PHYS 3313 – Section 001
Lecture #12

Monday, Feb. 24, 2014
Dr. Jaehoon Yu

- Rutherford Scattering Experiment and Rutherford Atomic Model
- The Classic Atomic Model
- The Bohr Model of the Hydrogen Atom

Announcements

- Quiz 2 results
  - Class average: 26.7/50
    - Equivalent to 53.4/100
      - Previous quiz: 30/100
  - Top score: 50/50
- Mid-term exam
  - In class on Wednesday, Mar. 5
  - Covers CH1.1 – what we finish on Monday, Mar. 3 + appendices
  - Mid-term exam constitutes 20% of the total
  - **Please do NOT miss the exam! You will get an F if you miss it.**
  - BYOF: You may bring a one 8.5x11.5 sheet (front and back) of handwritten formulae and values of constants for the exam
  - No derivations or solutions of any problems allowed!
  - No additional formulae or values of constants will be provided!

Reminder: Special Project #3

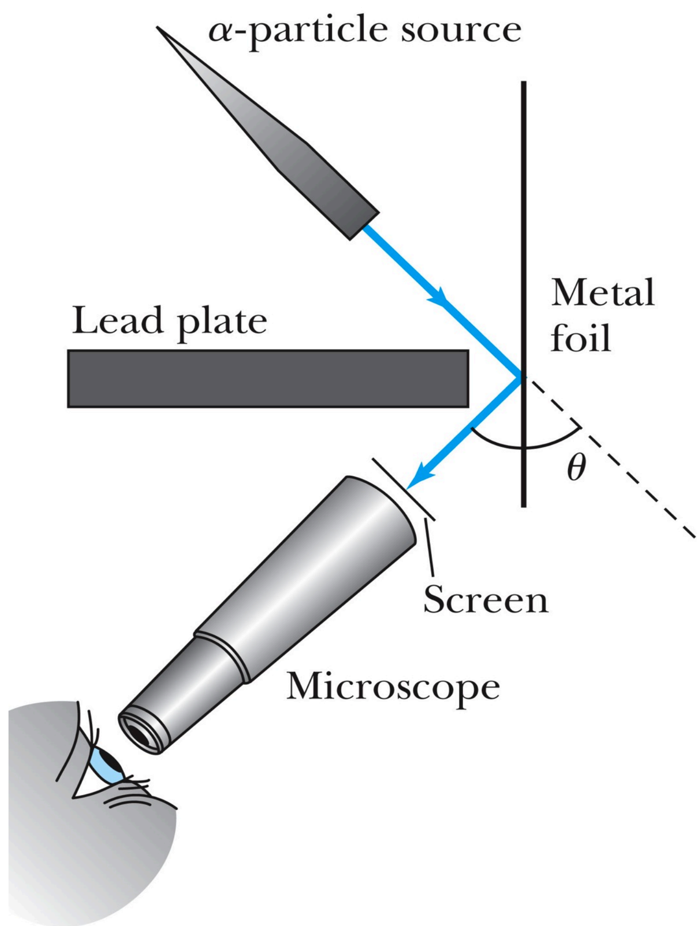
- A total of N_i incident projectile particle of atomic number Z_1 kinetic energy KE scatter on a target of thickness t and atomic number Z_2 and has n atoms per volume. What is the total number of scattered projectile particles at an angle θ? (20 points)
- Please be sure to clearly define all the variables used in your derivation! Points will be deducted for missing variable definitions.
- This derivation must be done on your own. Please do not copy the book, internet or your friends'.
- Due is this Wednesday, Feb.26 .

Prefixes, expressions and their meanings

- deca (da): 10^{1}
- hecto (h): 10^{2}
- kilo (k): 10^{3}
- mega (M): 10^{6}
- giga (G): 10^{9}
- tera (T): 10^{12}
- peta (P): 10^{15}
- exa (E): 10^{18}
- deci (d): 10^{-1}
- centi (c): 10^{-2}
- milli (m): 10^{-3}
- micro (μ): 10^{-6}
- nano (n): 10^{-9}
- pico (p): 10^{-12}
- femto (f): 10^{-15}
- atto (a): 10^{-18}

