PHYS 1441 – Section 002
Lecture #21

Monday, April 15, 2013
Dr. Jaehoon Yu

- Moment of Inertia
- Torque and Angular Acceleration
- Rotational Kinetic Energy

Today's homework is homework #11, due 11pm, Tuesday, Apr. 23!!

Announcements

- Quiz #4 results
  - Class average: 30.1/70
    - Equivalent to 43/100
    - Previous quizzes: 65/100, 60/100 and 52.5/100
  - Top score: 70/70
- Second non-comp term exam
  - Date and time: 4:00pm, this Wednesday, April 17 in class
  - Coverage: CH6.1 through what we finish today (CH8.9)
  - This exam could replace the first term exam if better
- Remember that the lab final exams are this week!!
- Special colloquium for 15 point extra credit!!
  - Wednesday, April 24, University Hall RM116
  - Class will be substituted by this colloquium

Moment of Inertia

Rotational Inertia: Measure of resistance of an object to changes in its rotational motion. Equivalent to mass in linear motion.

For a group of objects $\boxed{I \equiv \sum_i m_i r_i^2}$ For a rigid body $\boxed{I \equiv \int r^2 dm}$

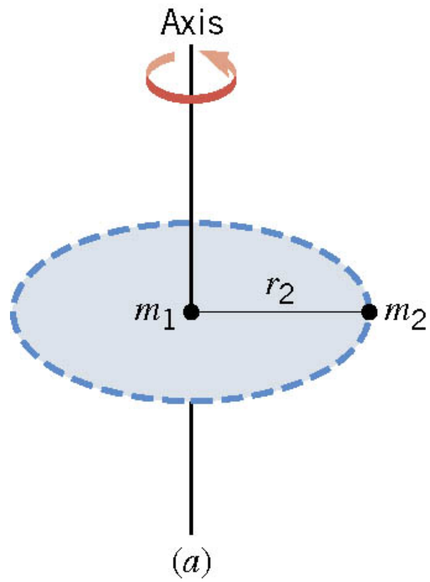
What are the dimension and unit of Moment of Inertia? $\left[ML^2\right]$ $kg \cdot m^2$

Determining Moment of Inertia is extremely important for computing equilibrium of a rigid body, such as a building.

Dependent on the axis of rotation!!!

Ex. The Moment of Inertia Depends on Where the Axis Is.

Two particles each have mass m_1 and m_2 and are fixed at the ends of a thin rigid rod. The length of the rod is *L*. Find the moment of inertia when this object rotates relative to an axis that is perpendicular to the rod at (a) one end and (b) the center.



(a) $I = \sum\left(mr^2\right) = m_1 r_1^2 + m_2 r_2^2$

$$m_1 = m_2 = m \qquad r_1 = 0 \quad r_2 = L$$

$$I = m(0)^2 + m(L)^2 = mL^2$$

(b) $I = \sum\left(mr^2\right) = m_1 r_1^2 + m_2 r_2^2$

$$m_1 = m_2 = m \qquad r_1 = L/2 \quad r_2 = L/2$$

$$I = m(L/2)^2 + m(L/2)^2 = \tfrac{1}{2} mL^2$$

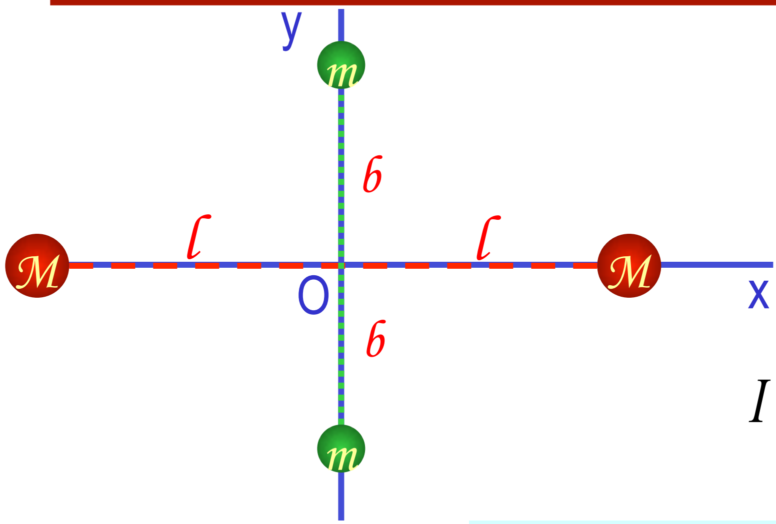
Which case is easier to spin?

