PHYS 1441 – Section 002
Lecture #20

Wednesday, April 10, 2013
Dr. Jaehoon Yu

- Equations of Rotational Kinematics
- Relationship Between Angular and Linear Quantities
- Rolling Motion of a Rigid Body
- Torque
- Moment of Inertia

Announcements

- Second non-comp term exam
  - Date and time: 4:00pm, Wednesday, April 17 in class
  - Coverage: CH6.1 through what we finish Monday, April 15
  - This exam could replace the first term exam if better
- Remember that the lab final exams are next week!!
- Special colloquium for 15 point extra credit!!
  - Wednesday, April 24, University Hall RM116
  - Class will be substituted by this colloquium
  - Dr. Ketevi Assamagan from Brookhaven National Laboratory on Higgs Discovery in ATLAS
  - Please mark your calendars!!

Rotational Kinematics

The first type of motion we have learned in linear kinematics was under a constant acceleration. We will learn about the rotational motion under constant angular acceleration (α), because these are the simplest motions in both cases.

Just like the case in linear motion, one can obtain

Angular velocity under constant angular acceleration:

$$\omega_f = \omega_0 + \alpha t$$

Linear kinematics $v = v_o + at$

Angular displacement under constant angular acceleration:

$$\theta_f = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

Linear kinematics $x_f = x_0 + v_o t + \frac{1}{2} a t^2$

One can also obtain

$$\omega_f^2 = \omega_0^2 + 2\alpha\left(\theta_f - \theta_0\right)$$

Linear kinematics $v_f^2 = v_o^2 + 2a\left(x_f - x_i\right)$

Rotational Kinematics Problem Solving Strategy

- Visualize the problem by drawing a picture.
- Write down the values that are given for any of the five kinematic variables and convert them to SI units.
  - Remember that the unit of the angle must be in radians!!
- Verify that the information contains values for at least three of the five kinematic variables. Select the appropriate equation.
- When the motion is divided into segments, remember that the final angular velocity of one segment is the initial velocity for the next.
- Keep in mind that there may be two possible answers to a kinematics problem.

Example for Rotational Kinematics

A wheel rotates with a constant angular acceleration of 3.50 rad/s^2. If the angular speed of the wheel is 2.00 rad/s at t_i=0, a) through what angle does the wheel rotate in 2.00s?

Using the angular displacement formula in the previous slide, one gets

$$\theta_f - \theta_i = \omega t + \frac{1}{2}\alpha t^2$$

$$= 2.00 \times 2.00 + \frac{1}{2} 3.50 \times (2.00)^2 = 11.0 rad$$

$$= \frac{11.0}{2\pi} rev. = 1.75 rev.$$

Example for Rotational Kinematics cnt'd

What is the angular speed at t=2.00s?

Using the angular speed and acceleration relationship

$$\omega_f = \omega_i + \alpha t \quad = 2.00 + 3.50 \times 2.00 = 9.00 rad/s$$

Find the angle through which the wheel rotates between t=2.00 s and t=3.00 s.

Using the angular kinematic formula

$$\theta_f - \theta_i = \omega t + \frac{1}{2}\alpha t^2$$

At t=2.00s

$$\theta_{t=2} = 2.00 \times 2.00 + \frac{1}{2} 3.50 \times 2.00 \quad = 11.0 rad$$

At t=3.00s

$$\theta_{t=3} = 2.00 \times 3.00 + \frac{1}{2} 3.50 \times (3.00)^2 \quad = 21.8 rad$$

Angular displacement

$$\Delta\theta = \theta_3 - \theta_2 = 10.8 rad \quad = \frac{10.8}{2\pi} rev. = 1.72 rev.$$

Ex. Blending with a Blender

The blade is whirling with an angular velocity of +375 rad/s when the "puree" button is pushed in. When the "blend" button is pushed, the blade accelerates and reaches a greater angular velocity after the blade has rotated through an angular displacement of +44.0 rad. The angular acceleration has a constant value of +1740 rad/s^2. Find the final angular velocity of the blade.

