PHYS 3313 – Section 001
Lecture #5

Wednesday, Jan. 29, 2014
Dr. Jaehoon Yu

- Length Contraction
- Relativistic Velocity Addition
- The Twin Paradox
- Space-time Diagram
- The Doppler Effect

Announcements

- Reminder for homework #1
  - Chapter 2 end of the chapter problems
  - 17, 21, 23, 24, 32, 59, 61, 66, 68, 81 and 96
  - Due is by the beginning of the class, Monday, Feb. 3
  - Work in study groups together with other students but PLEASE do write your answer in your own way!
- Colloquium at 4pm today in SH101
  - Dr. James LeBeau of NCSU

Physics Department
The University of Texas at Arlington
COLLOQUIUM

New Frontiers of Atomic scale characterization in the Electron Microscope

Dr. James M. LeBeau

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4:00 pm Wednesday January 29, 2014 room 101 SH

Abstract:

Within the past decade, electron microscopy has been revolutionized by the advent of the aberration corrector. Aberration correction dramatically improves spatial resolution into the sub-ångstrom regime, unlocking information about material defects. For example, when aberration correction is combined with state-of-the-art energy dispersive X-ray spectrometers, atomic resolution *chemical spectroscopy* becomes possible in the scanning transmission electron microscope (STEM). While these recent advances have proven essential to the atomic scale characterization of materials, measurement of atomic displacements and distances from STEM images is still hampered by the presence of sample drift. This limitation has obscured the capabilities to characterize minute changes to the atomic structure that ultimately control properties.

In this talk, I will introduce revolving scanning transmission electron microscopy (RevSTEM). The method uses a series of fast-acquisition STEM images, but with the scan coordinates rotated between successive frames. This scan rotation introduces a concomitant change in image distortion that we use to analyze the sample drift rate and direction. I will provide a theoretical basis for the approach and introduce the projective standard deviation (PSD) to quantify lattice vector angles in atomic resolution images. Multiple case studies will be presented to demonstrate the power of this new technique. In all cases, I will show that RevSTEM achieves near perfect image restoration without any prior knowledge about the sample structure. These results open a new world of atomic scale exploration that was previously just beyond our reach.

Beyond technique development, I will also show that aberration corrected STEM techniques have become indispensable for addressing a wide variety of materials science questions, and will explore the synergistic combination of atomic resolution STEM imaging and X-ray spectroscopy to study interfaces, defects, and phase evolution. As a prototypical example, I will discuss our recent study of the thin film topological insulator bismuth telluride, Bi_2Te_3, grown epitaxially on a GaAs substrate. Critical information from state-of-the-art microscopy provides a direct observation of the mechanisms responsible for van der Waals epitaxial growth.

Refreshments will be served at 3:30p.m in the Physics Lounge

Special Project #2

1. Derive the three Lorentz velocity transformation equations, starting from Lorentz coordinate transformation equations. (10 points)
2. Derive the three reverse Lorentz velocity transformation equations, starting from Lorentz coordinate transformation equations. (10 points)
3. Prove that the space-time invariant quantity $s^2=x^2-(ct)^2$ is indeed invariant, i.e. $s^2=s'^2$, in Lorentz Transformation. (5 points)
4. You must derive each one separately starting from the Lorentz spatial coordinate transformation equations to obtain any credit.
   - Just switching the signs and primes will be insufficient!
   - Must take the simplest form of the equations, using β and γ.
5. You MUST have your own, independent answers to the above three questions even if you worked together with others. All those who share the answers will get 0 credit if copied.

- Due for the submission is Wednesday, Feb. 5!

Length Contraction

- To understand *length contraction* the idea of **proper length** must be understood:
- Let an observer in each system K and K' have a meter stick at rest in ***their own system*** such that each measures the same length at rest.
- The length as measured at rest at the same time is called the **proper length**.

The Complete Lorentz Transformations

$$x' = \frac{x - vt}{\sqrt{1-\beta^2}}$$

$$x = \frac{x' + vt'}{\sqrt{1-\beta^2}}$$

$$y' = y$$

$$y = y'$$

$$z' = z$$

$$z = z'$$

$$t' = \frac{t - \left(vx/c^2\right)}{\sqrt{1-\beta^2}}$$

$$t = \frac{t' + \left(vx'/c^2\right)}{\sqrt{1-\beta^2}}$$

Length Contraction cont'd

Each observer lays the stick down along his or her respective *x* axis, putting the left end at x_ℓ (or x'_ℓ) and the right end at x_r (or x'_r).

