PHYS 1441 – Section 002
Lecture #23

Monday, April 29, 2013
Dr. Jaehoon Yu

- Conditions for Equilibrium
- Elastic Properties of Solids
  - Young's Modulus
  - Bulk Modulus
- Density and Specific Gravity
- Fluid and Pressure

Today's homework is NONE!!

Announcements

- Final comprehensive exam
  - Date and time: 2:00 – 4:30pm, Wednesday, May 8
  - Coverage: CH1.1 through what we finish this Wednesday, May 1, plus appendices
  - Please hit homeruns on this exam!!!
  - I will prepare a formula sheet for you this time!
- Planetarium extra credit
  - Deadline next Wednesday, May, 8
- Student Feedback Survey
  - Must be done by May 3
- No class next week!!

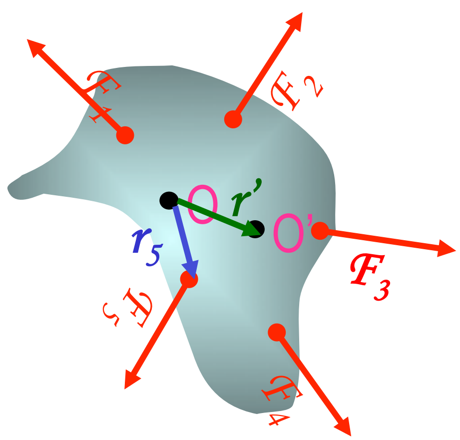
More on Conditions for Equilibrium

To simplify the problem, we will only deal with forces acting on x-y plane, giving torque only along z-axis. What do you think the conditions for equilibrium be in this case?

The six possible equations from the two vector equations turns to three equations.

$$\sum \vec{F} = 0 \Rightarrow \sum F_x = 0, \quad \sum F_y = 0 \quad \textbf{AND} \quad \sum \vec{\tau} = 0 \Rightarrow \sum \tau_z = 0$$

What happens if there are many forces exerting on an object?



If an object is at its translational static equilibrium, and if the net torque acting on the object is 0 about one axis, the net torque must be 0 about any arbitrary axis.

Why is this true?

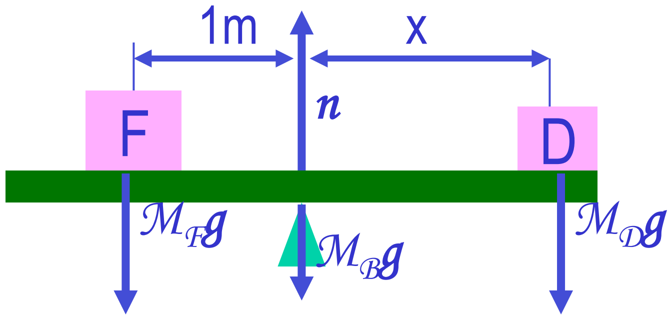
Because the object is ***not moving***, no matter what the rotational axis is, there should not be any motion. It is simply a matter of mathematical manipulation.

How do we solve static equilibrium problems?

1. Select the object to which the equations for equilibrium are to be applied.
2. Identify all the forces and draw a free-body diagram with them indicated on it with their directions and locations properly indicated
3. Choose a convenient set of x and y axes and write down the force equation for each x and y component with correct signs.
4. Apply the equations that specify the balance of forces at equilibrium. Set the net force in the x and y directions equal to 0.
5. Select the most optimal rotational axis for torque calculations ➔ Selecting the axis such that the torque of one or more of the unknown forces become 0 makes the problem much easier to solve.
6. Write down the torque equation with proper signs.
7. Solve the force and torque equations for the desired unknown quantities.

Example for Mechanical Equilibrium

A uniform 40.0 N board supports the father and the daughter each weighing 800 N and 350 N, respectively, and is not moving. If the support (or fulcrum) is under the center of gravity of the board, and the father is 1.00 m from the center of gravity (CoG), what is the magnitude of the normal force n exerted on the board by the support?



Since there is no linear motion, this system is in its translational equilibrium

$$\sum F_x = 0$$

$$\sum F_y = n - M_B g - M_F g - M_D g = 0$$

Therefore the magnitude of the normal force $n = 40.0 + 800 + 350 = 1190N$

Determine where the child should sit to balance the system.