θ	α	ω	ω_o	t
+44.0rad	+1740rad/s^2	?	+375rad/s	

Which kinematic eq? $\omega^2 = \omega_o^2 + 2\alpha\theta$

$$\omega = \pm\sqrt{\omega_o^2 + 2\alpha\theta}$$

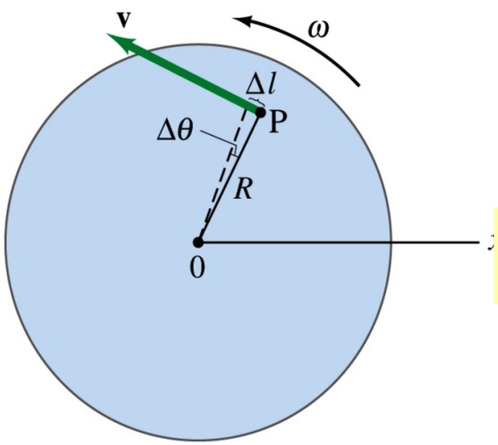
$$= \pm\sqrt{(375\,\text{rad/s})^2 + 2(1740\,\text{rad/s}^2)(44.0\,\text{rad})} = \pm 542\,\text{rad/s}$$

Which sign? $\omega = +542\,\text{rad/s}$ *Why? Because the blade is accelerating in counter-clockwise!*

Relationship Between Angular and Linear Quantities

What do we know about a rigid object that rotates about a fixed axis of rotation?

Every particle (or masslet) in an object moves in a circle centered at the same axis of rotation with the same angular velocity.



When a point rotates, it has both the linear and angular components in its motion.

What is the linear component of the motion you see?

Linear velocity along the tangential direction.

How do we related this linear component of the motion with angular component?

The direction of ω follows the right-hand rule.

The arc-length is $l = r\theta$ So the tangential speed v is $v = \frac{\Delta l}{\Delta t} = \frac{\Delta (r\theta)}{\Delta t} = r\frac{\Delta\theta}{\Delta t} = r\omega$

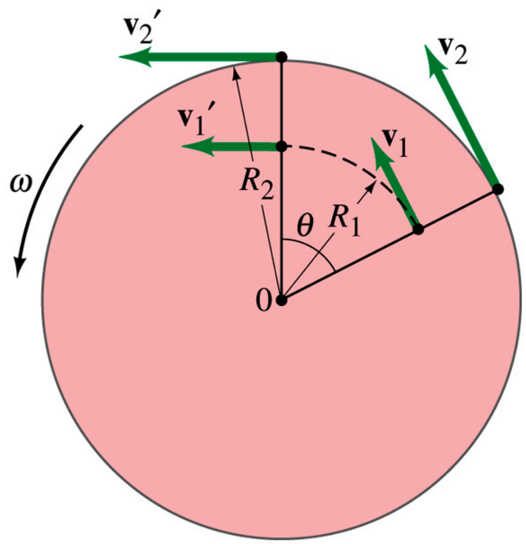
What does this relationship tell you about the tangential speed of the points in the object and their angular speed?

Although every particle in the object has the same angular speed, its tangential speed differs and is proportional to its distance from the axis of rotation.

The farther away the particle is from the center of rotation, the higher the tangential speed.

Is the lion faster than the horse?

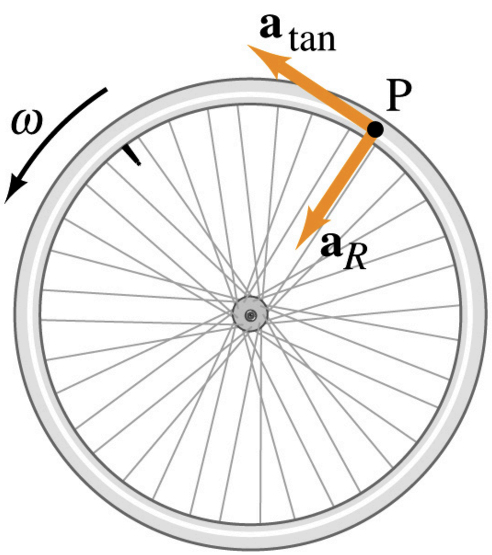
A rotating carousel has one child sitting on the horse near the outer edge and another child on the lion halfway out from the center. (a) Which child has the greater linear speed? (b) Which child has the greater angular speed?



(a) Linear speed is the distance traveled divided by the time interval. So the child sitting at the outer edge travels more distance within the given time than the child sitting closer to the center. Thus, the horse is faster than the lion.