- Thus, in the rest frame K, Frank measures his stick to be:

$$L_0 = x_r - x_l$$

- Similarly, in the moving frame K', Mary measures her stick at rest to be:

$$L'_0 = x'_r - x'_l$$

- Frank in his rest frame measures the moving length in Mary's frame moving with velocity v.
- Thus using the Lorentz transformations Frank measures the length of the stick in K' as:

$$x'_r - x'_l = \frac{(x_r - x_l) - v(t_r - t_l)}{\sqrt{1-\beta^2}}$$

Where both ends of the stick must be measured simultaneously, i.e, $t_r = t_\ell$

Here Mary's proper length is $L'_0 = x'_r - x'_\ell$

and Frank's measured length of Mary's stick is $L = x_r - x_\ell$

Measurement in Rest Frame

The observer in the rest frame measures the moving length as L given by

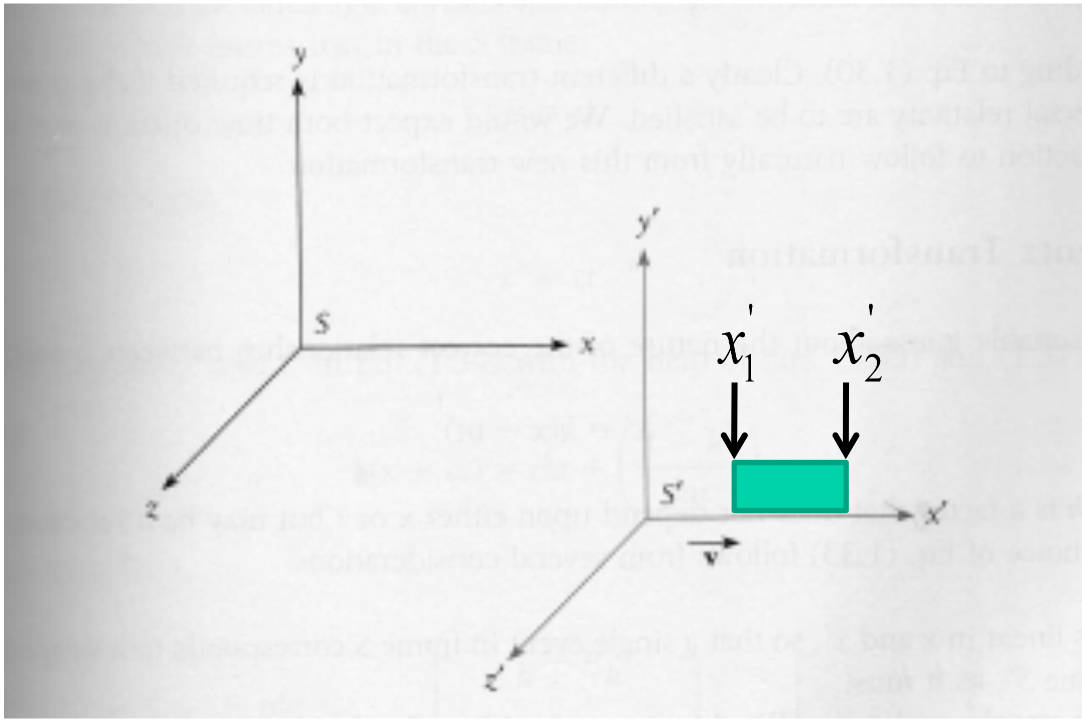
$$L'_0 = \frac{L}{\sqrt{1-\beta^2}} = \gamma L$$

but since both Mary and Frank in their respective frames measure $L'_0 = L_0$

$$L = L_0\sqrt{1-\beta^2} = \frac{L_0}{\gamma}$$

and $L_0 > L$, i.e. the moving stick shrinks

Length Contraction Summary



- Proper length (length of object in its own frame:

$$L_0 = x_2' - x_1'$$

- Length of object in observer's frame:

$$L = x_2 - x_1$$

$$L_0' = L_0 = x_2' - x_1' = \gamma(x_2 - vt) - \gamma(x_1 - vt) = \gamma(x_2 - x_1)$$

$L_0 = \gamma L \qquad L = L_0 / \gamma$

Since γ>1, the length is shorter in the direction of motion (length contraction!)