Case (b)

Why? Because the moment of inertia is smaller

Example for Moment of Inertia

In a system of four small spheres as shown in the figure, assuming the radii are negligible and the rods connecting the particles are massless, compute the moment of inertia and the rotational kinetic energy when the system rotates about the y-axis at angular speed w.



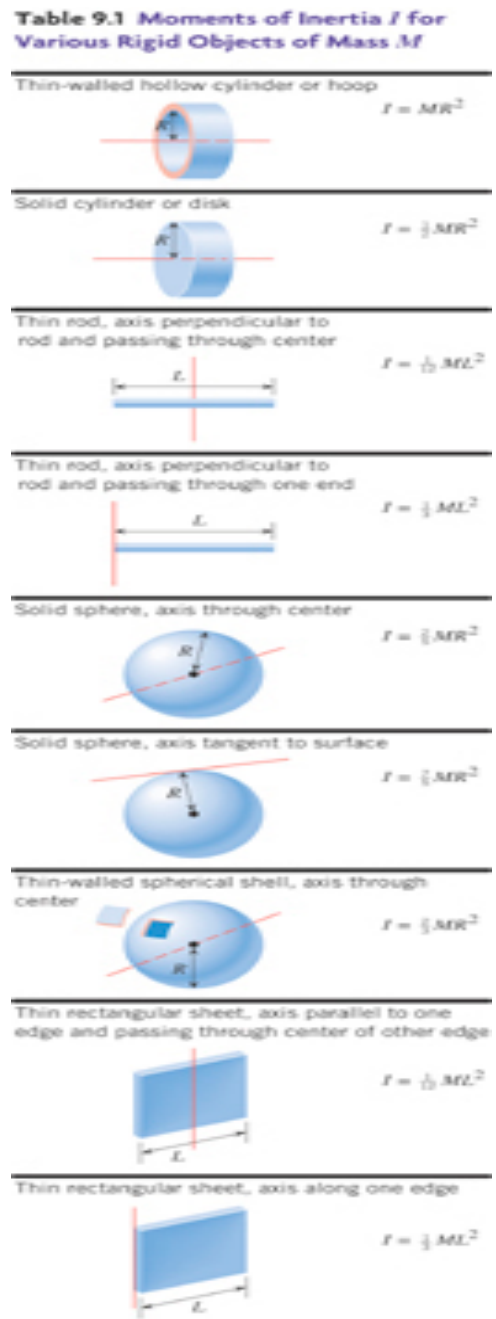
Since the rotation is about y axis, the moment of inertia about y axis, I_y, is

$$I = \sum_i m_i r_i^2 = Ml^2 + Ml^2 + m \cdot 0^2 + m \cdot 0^2 = 2Ml^2$$

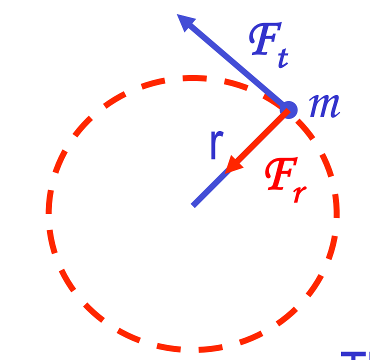
Why are some 0s?

This is because the rotation is done about the y axis, and the radii of the spheres are negligible.

Check out Figure 8 – 21 for moment of inertia for various shaped objects



Torque & Angular Acceleration



Let's consider a point object with mass m rotating on a circle.