(b) Angular speed is the angle traveled divided by the time interval. The angle both the children travel in the given time interval is the same. Thus, both the horse and the lion have the same angular speed.

How about the acceleration?



How many different linear acceleration components do you see in a circular motion and what are they? Two

Tangential, a_t, and the radial acceleration, a_r.

Since the tangential speed v is $v_t = r\omega$

The magnitude of tangential acceleration a_t is

$$a_t = \frac{v_{tf} - v_{t0}}{\Delta t} = \frac{r\omega_f - r\omega_0}{\Delta t} = r\frac{\omega_f - \omega_0}{\Delta t} = r\alpha$$

What does this relationship tell you?

Although every particle in the object has the same angular acceleration, its tangential acceleration differs proportional to its distance from the axis of rotation.

The radial or centripetal acceleration a_r is

$$a_r = \frac{v^2}{r} = \frac{(r\omega)^2}{r} = r\omega^2$$

What does this tell you?

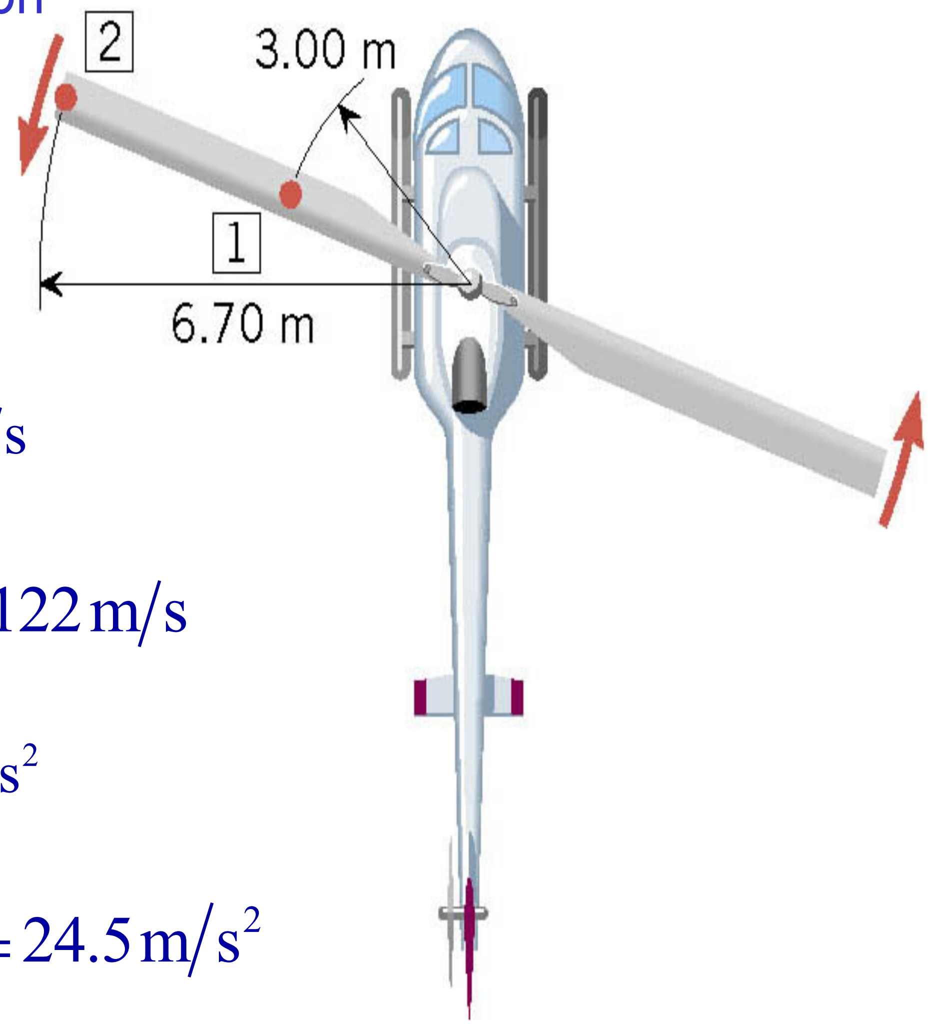
The father away the particle is from the rotation axis, the more radial acceleration it receives. In other words, it receives more centripetal force.