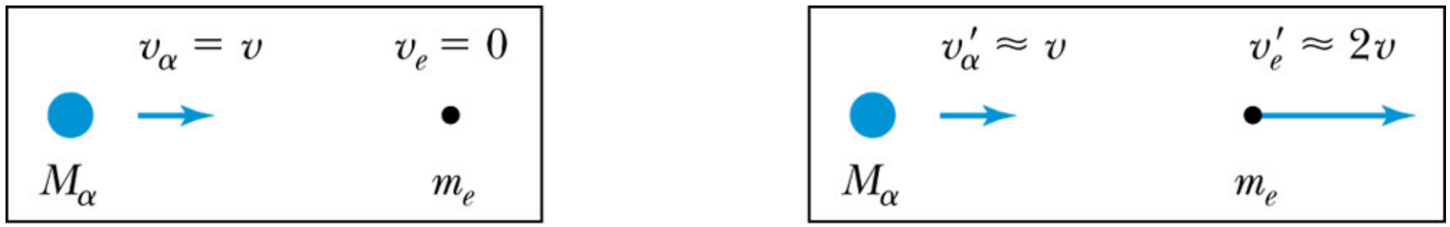
Experiments of Geiger and Marsden

- Rutherford, Geiger, and Marsden conceived a new technique for investigating the structure of matter by scattering a particle off atoms.
- Geiger showed that many particles were scattered from thin gold-leaf targets at backward angles greater than 90°.
- Time to do some calculations!



Ex 4.1: Maximum Scattering Angle

Geiger and Marsden (1909) observed backward-scattered (θ>=90^o) α particles when a beam of energetic α particles was directed at a piece of gold foil as thin as $6.0\text{x}10^{-7}$m. Assuming an α particle scatters from an electron in the foil, what is the maximum scattering angle?



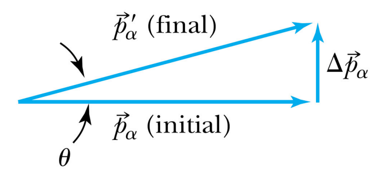
Before After

- The maximum scattering angle corresponds to the maximum momentum change
- Using the momentum conservation and the KE conservation for an elastic collision, the maximum momentum change of the α particle is

$$M_\alpha \vec{v}_\alpha = M_\alpha \vec{v}'_\alpha + m_e \vec{v}'_e$$
$$\frac{1}{2} M_\alpha v_\alpha^2 = \frac{1}{2} M_\alpha v_\alpha'^2 + \frac{1}{2} m_2 v_e^2$$

$$\Rightarrow \Delta \vec{p}_\alpha = M_\alpha \vec{v}_\alpha - M_\alpha \vec{v}'_\alpha = m_e \vec{v}'_e \Rightarrow \Delta p_{\alpha-\max} = 2 m_e v_\alpha$$

- Determine θ by letting Δp_{max} be perpendicular to the direction of motion.



$$\theta_{\max} = \frac{\Delta p_{\alpha-\max}}{p_\alpha} = \frac{2 m_e v_\alpha}{m_\alpha v_\alpha} = \frac{2 m_e}{m_\alpha} = 2.7 \times 10^{-4} rad = 0.016^\circ$$

Multiple Scattering from Electrons

- If an α particle were scattered by many electrons, then *N* electrons results in $\langle\theta\rangle_{total} \sim \sqrt{N}\theta$
- The number of atoms across the thin gold layer of 6 × 10^{-7} m:

$$\frac{N_{Molecules}}{cm^3} = N_{Avogadro}\left(molecules/mol\right)\times\left[\frac{1}{g-molecular-weight}\left(\frac{mol}{g}\right)\right]\cdot\left[\rho\left(\frac{g}{cm^3}\right)\right]$$

$$= 6.02\times10^{23}\left(\frac{molecules}{mol}\right)\cdot\left(\frac{1mol}{197g}\right)\cdot\left(19.3\frac{g}{cm^3}\right)$$

$$= 5.9\times10^{22}\frac{molecules}{cm^3} = 5.9\times10^{28}\frac{atoms}{m^3}$$

- Assume the distance between atoms is $d = \left(5.9\times10^{28}\right)^{-1/3} = 2.6\times10^{-10}\left(m\right)$

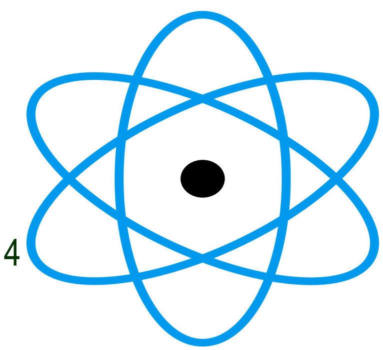
  and there are $N = \dfrac{6\times10^{-7}m}{2.6\times10^{-10}m} = 2300\left(atoms\right)$

  This gives $\langle\theta\rangle_{total} = \sqrt{2300}\left(0.016^\circ\right) = 0.8^\circ$

Rutherford's Atomic Model

- $<\theta>_{total}\sim 0.8*79=6.8^{o}$ even if the α particle scattered from all 79 electrons in each atom of gold

- The experimental results were inconsistent with Thomson's atomic model.
- Rutherford proposed that an atom has a positively charged core (nucleus) surrounded by the negative electrons.

