PHYS 3313 – Section 001
Lecture #4

Monday, Jan. 27, 2014
Dr. Jaehoon Yu

- Galilean Transformation

- Do we need Ether?

- Michelson-Morley Experiment

- Einstein's postulates

- Lorentz Transformations

- Time Dilation & Length Contraction

Announcements

- Quiz results
  - Class average: 19.4/65
    - Equivalent to 30/65
  - Top score: 65/65

- Reminder for homework #1
  - Chapter 2 end of the chapter problems
  - 17, 21, 23, 24, 32, 59, 61, 66, 68, 81 and 96
  - Due is by the beginning of the class, Monday, Feb. 3
  - Work in study groups together with other students but PLEASE do write your answer in your own way!

The Transition to Modern Relativity

- Although Newton's laws of motion had the same form under the Galilean transformation, Maxwell's equations did not.

- In 1905, Albert Einstein proposed a fundamental connection between space and time and that Newton's laws are only an approximation.

They Needed Ether!!

- The wave nature of light suggested that there existed a propagation medium called the *luminiferous ether* or just **ether**.

  – Provides an inertial reference frame

- The properties of ether

  – Very low density for planets to move through it without loss of energy

  – Sufficiently high elasticity to support the high velocity of light waves (c=?)

Ether as the Absolute Reference System

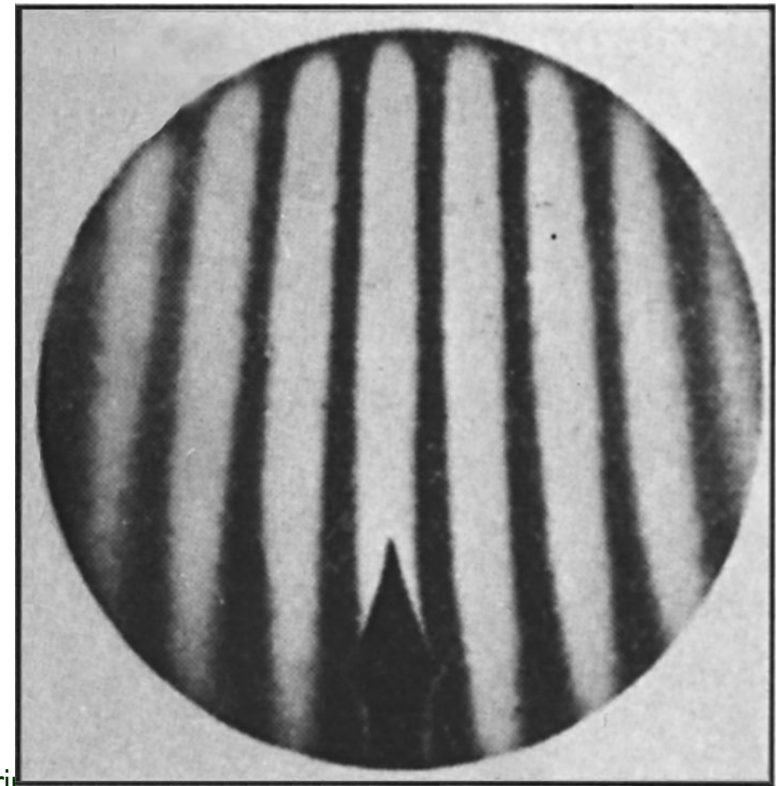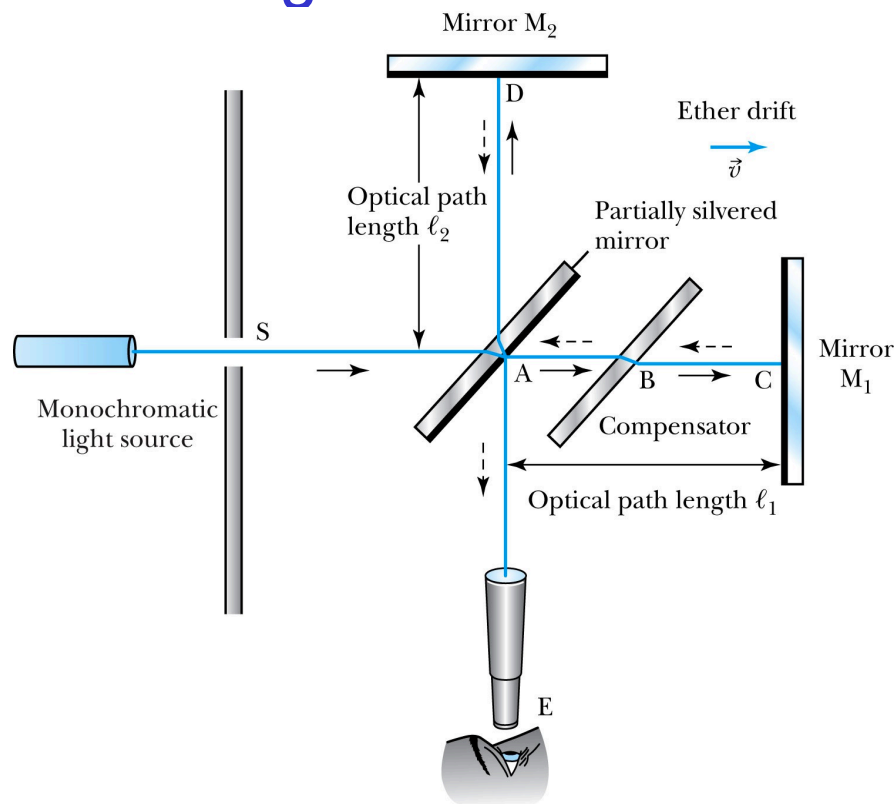- In Maxwell's theory, the speed of light is given by