Total linear acceleration is

$$a = \sqrt{a_t^2 + a_r^2} = \sqrt{(r\alpha)^2 + (r\omega^2)^2} = r\sqrt{\alpha^2 + \omega^4}$$

Ex. A Helicopter Blade

A helicopter blade has an angular speed of 6.50 rev/s and an angular acceleration of 1.30 rev/s^2. For point 1 on the blade, find the magnitude of (a) the tangential speed and (b) the tangential acceleration.



$$\omega = \left(6.50\ \frac{\text{rev}}{\text{s}}\right)\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right) = 40.8\ \text{rad/s}$$

$$v_T = r\varpi = (3.00\ \text{m})(40.8\,\text{rad/s}) = 122\,\text{m/s}$$

$$\alpha = \left(1.30\ \frac{\text{rev}}{\text{s}^2}\right)\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right) = 8.17\ \text{rad/s}^2$$

$$a_T = r\alpha = (3.00\ \text{m})(8.17\,\text{rad/s}^2) = 24.5\,\text{m/s}^2$$

Rolling Motion of a Rigid Body

What is a rolling motion?

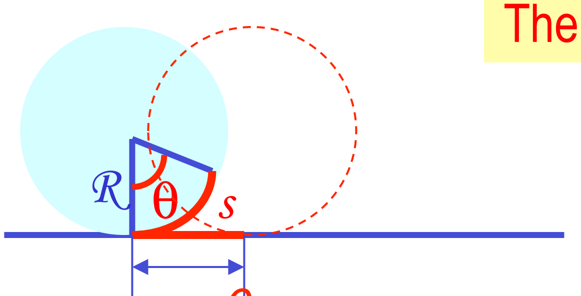
A more generalized case of a motion where the rotational axis moves together with an object

A rotational motion about a moving axis

To simplify the discussion, let's make a few assumptions

1. Limit our discussion on very symmetric objects, such as cylinders, spheres, etc
2. The object rolls on a flat surface

Let's consider a cylinder rolling on a flat surface, without slipping.



Under what condition does this "Pure Rolling" happen?

The total linear distance the CM of the cylinder moved is $s = R\theta$

Thus the linear speed of the CM is

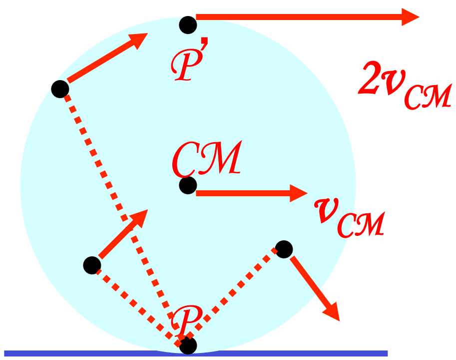
$$\bar{v}_{CM} = \frac{\Delta s}{\Delta t} = R\frac{\Delta \theta}{\Delta t} = R\omega$$

The condition for a "Pure Rolling motion"

More Rolling Motion of a Rigid Body

The magnitude of the linear acceleration of the CM is

$$a_{CM} = \frac{\Delta v_{CM}}{\Delta t} = R\frac{\Delta \omega}{\Delta t} = R\alpha$$



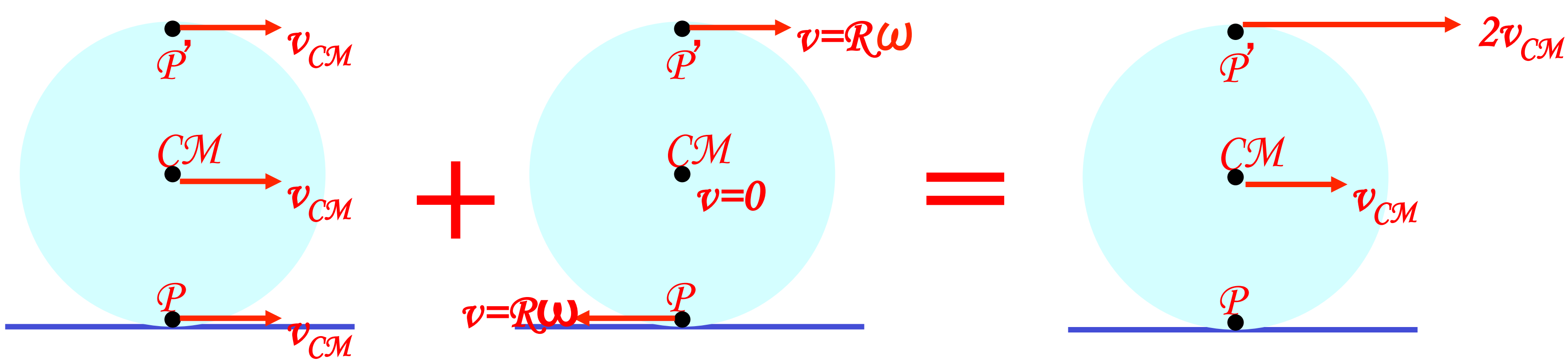
As we learned in rotational motion, all points in a rigid body moves at the same angular speed but at different linear speeds.

