pm, Wednesday, April 17 in class
 - Coverage: CH6.1 through what we finish Monday, April 15
 - This exam could replace the first term exam if better
- Special colloquium for 15 point extra credit
 - Wednesday, April 24, University Hall RM116
 - Class will be substituted by this colloquium
 - Dr. Ketevi Assamagan from Brookhaven National Laboratory on Higgs Discovery in ATLAS
 - Please mark your calendars!!



Rotational Motion and Angular Displacement

In the simplest kind of rotation, points on a rigid object move on circular paths around an **axis of rotation**.



The angle swept out by the line passing through any point on the body and intersecting the axis of rotation perpendicularly is called the **angular displacement**.

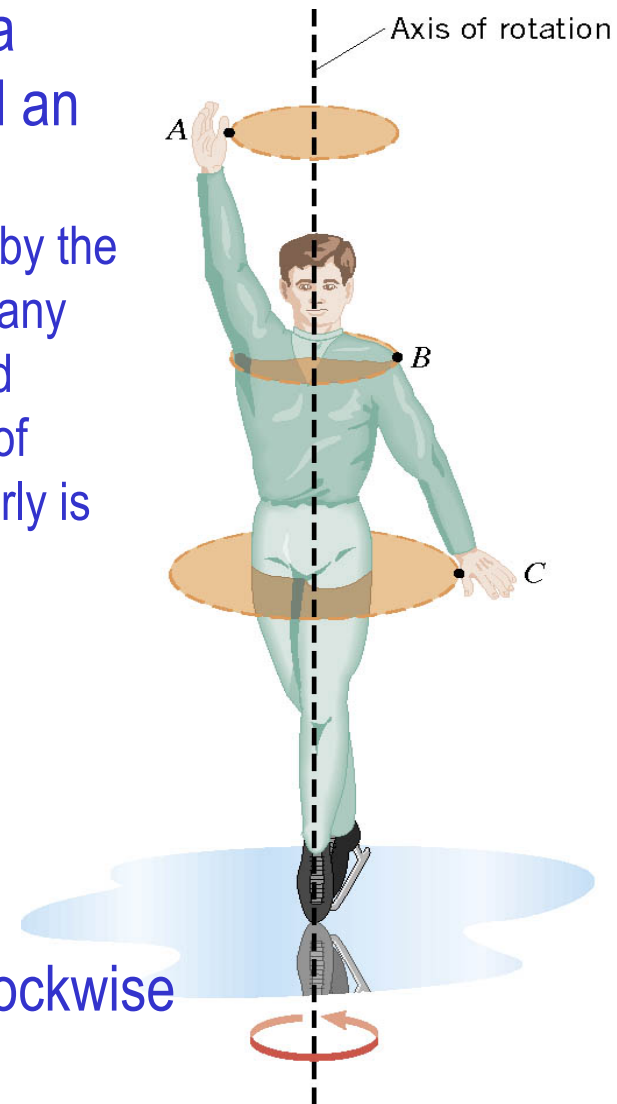
$$\Delta\theta = \theta - \theta_0$$

It's a vector!! So there must be a direction...

How do we define directions? +:if counter-clockwise
 -:if clockwise

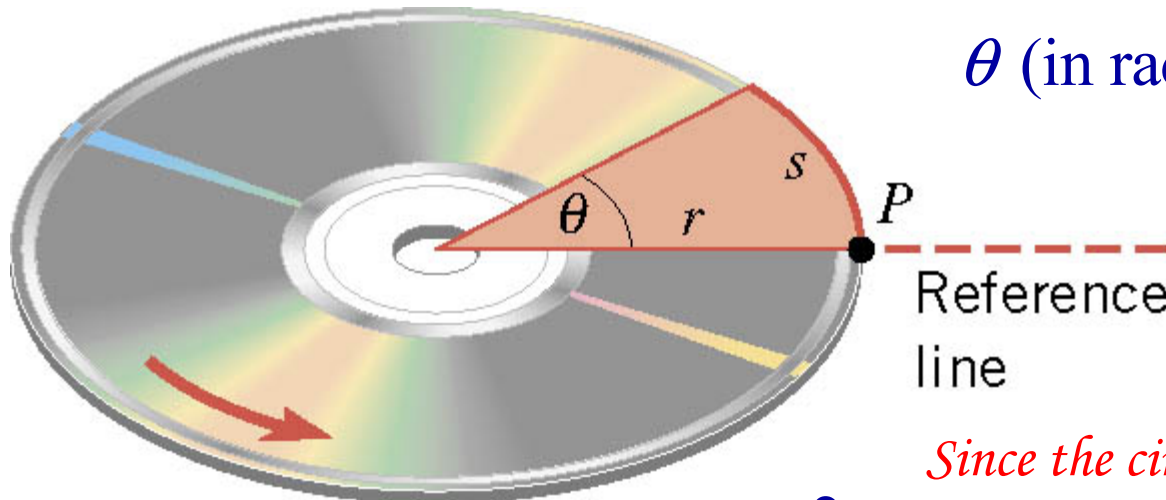
The direction vector gets determined based on the right-hand rule.