$$v = c = 1 \Big/ \sqrt{\varepsilon_0 \mu_0}$$

  - The velocity of light between moving systems must be a constant.
    - Can you see why?
  - Needed a system of medium that keeps this constant!

- Ether proposed as the absolute reference system in which the speed of light is constant and from which other measurements could be made.

- The Michelson-Morley experiment was an attempt to show the existence of ether.

The Michelson-Morley Experiment

- Albert Michelson (1852–1931) built an extremely precise device called the *interferometer* to measure the phase difference between two light waves traveling in orthogonal directions.

Mirror M$_2$

D

Ether drift

$\vec{v}$

Optical path length ℓ_2

Partially silvered mirror

S

A

B

C

Mirror M$_1$

Monochromatic light source

Compensator

Optical path length ℓ_1

E

How does Michelson Interferometer work?

1. AC is parallel to the motion of the Earth inducing an "ether wind"
2. Light from source S is split by mirror A and travels to mirrors C and D in mutually perpendicular directions
3. After reflection the beams recombine at A slightly out of phase due to the "ether wind" as viewed by telescope E.
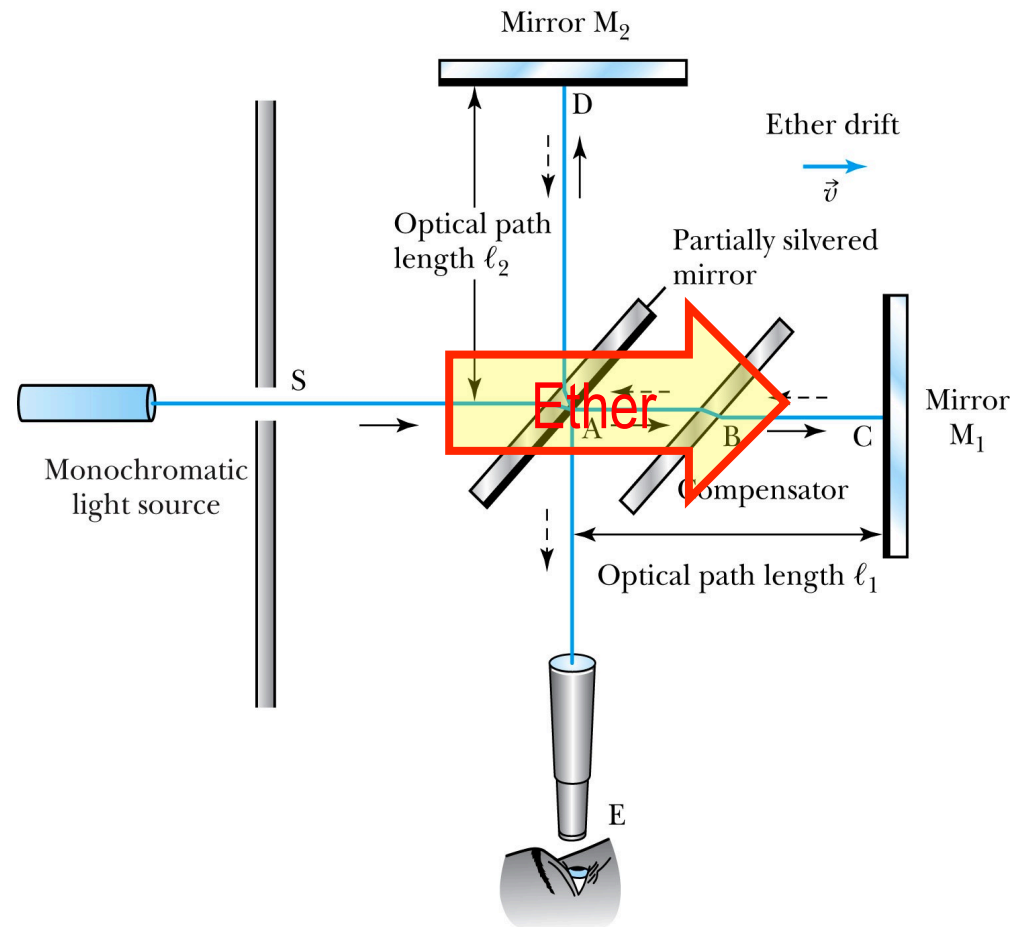
The analysis – Galilean X-formation