CM is moving at the same speed at all times.

At any given time, the point that comes to P has 0 linear speed while the point at P' has twice the speed of CM

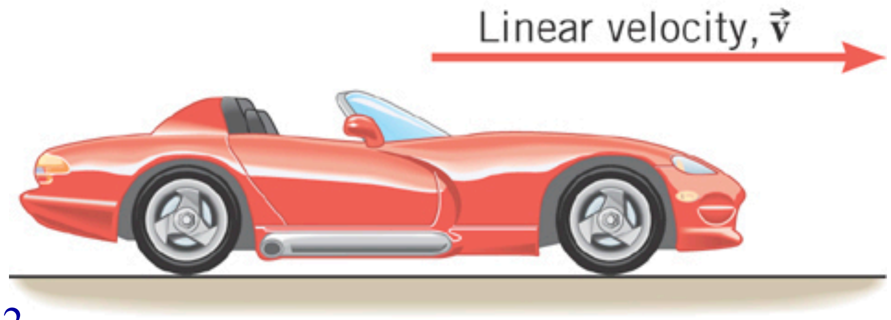
Why??

A rolling motion can be interpreted as the sum of Translation and Rotation



Ex. An Accelerating Car

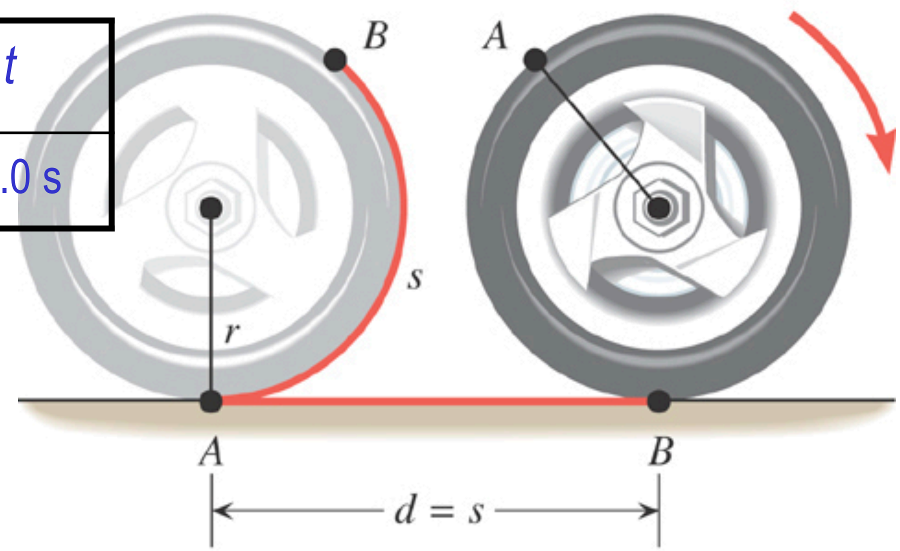
Starting from rest, the car accelerates for 20.0 s with a constant linear acceleration of 0.800 m/s^2. The radius of the tires is 0.330 m. What is the angle through which each wheel has rotated?