These are just conventions!!



SI Unit of the Angular Displacement

$$\theta \text{ (in radians)} = \frac{\text{Arc length}}{\text{Radius}} = \frac{s}{r}$$



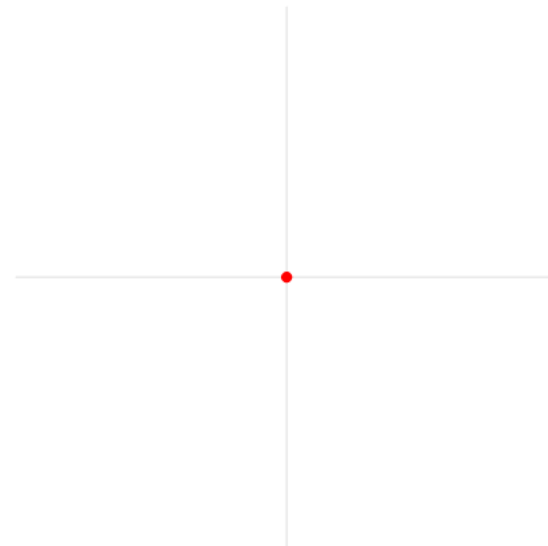
Dimension? None

For one full revolution:

Since the circumference of a circle is $2\pi r$

$$\theta = \frac{2\pi r}{r} = 2\pi \text{ rad} \quad \longrightarrow \quad 2\pi \text{ rad} = 360^\circ$$

One radian is an angle subtended by an arc of the same length as the radius!



Unit of the Angular Displacement

How many degrees are in one radian?

1 radian is $1 \text{ rad} = \frac{360^\circ}{2\pi \text{ rad}} \cdot 1 \text{ rad} = \frac{180^\circ}{\pi} \cong \frac{180^\circ}{3.14} \cong 57.3^\circ$

How radians is one degree?

And one degrees is $1^\circ = \frac{2\pi}{360^\circ} \cdot 1^\circ = \frac{\pi}{180^\circ} \cdot 1^\circ \cong \frac{3.14}{180^\circ} \cdot 1^\circ \cong 0.0175 \text{ rad}$

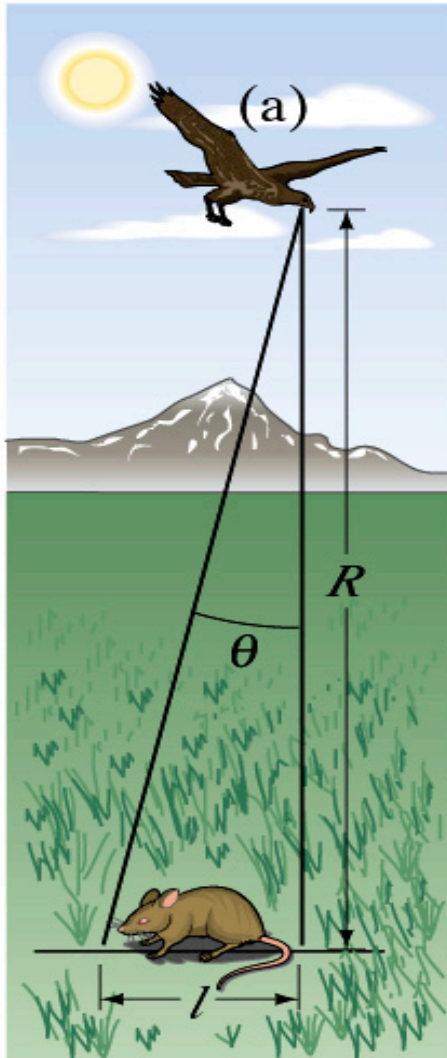
How many radians are in 10.5 revolutions?

$$10.5 \text{ rev} = 10.5 \text{ rev} \cdot 2\pi \frac{\text{rad}}{\text{rev}} = 21\pi (\text{rad})$$

Very important: In solving angular problems, all units, degrees or revolutions, must be converted to radians.