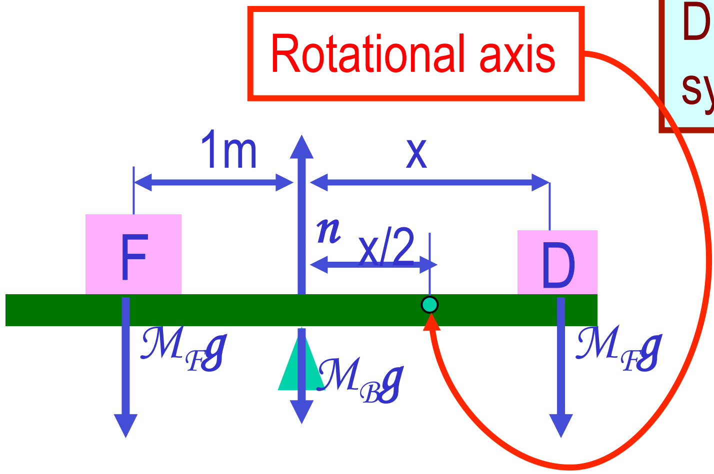
The net torque about the fulcrum by the three forces are

$$\tau = M_B g \cdot 0 + n \cdot 0 + M_F g \cdot 1.00 - M_D g \cdot x = 0$$

Therefore to balance the system the daughter must sit

$$x = \frac{M_F g}{M_D g} \cdot 1.00m = \frac{800}{350} \cdot 1.00m = 2.29m$$

Example for Mech. Equilibrium Cont'd



Determine the position of the child to balance the system for different position of axis of rotation.

The net torque about the axis of rotation by all the forces are

$$\tau = M_B g \cdot x/2 + M_F g \cdot (1.00 + x/2) - n \cdot x/2 - M_D g \cdot x/2 = 0$$

Since the normal force is $n = M_B g + M_F g + M_D g$

The net torque can be rewritten

$$\begin{aligned}\tau &= M_B g \cdot x/2 + M_F g \cdot (1.00 + x/2) \\ &\quad - (M_B g + M_F g + M_D g) \cdot x/2 - M_D g \cdot x/2 \\ &= M_F g \cdot 1.00 - M_D g \cdot x = 0\end{aligned}$$

Therefore

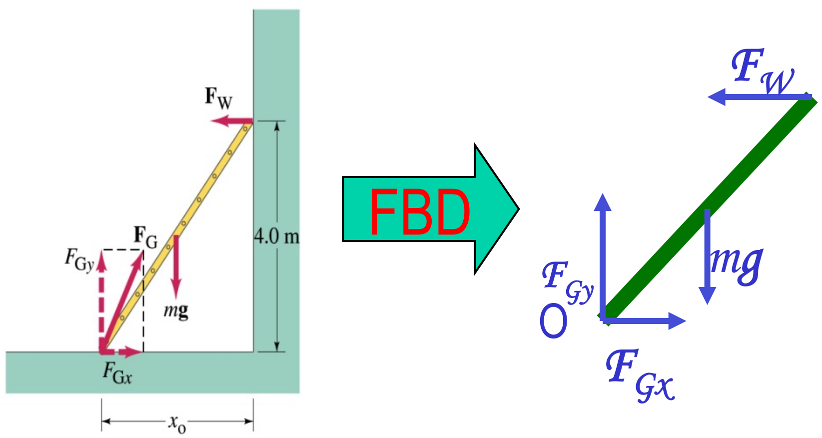
$$x = \frac{M_F g}{M_D g} \cdot 1.00m = \frac{800}{350} \cdot 1.00m = 2.29m$$

What do we learn?

No matter where the rotation axis is, net effect of the torque is identical.

Example 9 – 7

A 5.0 m long ladder leans against a wall at a point 4.0m above the ground. The ladder is uniform and has mass 12.0kg. Assuming the wall is frictionless (but ground is not), determine the forces exerted on the ladder by the ground and the wall.