(a)

$$\alpha = \frac{a}{r} = \frac{0.800\,\text{m/s}^2}{0.330\text{ m}} = 2.42\,\text{rad/s}^2$$

θ	α	ω	ω_o	t
?	-2.42 rad/s^2		0 rad/s	20.0 s

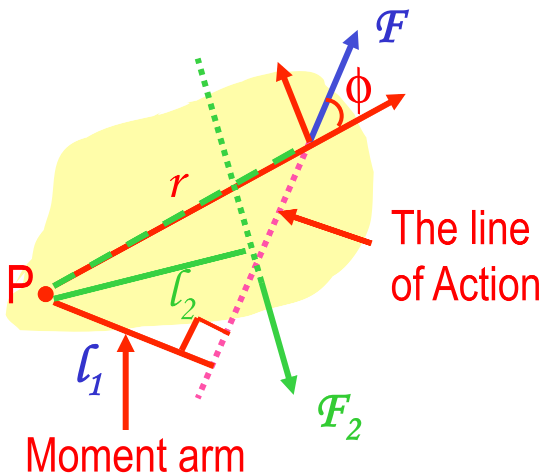


(b)

$$\theta = \omega_o t + \tfrac{1}{2}\alpha t^2$$
$$= \tfrac{1}{2}\left(-2.42\,\text{rad/s}^2\right)^2 (20.0\text{ s})^2$$
$$= -484\text{ rad}$$

Torque

Torque is the tendency of a force to rotate an object about an axis. Torque, τ, is a vector quantity.



Consider an object pivoting about the point P by the force F being exerted at a distance r from P.

The line that extends out of the tail of the force vector is called the line of action.

The perpendicular distance from the pivoting point P to the line of action is called the moment arm.

$$\left|\vec{\tau}\right| \equiv (\text{Magnitude of the Force}) \times (\text{Lever Arm}) = (F)(r\sin\phi) = Fl$$

Magnitude of torque is defined as the product of the force exerted on the object to rotate it and the moment arm.

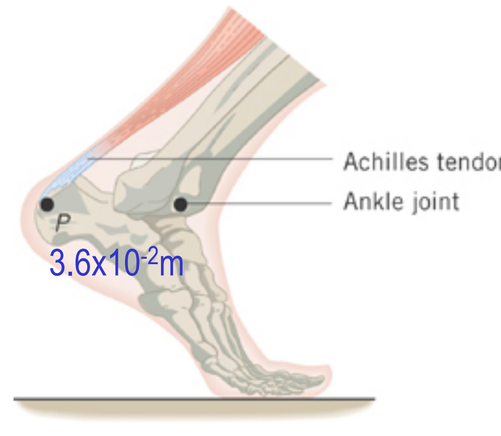
When there are more than one force being exerted on certain points of the object, one can sum up the torque generated by each force vectorially. The convention for sign of the torque is **positive if rotation is in counter-clockwise** and **negative if clockwise**.

$$\sum \tau = \tau_1 + \tau_2 = F_1 l_1 - F_2 l_2$$

Unit? $N \cdot m$

Ex. The Achilles Tendon

The tendon exerts a force of magnitude 790 N on the point P. Determine the torque (magnitude and direction) of this force about the ankle joint which is located 3.6×10^{-2}m away from point P.



(a)

First, let's find the lever arm length

$$\cos 55^{\circ} = \frac{l}{3.6\times10^{-2}\,\text{m}}$$

$$l = 3.6\times10^{-2}\cos 55^{\circ} =$$

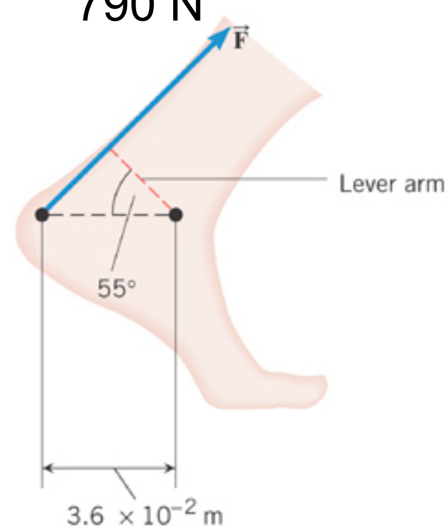
$$= 3.6\times10^{-2}\sin\left(90^{\circ}-55^{\circ}\right) = 2.1\times10^{-2}\,(m)$$

So the torque is

$$\tau = F\ell$$

$$= (720\text{ N})\left(3.6\times10^{-2}\,\text{m}\right)\cos 55^{\circ}$$

$$= (720\text{ N})\left(3.6\times10^{-2}\,\text{m}\right)\sin 35^{\circ} = 15\text{ N}\cdot\text{m}$$



(b)

Since the rotation is in clock-wise $\tau = -15N\cdot m$