Example 8-2

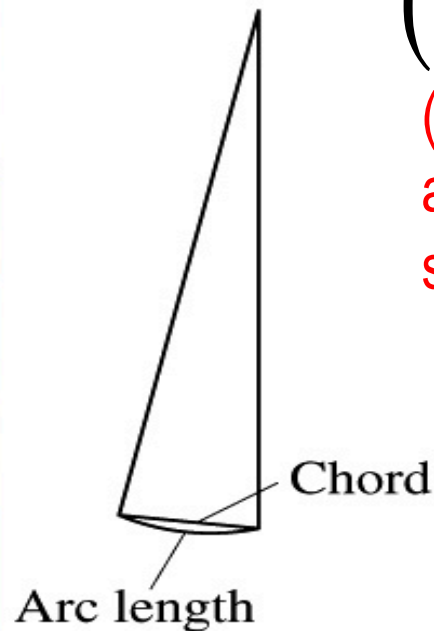
A particular bird's eyes can just distinguish objects that subtend an angle no smaller than about 3×10^{-4} rad. (a) How many degrees is this? (b) How small an object can the bird just distinguish when flying at a height of 100m?



(b) (a) One radian is $360^\circ/2\pi$. Thus

$$3 \times 10^{-4} \text{ rad} = (3 \times 10^{-4} \text{ rad}) \times \left(\frac{360^\circ}{2\pi \text{ rad}} \right) = 0.017^\circ$$

(b) Since $l = r\theta$ and for small angle arc length is approximately the same as the chord length.



$$l = r\theta = 100\text{m} \times 3 \times 10^{-4} \text{ rad} = 3 \times 10^{-2} \text{ m} = 3\text{cm}$$

Ex. Adjacent Synchronous Satellites

Synchronous satellites are put into an orbit whose radius is $4.23 \times 10^7 \text{ m}$. If the angular separation of the two satellites is 2.00 degrees, find the arc length that separates them.

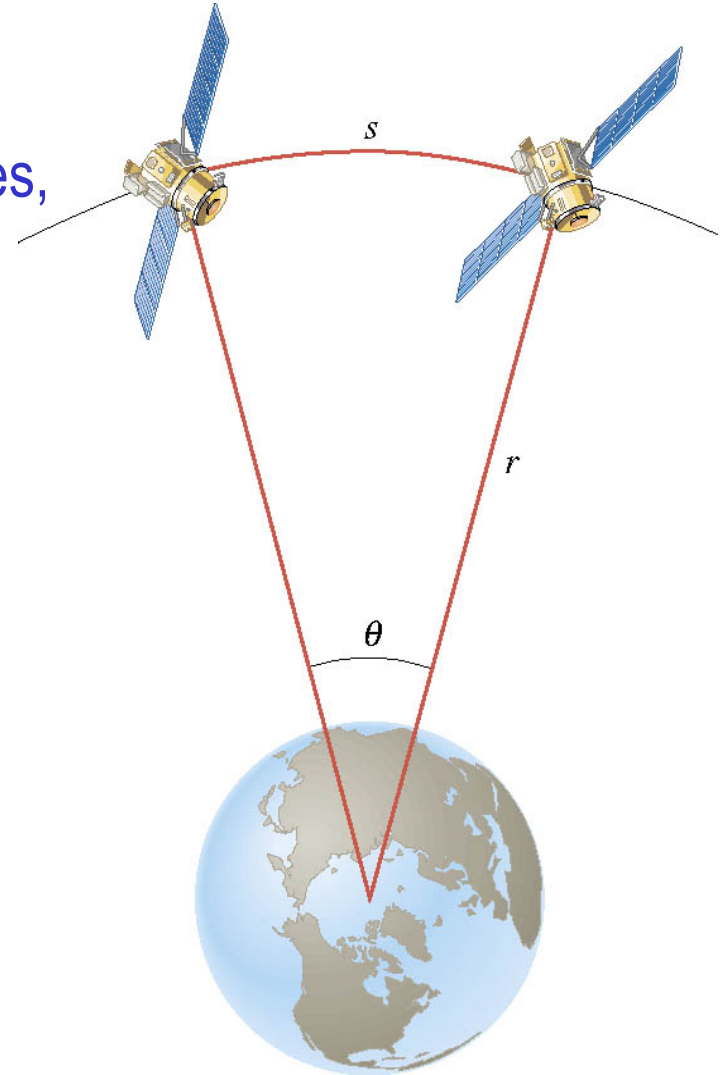
What do we need to find out? The Arc length!!!

