PHYS 3313 – Section 001
Lecture #7

Wednesday, Feb. 5, 2014
Dr. Jaehoon Yu

- Relativistic Momentum and Energy
- Relationship between relativistic quantities
- Quantization
- Discovery of the X-ray and the Electron
- Determination of Electron Charge

Announcements

- Reading assignments: CH 3.3 (special topic – the discovery of Helium) and CH3.7
- Colloquium today: Dr. Z. Musielak

Physics Department
The University of Texas at Arlington
COLLOQUIUM

The Three-Body Problem

Dr. Zdzislaw Musielak
Department of Physics
The University of Texas at Arlington

4:00 pm Wednesday February 5, 2014
Room 101 SH

Abstract:

The three-body problem, which describes three masses interacting through Newtonian gravity without any restrictions imposed on the initial positions and velocities of these masses, has attracted the attention of many scientists and mathematicians for more than 300 years. A broad review of the three-body problem in the context of both historical and modern developments will be presented, with a special emphasis given to its applications to newly discovered extra-solar planetary systems.

Refreshments will be served at 3:30p.m in the Physics Lounge

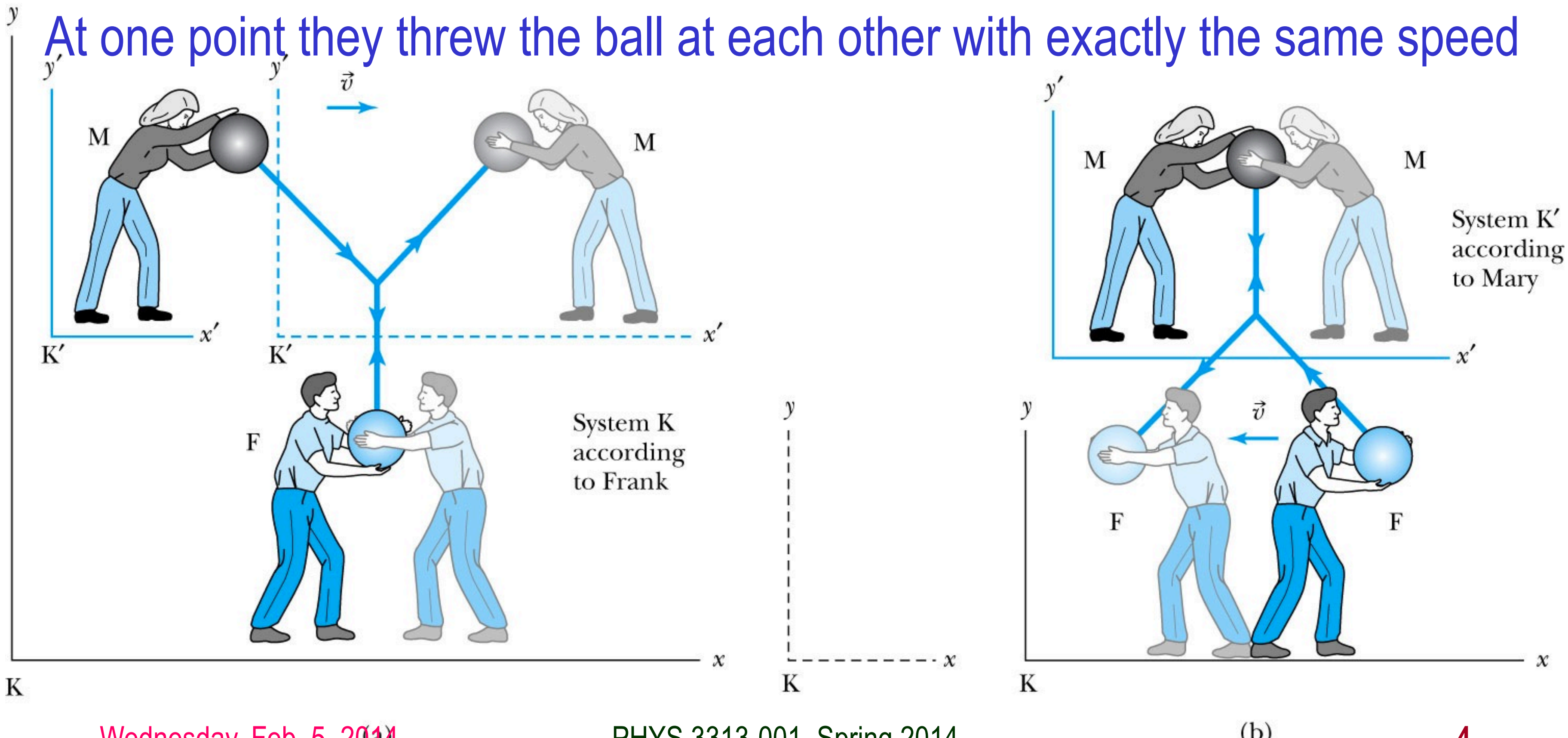
Relativistic Momentum

The most fundamental principle used here is the momentum conservation!