- Travel time t_1 for a round trip over AC (the ether direction) is

$$t_1 = \frac{l_1}{c+v} + \frac{l_1}{c-v} = \frac{2l_1 c}{c^2 - v^2} = \frac{2l_1}{c} \frac{1}{1 - v^2/c^2}$$

- Travel time t_2 for a round trip over AD (perpendicular direction to ether) is

$$t_2 = \frac{2l_2}{\sqrt{c^2 - v^2}} = \frac{2l_2}{c} \frac{1}{\sqrt{1 - v^2/c^2}}$$

- The time difference is

$$\Delta t = t_2 - t_1 = \frac{2}{c}\left(\frac{l_2}{\sqrt{1 - v^2/c^2}} - \frac{l_1}{1 - v^2/c^2} \right)$$

The analysis

- After rotating the machine by 90°, the time difference becomes

$$\Delta t' = t_2' - t_1' = \frac{2}{c}\left(\frac{l_2}{1 - v^2/c^2} - \frac{l_1}{\sqrt{1 - v^2/c^2}} \right)$$

- The difference of the time differences

$$\Delta t' - \Delta t = \frac{2}{c}\left(\frac{l_1 + l_2}{1 - v^2/c^2} - \frac{l_1 + l_2}{\sqrt{1 - v^2/c^2}} \right) = \frac{2}{c}(l_1 + l_2)\left(\frac{1}{1 - v^2/c^2} - \frac{1}{\sqrt{1 - v^2/c^2}} \right)$$

- Since v (the Earth's speed) is 10^{-4} of c, we can do binomial expansion of the above

$$\Delta t' - \Delta t = \frac{2}{c}(l_1 + l_2)\left[\left(1 + \frac{v^2}{c^2} + \cdots \right) - \left(1 + \frac{v^2}{2c^2} + \cdots \right) \right] \approx \frac{v^2}{c^3}(l_1 + l_2)$$

The Results

- Using the Earth's orbital speed as:

$$V = 3 \times 10^4 \text{ m/s}$$

together with

$$\ell_1 \approx \ell_2 = 1.2 \text{ m}$$

So that the time difference becomes

$$\Delta t' - \Delta t \approx v^2(\ell_1 + \ell_2)/c^3 = 8 \times 10^{-17} \text{ s}$$

- Although a very small number, it was within the experimental range of measurement for light waves.

- Later with Morley, they increased the path lengths to 11m and improved precision better than a factor of 10

- Yet, Michelson FAILED to "see" the expected interference pattern

Conclusions of Michelson Experiment

- Michelson noted that he should be able to detect a phase shift of light due to the time difference between path lengths but found none.

- He thus concluded that the hypothesis of the stationary ether must be incorrect.

- After several repeats and refinements with assistance from Edward Morley (1893-1923), again *a null result.*

- ***Thus, ether does not seem to exist!***

- Many explanations ensued afterward but none worked out!