What forces do you see in this motion?

The tangential force $\mathcal{F}_t$ and the radial force $\mathcal{F}_r$

The tangential force $\mathcal{F}_t$ is $F_t = ma_t = mr\alpha$

The torque due to tangential force $\mathcal{F}_t$ is $\tau = F_t r = ma_t r = mr^2\alpha = I\alpha$

What do you see from the above relationship? $\tau = I\alpha$

What does this mean? Torque acting on a particle is proportional to the angular acceleration.

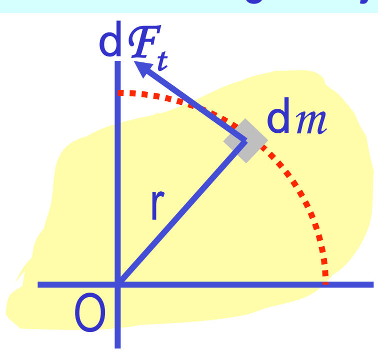
What law do you see from this relationship? Analogs to Newton's 2nd law of motion in rotation.

How about a rigid object?

The external tangential force $\delta\mathcal{F}_t$ is $\delta F_t = \delta m a_t = \delta m r\alpha$

The torque due to tangential force $\mathcal{F}_t$ is $\delta\tau = \delta F_t r = \left(r^2\delta m\right)\alpha$

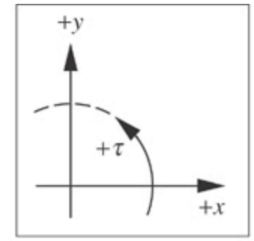
The total torque is $\sum\delta\tau = \alpha\sum r^2\delta m = I\alpha$



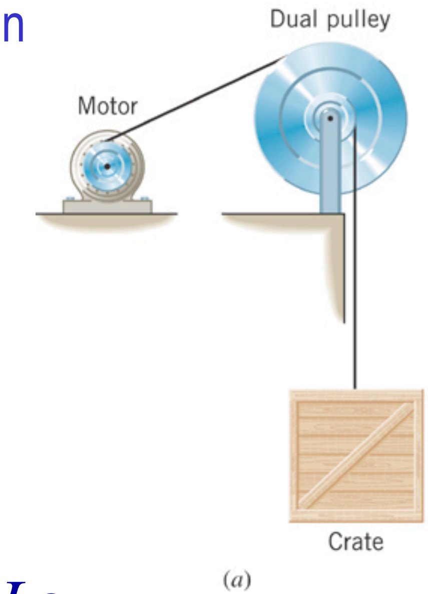
What is the contribution due to radial force and why?

Contribution from radial force is 0, because its line of action passes through the pivoting point, making the moment arm 0.

Ex. Hoisting a Crate



The combined moment of inertia of the dual pulley is 50.0 kg·m^2. The crate weighs 4420 N. A tension of 2150 N is maintained in the cable attached to the motor. Find the angular acceleration of the dual pulley.



$$\sum F_y = T_2' - mg = ma_y$$

$$T_2' = mg + ma_y$$

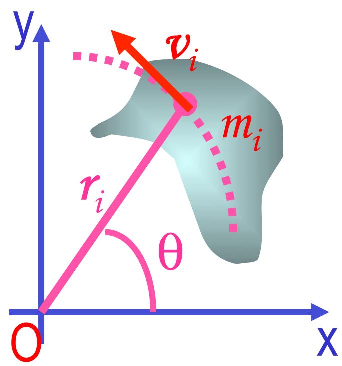
$$\sum \tau = T_1 \ell_1 - T_2' \ell_2 = I\alpha$$

$$T_1 \ell_1 - \left(mg + ma_y\right)\ell_2 = I\alpha$$

$$a_y = \ell_2 \alpha \qquad T_1 \ell_1 - \left(mg + m\ell_2 \alpha\right)\ell_2 = I\alpha$$

$$\alpha = \frac{T_1 \ell_1 - mg\ell_2}{I + m\ell_2^2} = \frac{(2150\text{ N})(0.600\text{ m}) - (451\text{ kg})\left(9.80\,\text{m/s}^2\right)(0.200\text{ m})}{46.0\text{ kg}\cdot\text{m}^2 + (451\text{ kg})(0.200\text{ m})^2} = 6.3\,\text{rad/s}^2$$

Rotational Kinetic Energy



What do you think the kinetic energy of a rigid object that is undergoing a circular motion is?