Frank is at rest in system K holding a ball of mass *m*.

Mary holds a similar ball in system K' that is moving in the *x* direction with velocity *v* with respect to system K.

At one point they threw the ball at each other with exactly the same speed



Relativistic Momentum

- If we use the definition of momentum, the momentum of the ball thrown by Frank is entirely in the *y* direction

$$p_{Fy} = mu_0$$

- The change of momentum as observed by Frank is

$$\Delta p_F = \Delta p_{Fy} = -2mu_0$$

- Mary measures the initial velocity of her own ball to be

$$u'_{Mx} = 0 \text{ and } u'_{My} = -u_0.$$

- In order to determine the velocity of Mary's ball as measured by Frank we use the velocity transformation equations:

$$u_{Mx} = v \qquad u_{My} = -u_0\sqrt{1 - v^2/c^2}$$

Relativistic Momentum

Before the collision, the momentum of Mary's ball as measured by Frank (in the **F**ixed frame) with the Lorentz velocity X-formation becomes

$$p_{Mx} = mv \qquad p_{My} = -mu_0\sqrt{1 - v^2/c^2}$$

For a perfectly elastic collision, the momentum after the collision is

$$p_{Mx} = mv \qquad p_{My} = +mu_0\sqrt{1 - v^2/c^2}$$

Thus the change in momentum of Mary's ball according to Frank is

$$\Delta p_M = \Delta p_{My} = 2mu_0\sqrt{1-\beta^2} \neq -\Delta p_{Fy}$$

OMG! The linear momentum is not conserved even w/o an external force!!