First the translational equilibrium, using components

$$\sum F_x = F_{Gx} - F_W = 0$$

$$\sum F_y = -mg + F_{Gy} = 0$$

Thus, the y component of the force by the ground is

$$F_{Gy} = mg = 12.0 \times 9.8N = 118N$$

The length x_0 is, from Pythagorian theorem

$$x_0 = \sqrt{5.0^2 - 4.0^2} = 3.0m$$

Example 9 – 7 cont'd

From the rotational equilibrium $\sum \tau_O = -mg\,x_0/2 + F_W\,4.0 = 0$

Thus the force exerted on the ladder by the wall is

$$F_W = \frac{mg\,x_0/2}{4.0} = \frac{118 \cdot 1.5}{4.0} = 44N$$

The x component of the force by the ground is

$$\sum F_x = F_{Gx} - F_W = 0$$

Solve for F_{Gx}

$$F_{Gx} = F_W = 44N$$

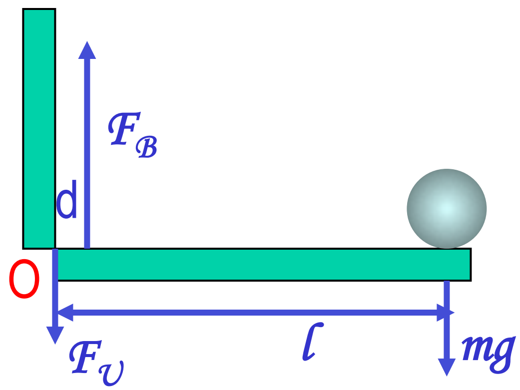
Thus the force exerted on the ladder by the ground is

$$F_G = \sqrt{F_{Gx}^2 + F_{Gy}^2} = \sqrt{44^2 + 118^2} \approx 130N$$

The angle between the ground force to the floor $\theta = \tan^{-1}\left(\frac{F_{Gy}}{F_{Gx}}\right) = \tan^{-1}\left(\frac{118}{44}\right) = 70^{o}$

Ex. 9.8 for Mechanical Equilibrium

A person holds a 50.0N sphere in his hand. The forearm is horizontal. The biceps muscle is attached 3.00 cm from the joint, and the sphere is 35.0cm from the joint. Find the upward force exerted by the biceps on the forearm and the downward force exerted by the upper arm on the forearm and acting at the joint. Neglect the weight of forearm.



Since the system is in equilibrium, from the translational equilibrium condition

$$\sum F_x = 0$$

$$\sum F_y = F_B - F_U - mg = 0$$

From the rotational equilibrium condition $\sum \tau = F_U \cdot 0 + F_B \cdot d - mg \cdot l = 0$

Thus, the force exerted by the biceps muscle is

$$F_B \cdot d = mg \cdot l$$

$$F_B = \frac{mg \cdot l}{d} = \frac{50.0 \times 35.0}{3.00} = 583N$$

Force exerted by the upper arm is $F_U = F_B - mg = 583 - 50.0 = 533N$

Elastic Properties of Solids

We have been assuming that the objects do not change their shapes when external forces are exerting on it. It this realistic?

No. In reality, the objects get deformed as external forces act on it, though the internal forces resist the deformation as it takes place.

Deformation of solids can be understood in terms of Stress and Strain

Stress: A quantity proportional to the force causing the deformation.
Strain: Measure of the degree of deformation
Elastic Limit: Point of elongation under which an object returns to its original shape