More about Muons

- Rate: 1/cm^2/minute at Earth's surface (so for a person with 600 cm^2 surface area, the rate would be 600/60=10 muons/sec passing through the body!)
- They are typically produced in atmosphere about 6 km above surface of Earth and often have velocities that are a substantial fraction of speed of light, v=.998 c for example and life time 2.2 μ sec $vt_0 = 2.994 \times 10^8 \frac{m}{\sec} \cdot 2.2 \times 10^{-6} \sec = 0.66 km$
- How do they reach the Earth if they only go 660 m and not 6000 m?
- The time dilation stretches life time to t=35 μ sec not 2.2 μ sec, thus they can travel 16 times further, or about 10 km, implying they easily reach the ground
- But riding on a muon, the trip takes only 2.2 μ sec, so how do they reach the ground???
- Muon-rider sees the ground moving towards him, so the length he has to travel contracts and is only $L_0 / \gamma = 6/16 = 0.38 km$
- At 1000 km/sec, it would take 5 seconds to cross U.S., pretty fast, but does it give length contraction? $L = .999994 L_0$ {not much contraction}
  (for v=0.9c, the length is reduced by 44%)

Addition of Velocities

How do we add velocities in a relativistic case?

Taking differentials of the Lorentz transformation, relative velocities may be calculated:

$$dx = \gamma\left(dx' + v\,dt'\right)$$

$$dy = dy'$$

$$dz = dz'$$

$$dt = \gamma\left[dt' + \left(v/c^2\right)dx'\right]$$

So that…

defining velocities as: $u_x = dx/dt$, $u_y = dy/dt$, $u'_x = dx'/dt'$, etc. it can be shown that:

$$u_x = \frac{dx}{dt} = \frac{\gamma\left[dx' + v\,dt'\right]}{\gamma\left[dt' + \frac{v}{c^2}dx'\right]} = \frac{u'_x + v}{1 + \left(v/c^2\right)u'_x}$$

With similar relations for u_y and u_z:

$$u_y = \frac{dy}{dt} = \frac{u'_y}{\gamma\left[1 + \left(v/c^2\right)u'_x\right]} \qquad u_z = \frac{dz}{dt} = \frac{u'_z}{\gamma\left[1 + \left(v/c^2\right)u'_x\right]}$$

The Lorentz Velocity Transformations

In addition to the previous relations, the **Lorentz velocity transformations** for u'_x, u'_y, and u'_z can be obtained by switching primed and unprimed and changing v to $-v$:

$$u'_x = \frac{u_x - v}{1 - \left(v/c^2\right)u'_x}$$

$$u'_y = \frac{u_y}{\gamma\left[1 - \left(v/c^2\right)u'_x\right]}$$

$$u'_z = \frac{u_z}{\gamma\left[1 - \left(v/c^2\right)u'_x\right]}$$

Velocity Addition Summary

- Galilean Velocity addition $v_x = v_x' + v$ where $v_x = \frac{dx}{dt}$ and $v_x' = \frac{dx'}{dt'}$
- From inverse Lorentz transform $dx = \gamma(dx' + v dt')$ and $dt = \gamma(dt' + \frac{v}{c^2}dx')$
- So $$v_x = \frac{dx}{dt} = \frac{\gamma(dx' + v dt')}{\gamma(dt' + \frac{v}{c^2}dx')} \div \frac{dt'}{dt'} = \frac{\frac{dx'}{dt'} + v}{1 + \frac{v}{c^2}\frac{dx'}{dt'}} = \frac{v_x' + v}{1 + \frac{v v_x'}{c^2}}$$
- Thus $$\boxed{v_x = \frac{v_x' + v}{1 + \frac{v v_x'}{c^2}}}$$
- What would be the measured speed of light in S frame?
  - Since $v_x' = c$ we get $$v_x = \frac{c + v}{1 + \frac{v^2}{c^2}} = \frac{c^2(c+v)}{c(c+v)} = c$$

Observer in S frame measures c too! Strange but true!

Velocity Addition Example

- Yu Darvish is riding his bike at 0.8c relative to observer. He throws a ball at 0.7c in the direction of his motion. What speed does the observer see?

$$\boxed{v_x = \frac{v_x' + v}{1 + \frac{v v_x'}{c^2}}}$$

$$v_x = \frac{.7c + .8c}{1 + \frac{.7 \times .8c^2}{c^2}} = 0.962c$$

- What if he threw it just a bit harder?
- Doesn't help—asymptotically approach c, can't exceed (it's not just a postulate it's the law)

A test of Lorentz velocity addition: π^0 decay

- How can one test experimentally the correctness of the Lorentz velocity transformation vs Galilean one?
- In 1964, T. Alvager and company performed a measurements of the arrival time of two photons resulting from the decay of a π^0 in two detectors separated by 30m.
- Each photon has a speed of 0.99975c. What are the speed predicted by Galilean and Lorentz x-mation?
  - $v_G = c + 0.99975c = 1.99975c$
  - $v_L = \dfrac{c + 0.99975c}{1 + 0.99975c^2/c^2} = \quad \approx c$
- How much time does the photon take to arrive at the detector?