- This experiment shattered the popular belief of light being waves

The Lorentz-FitzGerald Contraction

- Another hypothesis proposed independently by both H. A. Lorentz and G. F. FitzGerald suggested that the length ℓ_1, in the direction of the motion was *contracted* by a factor of

$$\sqrt{1 - v^2/c^2}$$

- Thus making the path lengths equal to account for the zero phase shift.

  - This, however, was an ad hoc assumption that could not be experimentally tested.

Einstein's Postulates

- Fundamental assumption: Maxwell's equations must be valid in all inertial frames

- **The principle of relativity**: The laws of physics are the same in all inertial systems. There is no way to detect absolute motion, and no preferred inertial system exists

    – Published a paper in 1905 at the age 26

    – Believed to be fundamental

- **The constancy of the speed of light**: Observers in all inertial systems measure the same value for the speed of light in a vacuum.

The Lorentz Transformations

General linear transformation relationship between P=(x, y, z, t) in frame S and P'=(x',y',z',t') in frame S' ➜ these assume measurements are made in S frame and transferred to S' frame

- preserve the constancy of the speed of light between inertial observers

- account for the problem of simultaneity between these observers

$$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}} \quad y' = y \quad z' = z \quad t' = \frac{t - \left(vx/c^2\right)}{\sqrt{1 - v^2/c^2}}$$

- With the definitions $\beta \equiv v/c$ and $\gamma \equiv 1/\sqrt{1 - \beta^2}$

$$x' = \gamma\left(x - \beta ct\right) \quad y' = y \quad z' = z \quad t' = \gamma\left(1 - \beta x/c\right)$$
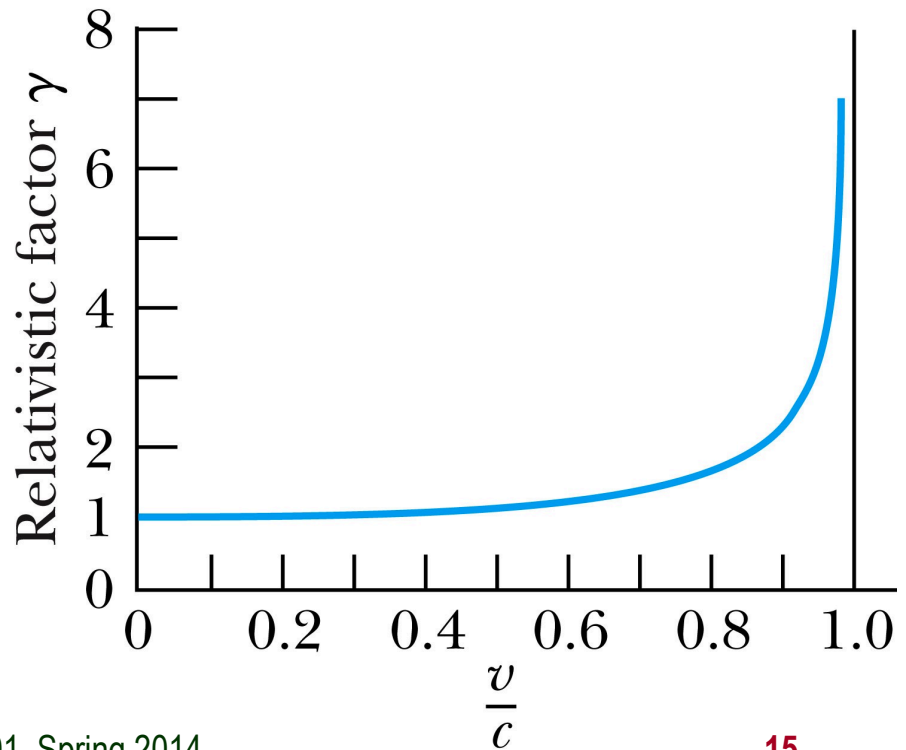
Properties of the Relativistic Factor γ

What is the property of the relativistic factor, γ?

Is it bigger or smaller than 1?