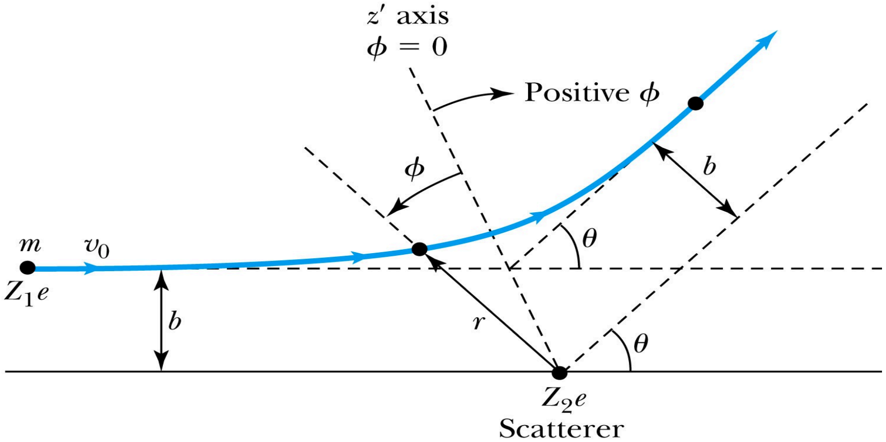


Assumptions of Rutherford Scattering

1. The scatterer is so massive that it does not recoil significantly; therefore the initial and final KE of the α particle are practically equal.
2. The target is so thin that only a single scattering occurs.
3. The bombarding particle and target scatterer are so small that they may be treated as point masses with electrical charges.
4. Only the Coulomb force is effective.