$$\theta \text{ (in radians)} = \frac{\text{Arc length}}{\text{Radius}} = \frac{s}{r}$$

Convert
degrees to
radians

$$2.00 \text{ deg} \left(\frac{2\pi \text{ rad}}{360 \text{ deg}} \right) = 0.0349 \text{ rad}$$

$$s = r\theta = (4.23 \times 10^7 \text{ m})(0.0349 \text{ rad}) \\ = 1.48 \times 10^6 \text{ m} \text{ (920 miles)}$$

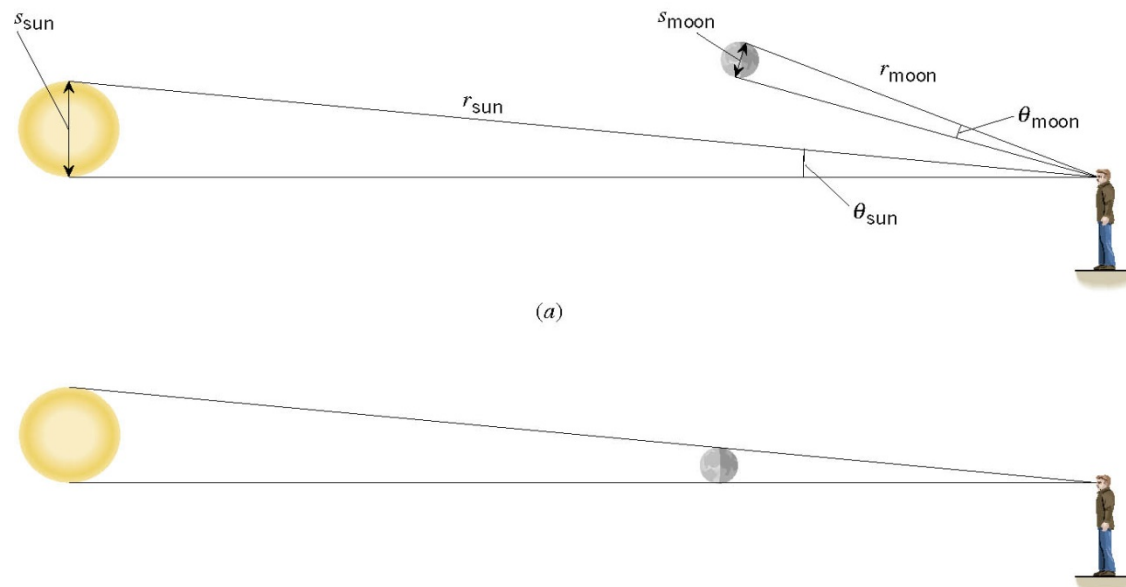


Ex. A Total Eclipse of the Sun

The diameter of the sun is about 400 times greater than that of the moon. By coincidence, the sun is also about 400 times farther from the earth than is the moon. For an observer on the earth, compare the angle subtended by the moon to the angle subtended by the sun and explain why this result leads to a total solar eclipse.

θ (in radians) =

$$\frac{\text{Arc length}}{\text{Radius}} = \frac{s}{r}$$



I can even cover the entire sun with my thumb!! Why?

Monday, April 8, 2013



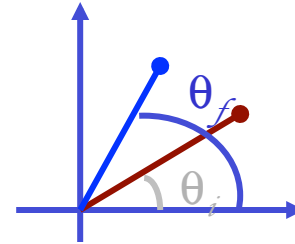
Because the distance ^(b) (r) from my eyes to my thumb is far shorter than that to the sun.

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Angular Displacement, Velocity, and Acceleration

Angular displacement is defined as

$$\Delta\theta = \theta_f - \theta_i$$



How about the average angular velocity, the rate of change of angular displacement?

$$\bar{\omega} \equiv \frac{\theta_f - \theta_i}{t_f - t_i} = \frac{\Delta\theta}{\Delta t}$$

Unit? rad/s Dimension? $[T^{-1}]$

By the same token, the average angular acceleration, rate of change of the angular velocity, is defined as...

$$\bar{\alpha} \equiv \frac{\omega_f - \omega_i}{t_f - t_i} = \frac{\Delta\omega}{\Delta t}$$

Unit? rad/s^2 Dimension? $[T^{-2}]$

When rotating about a fixed axis, every particle on a rigid object rotates through the same angle and has the same angular speed and angular acceleration.