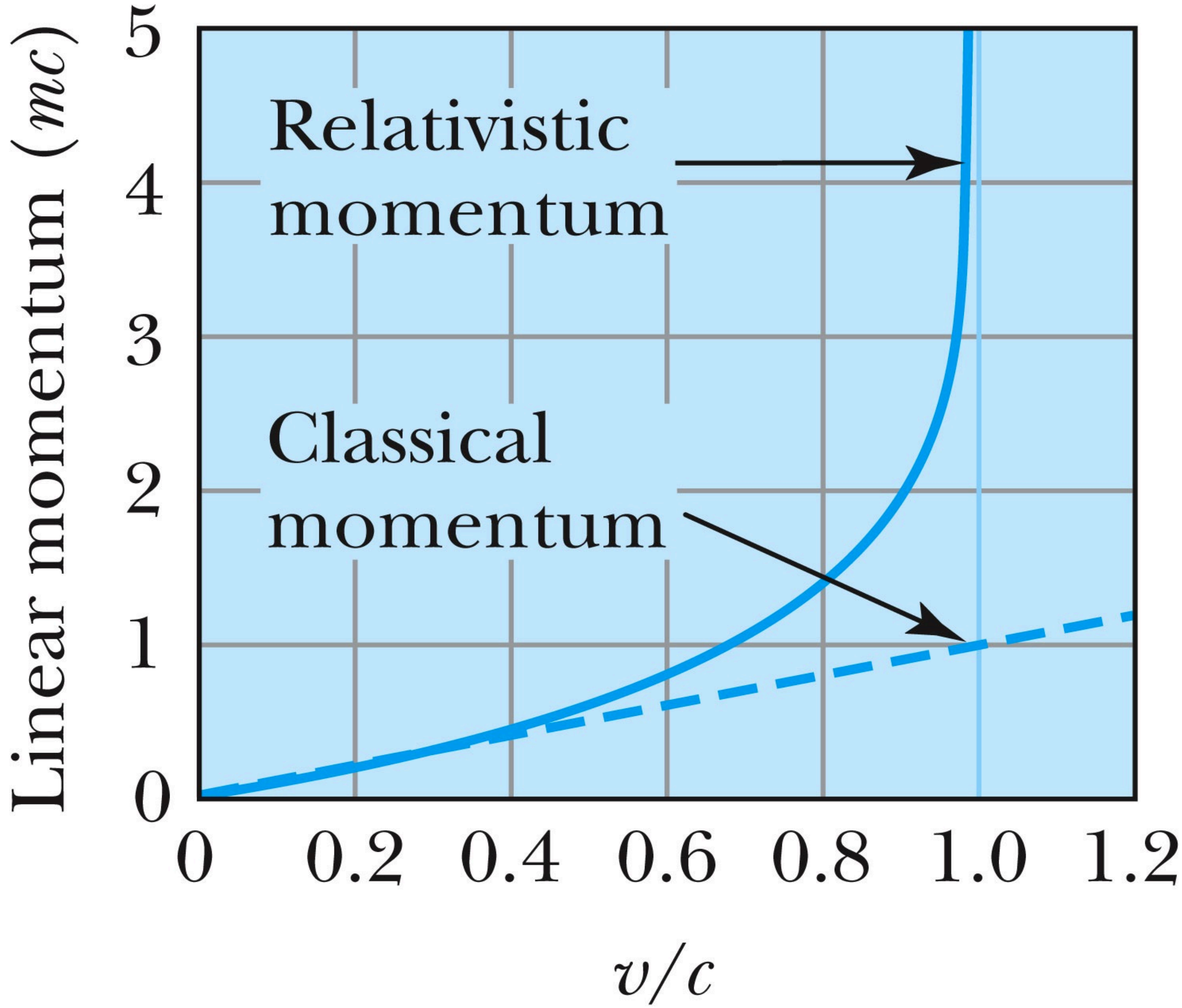
What do we do?

➔Redefine the momentum in a fashion $\vec{p} = m\frac{d\left(\gamma_u \vec{r}\right)}{dt} = m\gamma_u \vec{u}$

➔Something has changed. Mass is now, mγ!! The relativistic mass!!

➔Mass as the fundamental property of matter is called the "rest mass", m_0!

Relativistic and Classical Linear Momentum



How do we keep momentum conserved in a relativistic case?

Redefine the classical momentum in the form:

$$\vec{p} = \Gamma(u) m\vec{u} = \frac{1}{\sqrt{1 - u^2/c^2}} m\vec{u}$$

This $\Gamma(u)$ is different than the γ factor since it uses the particle's speed u

➔ What? How does this make sense?

➔ Well the particle itself is moving with a relativistic speed, thus that must impact the measurements by the observer in the rest frame!!

Now, the agreed form of the momentum in all frames is:

$$\vec{p} = m\frac{d\vec{r}}{d\tau} = m\frac{d\vec{r}}{dt}\frac{dt}{d\tau} = m\vec{u}\gamma = \frac{1}{\sqrt{1 - u^2/c^2}} m\vec{u}$$

Resulting in the new relativistic definition of the momentum:

$$\vec{p} = m\gamma\vec{u}$$

Relativistic Energy

- Due to the new idea of relativistic mass, we must now redefine the concepts of work and energy.
  - Modify Newton's second law to include our new definition of linear momentum, and force becomes:

$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d}{dt}\left(\gamma m\vec{u}\right) = \frac{d}{dt}\left(\frac{m\vec{u}}{\sqrt{1-u^2/c^2}}\right)$$

- The work *W* done by a force **F** to move a particle from rest to a certain kinetic energy is $W = K = \int \frac{d}{dt}\left(\gamma m\vec{u}\right)\cdot\vec{u}dt$
- Resulting relativistic kinetic energy becomes

$$K = \int_0^{\gamma u} ud\left(\gamma u\right) = \gamma mc^2 - mc^2 = \left(\gamma - 1\right)mc^2$$

- Why doesn't this look anything like the classical KE?

Big note on Relativistic KE

- Only $K = (\gamma - 1)mc^2$ is right!

- $K = \frac{1}{2}mu^2$ and $K = \frac{1}{2}\gamma mu^2$ are wrong!

Total Energy and Rest Energy

Rewriting the relativistic kinetic energy:

$$\gamma mc^2 = \frac{mc^2}{\sqrt{1-u^2/c^2}} = K + mc^2$$

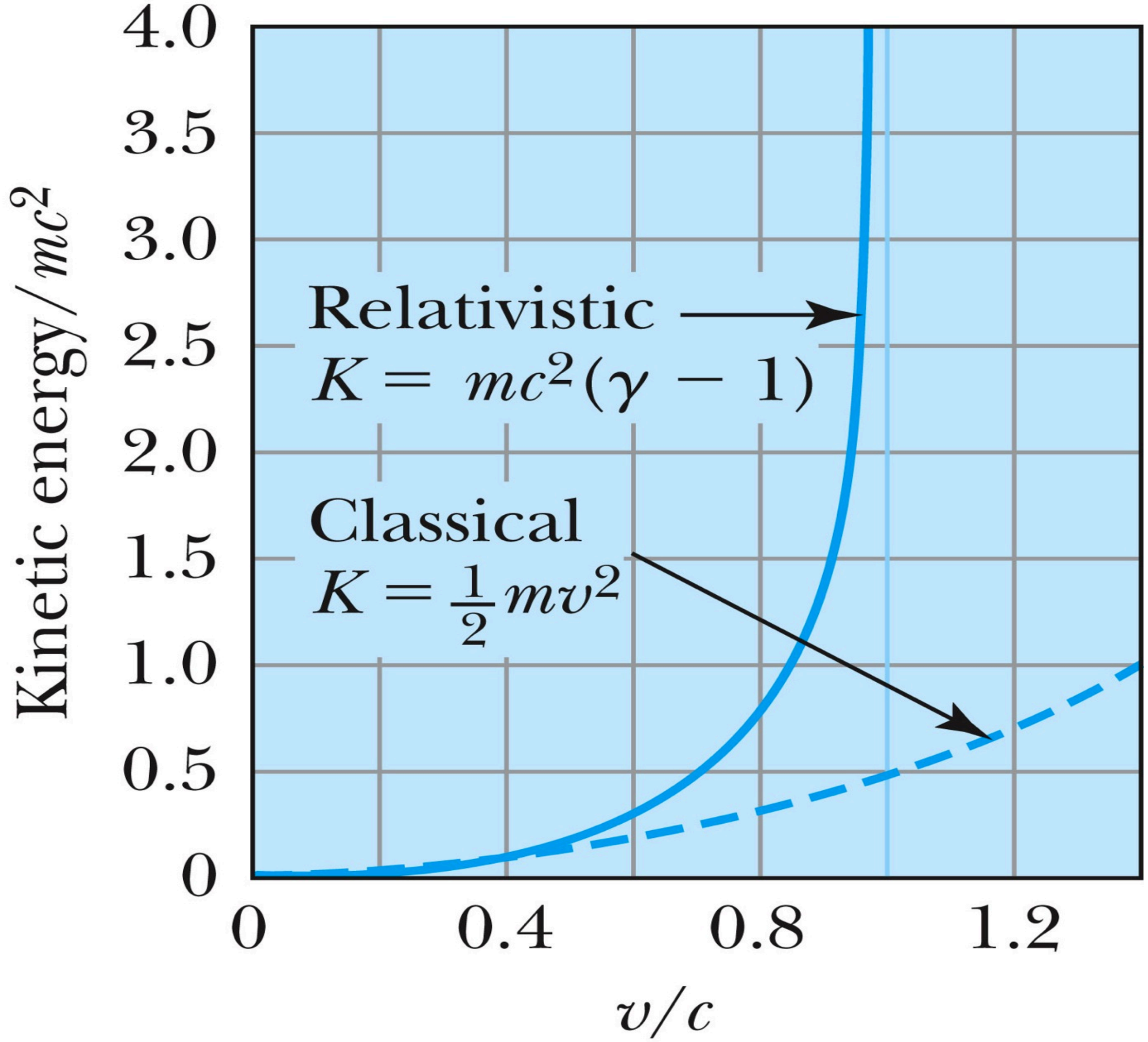
The term mc^2 is called the rest energy and is denoted by E_0.