Recall Einstein's postulate, $\beta = v/c < 1$ for all observers

- $\gamma = 1$ only when $v = 0$

$$\gamma = 1 \Big/ \sqrt{1 - \beta^2} \geq 1$$

The complete Lorentz Transformations

$$x' = \frac{x - vt}{\sqrt{1-\beta^2}}$$

$$x = \frac{x' + vt'}{\sqrt{1-\beta^2}}$$

$$y' = y$$

$$y = y'$$

$$z' = z$$

$$z = z'$$

$$t' = \frac{t - \left(vx/c^2\right)}{\sqrt{1-\beta^2}}$$

$$t = \frac{t' + \left(vx'/c^2\right)}{\sqrt{1-\beta^2}}$$

- Some things to note
  - What happens when $\beta \sim 0$ (or $v \sim 0$)?
    - The Lorentz x-formation becomes Galilean x-formation
  - Space-time are not separated
  - For non-imaginary x-formations, the frame speed cannot exceed c!

Time Dilation and Length Contraction

Direct consequences of the Lorentz Transformation:

- **Time Dilation**:

  Clocks in a moving inertial reference frame K' run slower with respect to stationary clocks in K.
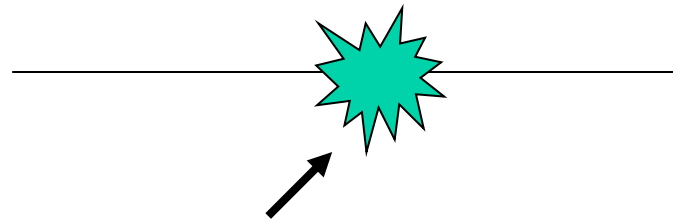
- **Length Contraction**:

  Lengths measured in a moving inertial reference frame K' are shorter with respect to the same lengths stationary in K.

Time Dilation

To understand *time dilation* the idea of **proper time** must be understood:

- **proper time**, T_0, is the time difference between two events occurring at the same position in a system as measured by a clock at that position.

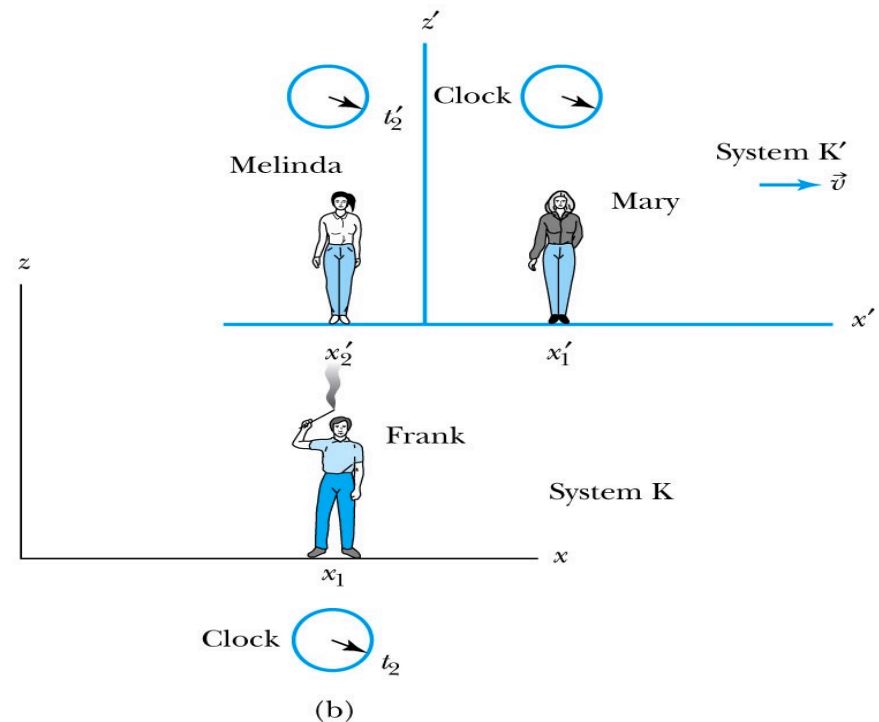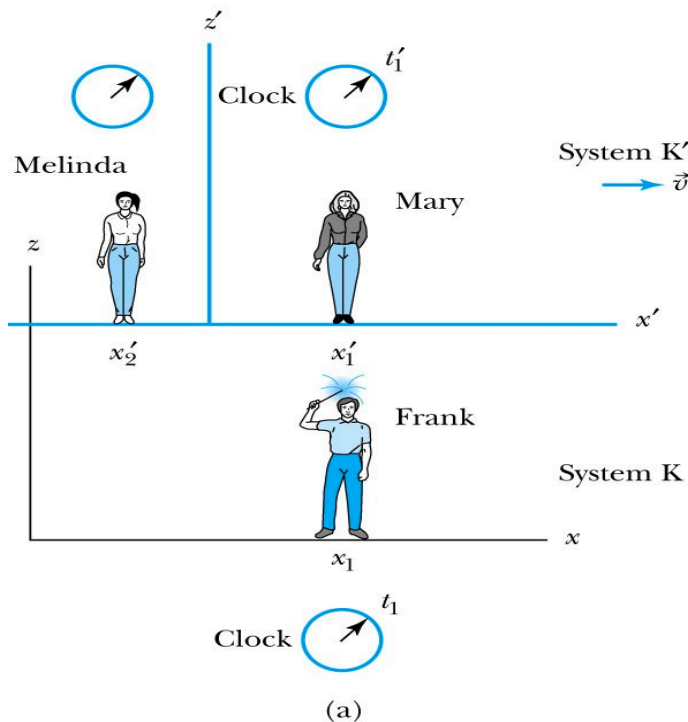Same location (spark "on" then off")