Kinetic energy of a masslet, m_i, moving at a tangential speed, v_i, is

$$K_i = \frac{1}{2} m_i v_i^2 = \frac{1}{2} m_i r_i^2 \omega^2$$

Since a rigid body is a collection of masslets, the total kinetic energy of the rigid object is

$$K_R = \sum_i K_i = \frac{1}{2} \sum_i m_i r_i^2 \omega^2 = \frac{1}{2} \boxed{\left(\sum_i m_i r_i^2 \right)} \omega^2$$

Since moment of Inertia, I, is defined as

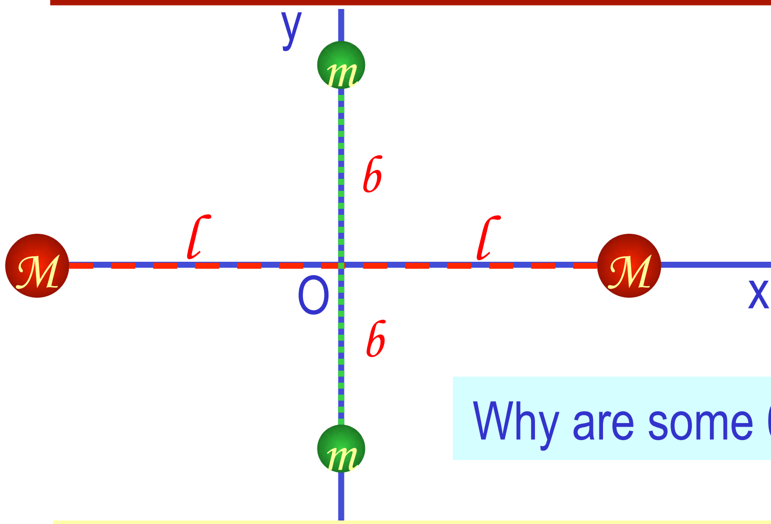
$$I = \sum_i m_i r_i^2$$

The above expression is simplified as

$$K_R = \frac{1}{2} I \omega^2$$

Example for Moment of Inertia

In a system of four small spheres as shown in the figure, assuming the radii are negligible and the rods connecting the particles are massless, compute the moment of inertia and the rotational kinetic energy when the system rotates about the y-axis at angular speed w.



Since the rotation is about y axis, the moment of inertia about y axis, I_y, is

$$I = \sum_i m_i r_i^2 = Ml^2 + Ml^2 + m \cdot 0^2 + m \cdot 0^2 = 2Ml^2$$

Why are some 0s?

This is because the rotation is done about y axis, and the radii of the spheres are negligible.

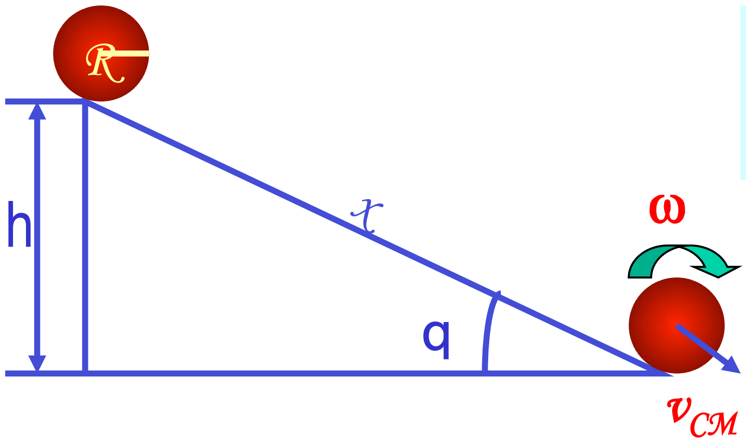
Thus, the rotational kinetic energy is

$$K_R = \frac{1}{2} I\omega^2 = \frac{1}{2}\left(2Ml^2\right)\omega^2 = Ml^2\omega^2$$