Ex. Gymnast on a High Bar

A gymnast on a high bar swings through two revolutions in a time of 1.90 s. Find the average angular velocity of the gymnast.

What is the angular displacement?

$$\Delta\theta = \ominus 2.00 \text{ rev} \left(\frac{2\pi \text{ rad}}{1 \text{ rev}} \right) \\ = \ominus 12.6 \text{ rad}$$

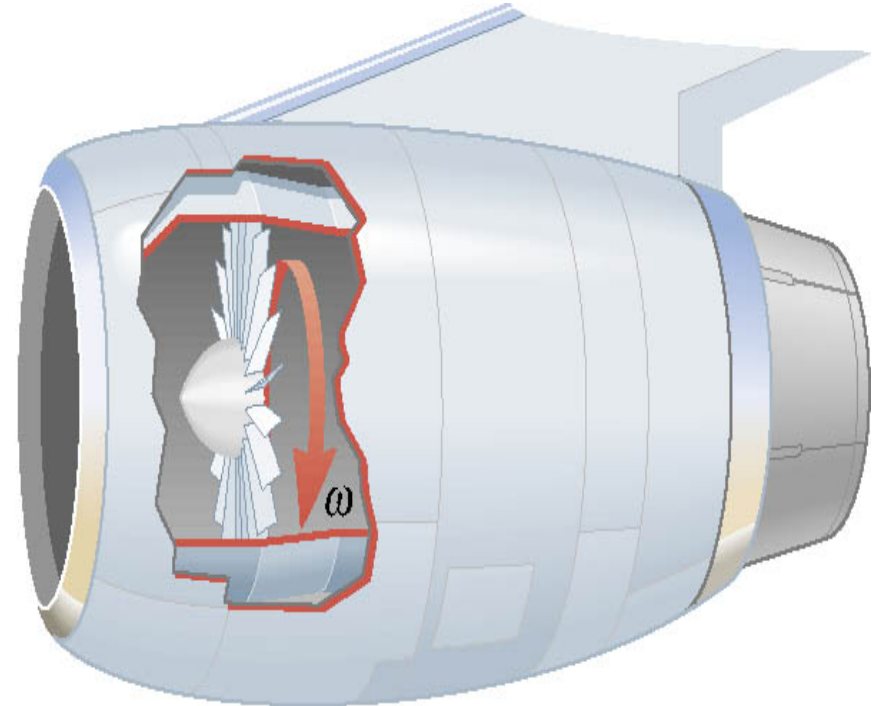
Why negative? Because he is rotating clockwise!!

$$\bar{\omega} = \frac{-12.6 \text{ rad}}{1.90 \text{ s}} = -6.63 \text{ rad/s}$$



Ex. A Jet Revving Its Engines

As seen from the front of the engine, the fan blades are rotating with an angular speed of -110 rad/s . As the plane takes off, the angular velocity of the blades reaches -330 rad/s in a time of 14 s . Find the angular acceleration, assuming it to be constant.



$$\begin{aligned}\bar{\alpha} &= \frac{\omega_f - \omega_i}{t_f - t_i} = \frac{\Delta\omega}{\Delta t} \\ &= \frac{(-330 \text{ rad/s}) - (-110 \text{ rad/s})}{14 \text{ s}} = -16 \text{ rad/s}^2\end{aligned}$$

Rotational Kinematics

The first type of motion we have learned in linear kinematics was under a constant acceleration. We will learn about the rotational motion under constant angular acceleration (α), because these are the simplest motions in both cases.

Just like the case in linear motion, one can obtain

Angular velocity under constant angular acceleration:

$$\omega_f = \omega_0 + \alpha t$$

Linear kinematics $v = v_o + at$

Angular displacement under constant angular acceleration:

$$\theta_f = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$$

Linear kinematics $x_f = x_0 + v_o t + \frac{1}{2} at^2$

One can also obtain

Linear kinematics $v_f^2 = v_o^2 + 2a(x_f - x_i)$

$$\omega_f^2 = \omega_0^2 + 2\alpha(\theta_f - \theta_0)$$



Problem Solving Strategy

- Visualize the problem by drawing a picture.
- Write down the values that are given for any of the five kinematic variables and convert them to SI units.
 - Remember that the unit of the angle must be in radians!!
- Verify that the information contains values for at least three of the five kinematic variables. Select the appropriate equation.
- When the motion is divided into segments, remember that the final angular velocity of one segment is the initial velocity for the next.
- Keep in mind that there may be two possible answers to a kinematics problem.