Time Dilation

Is this a Proper Time?

spark "on"   then spark "off"

Beginning and ending of the event occur at different positions

Time Dilation with Mary, Frank, and Melinda

Frank's clock is at the same position in system K when the sparkler is lit in (a) (t=t_1) and when it goes out in (b) (t=t_2). ➔ The proper time $T_0=t_2-t_1$

Mary, in the moving system K', is beside the sparkler when it was lit (t=t_1')

Melinda then moves into the position where and when the sparkler extinguishes (t=t_2')

Thus, Melinda, at the new position, measures the time in system K' when the sparkler goes out in (b).

According to Mary and Melinda…

- Mary and Melinda measure the two times for the sparkler to be lit and to go out in system K' as times t_1' and t_2' so that by the Lorentz transformation:

$$t'_2 - t'_1 = \frac{(t_1 - t_2) - (v/c^2)(x_1 - x_2)}{\sqrt{1 - \beta^2}}$$

  – Note here that Frank records $x_2 - x_1 = 0$ in K with a proper time: $T_0 = t_2 - t_1$ or

$$T' = t'_2 - t'_1 = \frac{T}{\sqrt{1 - \beta^2}} = \gamma T$$

Time Dilation: Moving Clocks Run Slow

1) $T' > T_0$ or the time measured between two events at *different positions* is greater than the time between the same events at *one position:* **time dilation.**

   The proper time is always the shortest time!!

2) The events do not occur at the same space and time coordinates in the two systems

3) System K requires 1 clock and K' requires 2 clocks.

Time Dilation Example: muon lifetime

- Muons are essentially heavy electrons (~200 times heavier)
- Muons are typically generated in collisions of cosmic rays in upper atmosphere and, unlike electrons, decay ($t_0 = 2.2$ μ sec)
- For a muon incident on Earth with v=0.998c, an observer on Earth would see what lifetime of the muon?
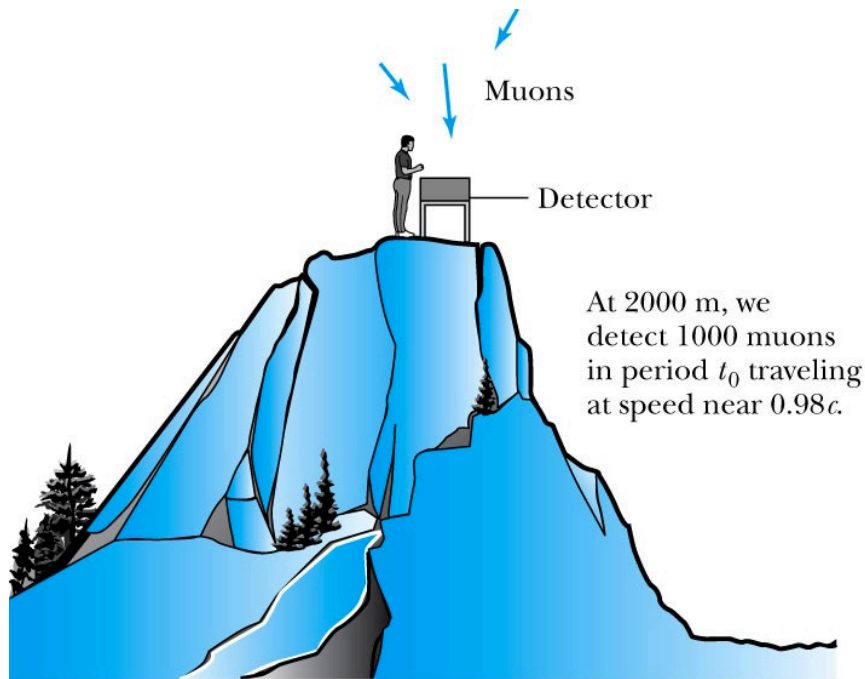- 2.2 μ sec?