$$E_0 = mc^2$$

The sum of the kinetic energy and rest energy is interpreted as the total energy of the particle.

$$E = \gamma mc^2 = \frac{mc^2}{\sqrt{1-u^2/c^2}} = \frac{E_0}{\sqrt{1-u^2/c^2}} = K + E_0$$

Relativistic and Classical Kinetic Energies



Relationship of Energy and Momentum

$$p = \gamma m u = \frac{mu}{\sqrt{1 - u^2/c^2}}$$

We square this result, multiply by c^2, and rearrange the result.

$$p^2c^2 = \gamma^2 m^2 u^2 c^2 = \gamma^2 m^2 c^4 \left(\frac{u^2}{c^2}\right) = \gamma^2 m^2 c^4 \beta^2$$

$$\beta^2 = 1 - \frac{1}{\gamma^2} \Rightarrow p^2c^2 = \gamma^2 m^2 c^4 \left(1 - \frac{1}{\gamma^2}\right) = \gamma^2 m^2 c^4 - m^2 c^4$$

Rewrite

$$p^2c^2 = E^2 - E_0^2$$

Rewrite

$$E^2 = p^2c^2 + E_0^2 = p^2c^2 + m^2c^4$$

Massless Particles have a speed equal to the speed of light c

- Recall that a photon has "zero" rest mass and the equation from the last slide reduces to: E = pc and we may conclude that:

$$E = \gamma mc^2 = pc = \gamma muc$$

- Thus the velocity, u, of a massless particle must be c since, as $m \to 0, \gamma \to \infty$ and it follows that: u = c.

Units of Work, Energy and Mass

- The work done in accelerating a charge through a potential difference V is $W = qV$.
  - For a proton, with the charge $e = 1.602 \times 10^{-19}$ C being accelerated across a potential difference of 1 V, the work done is

    $$1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}$$

    $$W = (1.602 \times 10^{-19})(1 \text{ V}) = 1.602 \times 10^{-19} \text{ J}$$

- eV is also used as a unit of energy.

Other Units

1) Rest energy of a particle:
   Example: Rest energy, E_0, of proton

$$E_0(proton) = m_p c^2 = \left(1.67\times10^{-27}kg\right)\cdot\left(3.00\times10^{8}\,m/s\right) = 1.50\times10^{-10}J$$

$$= 1.50\times10^{-10}J\cdot\frac{1eV}{1.602\times10^{-19}J} = 9.38\times10^{8}eV$$

2) **Atomic mass unit** (amu): Example: carbon-12

$$M\left({}^{12}C\ \text{atom}\right) = \frac{12\,g/mole}{6.02\times10^{23}\,atoms/mole}$$

$$= 1.99\times10^{-23}\,g/atom$$

$$M\left({}^{12}C\ \text{atom}\right) = 1.99\times10^{-26}\,kg/atom = 12\,u/atom$$