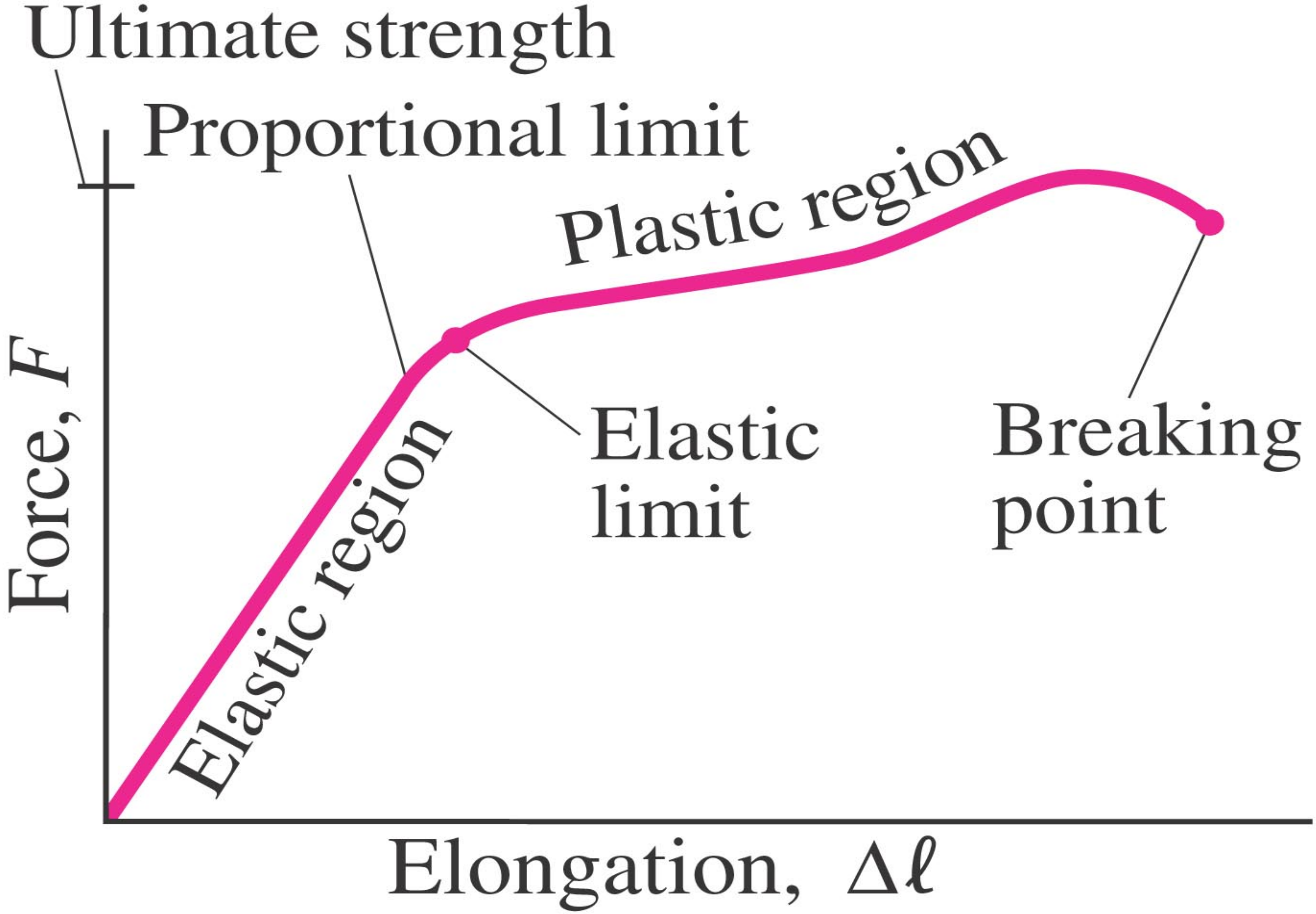
It is empirically known that for small stresses, strain is proportional to stress

The constants of proportionality are called Elastic Modulus $\text{Elastic Modulus} \equiv \dfrac{\text{stress}}{\text{strain}}$

Three types of Elastic Modulus

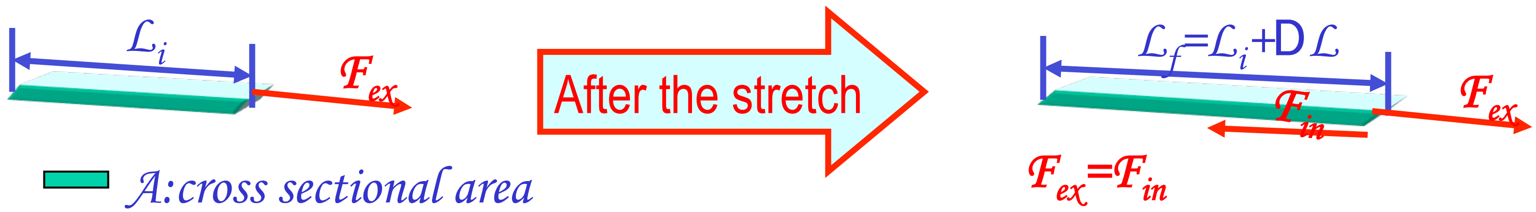
1. **Young's modulus**: Measure of the elasticity in a length
2. **Shear modulus**: Measure of the elasticity in an area
3. **Bulk modulus**: Measure of the elasticity in a volume

Applied force vs elongation



Young's Modulus

Let's consider a long bar with cross sectional area A and initial length L_i.



Tensile stress $\text{Tensile Stress} \equiv \frac{F_{ex}}{A}$ Tensile strain $\text{Tensile Strain} \equiv \frac{\Delta L}{L_i}$

Young's Modulus is defined as

$$Y \equiv \frac{\text{Tensile Stress}}{\text{Tensile Strain}} = \frac{F_{ex}/A}{\Delta L/L_i}$$

Used to characterize a rod or wire stressed under tension or compression

What is the unit of Young's Modulus?