$$\gamma = \frac{1}{\sqrt{1 - \dfrac{v^2}{c^2}}} \approx 16$$
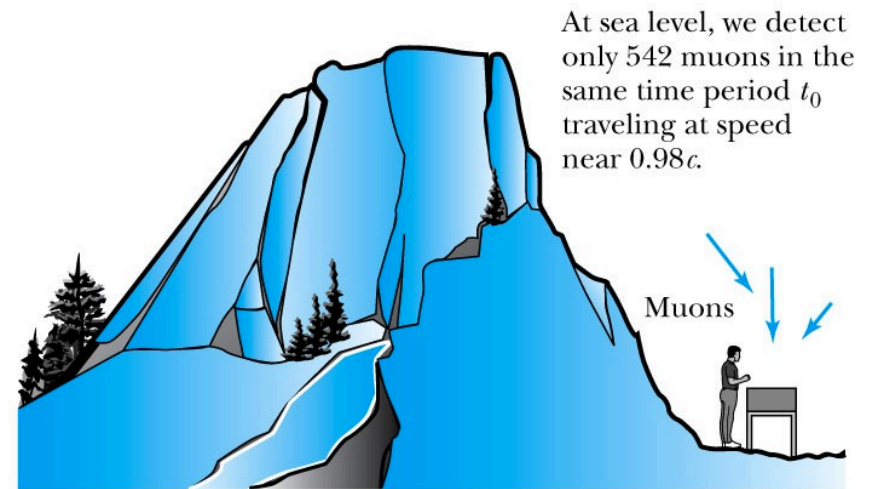
- t=35 μ sec
- Moving clocks run slow so when an outside observer measures, they see a longer time than the muon itself sees.

Experimental Verification of Time Dilation
Arrival of Muons on the Earth's Surface

Muons

Detector

At 2000 m, we detect 1000 muons in period t_0 traveling at speed near 0.98c.

At sea level, we detect only 542 muons in the same time period t_0 traveling at speed near 0.98c.

Muons

(a)

(b)

The number of muons detected with speeds near 0.98c is much different (a) on top of a mountain than

(b) at sea level, because of the muon's decay. The experimental result agrees with our time dilation equation.

Length Contraction

To understand *length contraction* the idea of **proper length** must be understood:

- Let an observer in each system K and K' have a meter stick at rest in ***their own system*** such that each measures the same length at rest.

- The length as measured at rest at the same time is called the **proper length**.

Length Contraction cont'd

Each observer lays the stick down along his or her respective x axis, putting the left end at x_ℓ (or x'_ℓ) and the right end at x_r (or x'_r).

- Thus, in the rest frame K, Frank measures his stick to be:

$$L_0 = x_r - x_l$$

- Similarly, in the moving frame K', Mary measures her stick at rest to be:

$$L'_0 = x'_r - x'_l$$

• Frank in his rest frame measures the moving length in Mary's frame moving with velocity v.

• Thus using the Lorentz transformations Frank measures the length of the stick in K' as:

$$x'_r - x'_l = \frac{(x_r - x_l) - v(t_r - t_l)}{\sqrt{1 - \beta^2}}$$

Where both ends of the stick must be measured simultaneously, i.e, $t_r = t_\ell$

Here Mary's proper length is $L'_0 = x'_r - x'_\ell$

and Frank's measured length is $L = x_r - x_\ell$

Measurement in Rest Frame

The observer in the rest frame measures the moving length as L given by

$$L_0' = \frac{L}{\sqrt{1 - \beta^2}} = \gamma L$$

but since both Mary and Frank in their respective frames measure $L'_0 = L_0$

$$L = L_0 \sqrt{1 - \beta^2} = \frac{L_0}{\gamma}$$

and $L_0 > L$, i.e. the moving stick shrinks