Find the moment of inertia and rotational kinetic energy when the system rotates on the x-y plane about the z-axis that goes through the origin O.

$$I = \sum_i m_i r_i^2 = Ml^2 + Ml^2 + mb^2 + mb^2 = 2\left(Ml^2 + mb^2\right) \quad K_R = \frac{1}{2} I\omega^2 = \frac{1}{2}\left(2Ml^2 + 2mb^2\right)\omega^2 = \left(Ml^2 + mb^2\right)\omega^2$$

Kinetic Energy of a Rolling Sphere



Let's consider a sphere with radius R rolling down the hill without slipping.

$$K = \frac{1}{2}I_{CM}\omega^2 + \frac{1}{2}MR^2\omega^2$$

$$= \frac{1}{2}I_{CM}\left(\frac{v_{CM}}{R}\right)^2 + \frac{1}{2}Mv_{CM}^2$$

$$= \frac{1}{2}\left(\frac{I_{CM}}{R^2} + M\right)v_{CM}^2$$

Since $v_{CM}=R\omega$

What is the speed of the CM in terms of known quantities and how do you find this out?

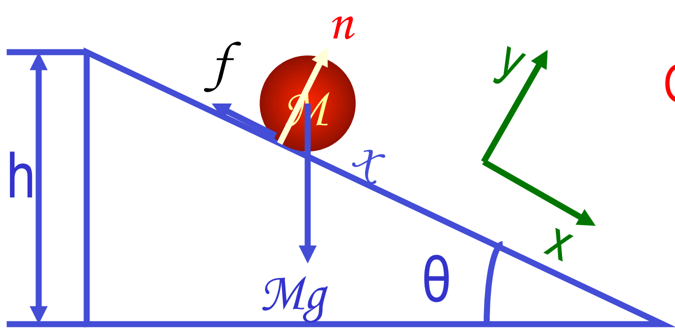
Since the kinetic energy at the bottom of the hill must be equal to the potential energy at the top of the hill

$$K = \frac{1}{2}\left(\frac{I_{CM}}{R^2} + M\right)v_{CM}^2 = Mgh$$

$$v_{CM} = \sqrt{\frac{2gh}{1 + I_{CM}/MR^2}}$$

Example for Rolling Kinetic Energy

For solid sphere as shown in the figure, calculate the linear speed of the CM at the bottom of the hill and the magnitude of linear acceleration of the CM. Solve this problem using Newton's second law, the dynamic method.



What are the forces involved in this motion?

Gravitational Force, Frictional Force, Normal Force

Newton's second law applied to the CM gives

$$\sum F_x = Mg\sin\theta - f = Ma_{CM}$$

$$\sum F_y = n - Mg\cos\theta = 0$$

Since the forces $\mathcal{M}g$ and **n** go through the CM, their moment arm is 0 and do not contribute to torque, while the static friction f causes torque

$$\tau_{CM} = fR = I_{CM}\alpha$$

We know that

$$I_{CM} = \frac{2}{5}MR^2$$

$$a_{CM} = R\alpha$$

We obtain

$$f = \frac{I_{CM}\alpha}{R} = \frac{\frac{2}{5}MR^2}{R}\left(\frac{a_{CM}}{R}\right) = \frac{2}{5}Ma_{CM}$$

Substituting f in dynamic equations

$$Mg\sin\theta = \frac{7}{5}Ma_{CM} \qquad a_{CM} = \frac{5}{7}g\sin\theta$$