Force per unit area

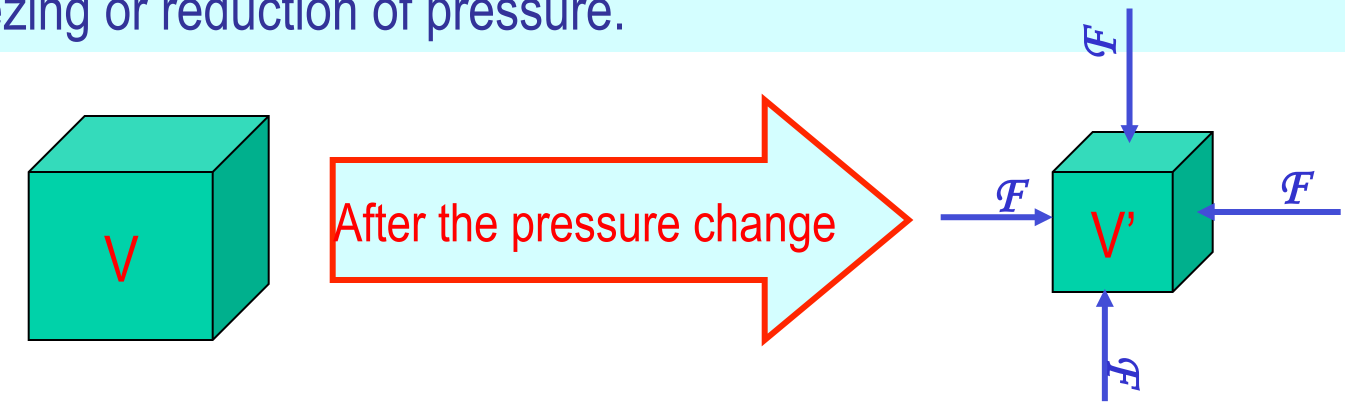
Experimental Observations

1. For a fixed external force, the change in length is proportional to the original length
2. The necessary force to produce the given strain is proportional to the cross sectional area

Elastic limit: Maximum stress that can be applied to the substance before it becomes permanently deformed

Bulk Modulus

Bulk Modulus characterizes the response of a substance to uniform squeezing or reduction of pressure.



Volume stress =pressure

$$\text{Pressure} \equiv \frac{\text{Normal Force}}{\text{Surface Area the force applies}} = \frac{F}{A}$$

If the pressure on an object changes by DP=DF/A, the object will undergo a volume change DV.

Bulk Modulus is defined as

$$B \equiv \frac{\text{Volume Stress}}{\text{Volume Strain}} = -\frac{\Delta F / A}{\Delta V / V_i} = -\frac{\Delta P}{\Delta V / V_i}$$

Because the change of volume is reverse to change of pressure.

Compressibility is the reciprocal of Bulk Modulus

Example for Solid's Elastic Property

A solid brass sphere is initially under normal atmospheric pressure of $1.0\text{x}10^5\text{N/m}^2$. The sphere is lowered into the ocean to a depth at which the pressures is $2.0\text{x}10^7\text{N/m}^2$. The volume of the sphere in air is 0.5m^3. By how much its volume change once the sphere is submerged?

Since bulk modulus is $$\mathrm{B} = -\frac{\Delta P}{\Delta V / V_i}$$

The amount of volume change is $$\Delta V = -\frac{\Delta P V_i}{\mathrm{B}}$$

From table 12.1, bulk modulus of brass is $6.1\text{x}10^{10}$ N/m^2

The pressure change DP is $$\Delta P = P_f - P_i = 2.0\times10^7 - 1.0\times10^5 \approx 2.0\times10^7$$

Therefore the resulting volume change DV is $$\Delta V = V_f - V_i = -\frac{2.0\times10^7\times0.5}{6.1\times10^{10}} = -1.6\times10^{-4}\,m^3$$

The volume has decreased.

Density and Specific Gravity

Density, ρ(rho), of an object is defined as mass per unit volume

$$\rho \equiv \frac{M}{V}$$

Unit? kg/m^3

Dimension? $[ML^{-3}]$

Specific Gravity of a substance is defined as the ratio of the density of the substance to that of water at 4.0 °C (ρ_{H2O}=1.00g/cm^3).

$$SG \equiv \frac{\rho_{\text{substance}}}{\rho_{H_2O}}$$

Unit? None

Dimension? None

What do you think would happen of a substance in the water dependent on SG?

$SG > 1$ Sink in the water

$SG < 1$ Float on the surface