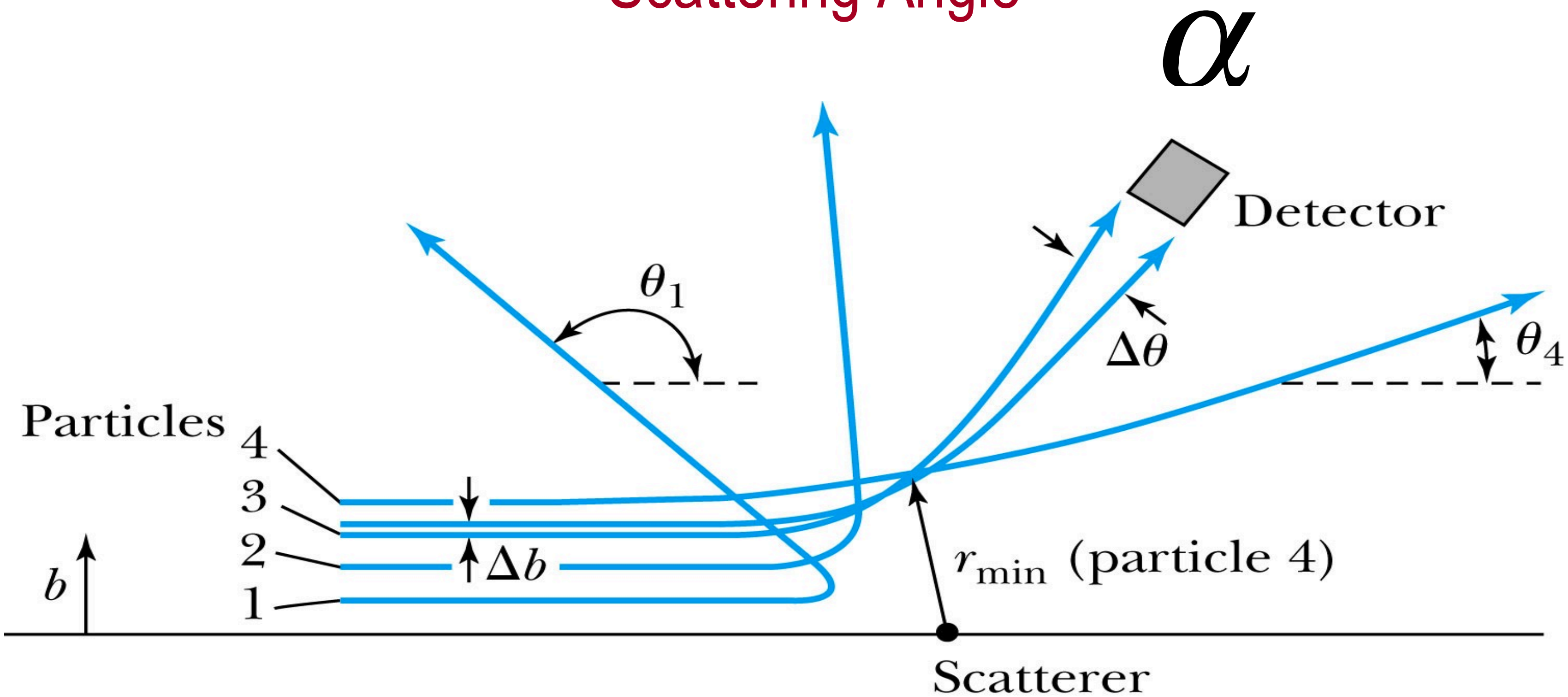
Rutherford Scattering

- Scattering experiments help us study matter too small to be observed directly by measuring the angular distributions of the scattered particles
  - What is the force acting in this scattering?
- There is a relationship between the impact parameter *b* and the scattering angle *θ*.



- When *b* is small,
- *r* gets small.
- Coulomb force gets large.
- *θ* can be large and the particle can be repelled backward.

The Relationship Between the Impact Parameter b and the Scattering Angle



The relationship between the impact parameter b and scattering angle θ: Particles with small impact parameters approach the nucleus most closely (r_{min}) and scatter to the largest angles. Particles within a certain range of impact parameters b will be scattered within the window Δθ.

Rutherford Scattering

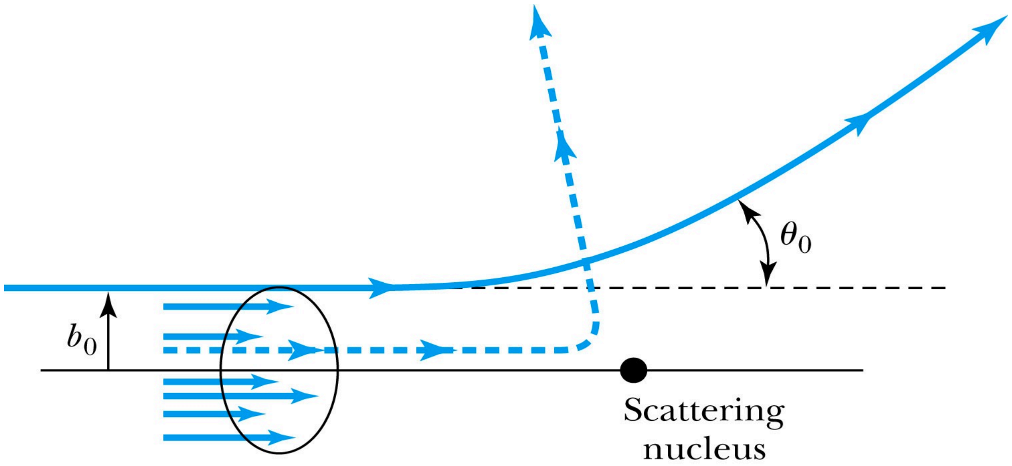
- What are the quantities that can affect the scattering?
  - What was the force again?
    - The Coulomb force

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{Z_1 Z_2 e^2}{r^2}\hat{r}_e$$

  - The charge of the incoming particle (Z_1e)
  - The charge of the target particle (Z_2e)
  - The minimum distance the projectile approaches the target (r)
- Using the fact that this is a totally elastic scattering under a central force, we know that
  - Linear momentum is conserved $\vec{p}_i^{\,\alpha} = \vec{p}_f^{\,\alpha} + \vec{p}^{\,N}$
  - KE is conserved $\frac{1}{2}mv_{\alpha i}^2 = \frac{1}{2}mv_{\alpha f}^2 + \frac{1}{2}mv_n^2$
  - Angular momentum is conserved $mr^2\varpi = mv_{\alpha i}b$
- From this, impact parameter $b = \frac{Z_1 Z_2 e^2}{4\pi\varepsilon_0 m v_{\alpha i}^2}\cot\frac{\theta}{2} = \frac{Z_1 Z_2 e^2}{8\pi\varepsilon_0 KE_i}\cot\frac{\theta}{2}$

Rutherford Scattering - probability

- Any particle inside the circle of area πb_0^2 will be similarly scattered.



- The **cross section** $\sigma = \pi b^2$ is related to the **probability** for a particle being scattered by a nucleus.

$$nt\pi b^2 = \pi nt\left(\frac{Z_1 Z_2 e^2}{8\pi\varepsilon_0 KE_i}\cot\frac{\theta}{2}\right)^2$$

t: target thickness
n: atomic number density

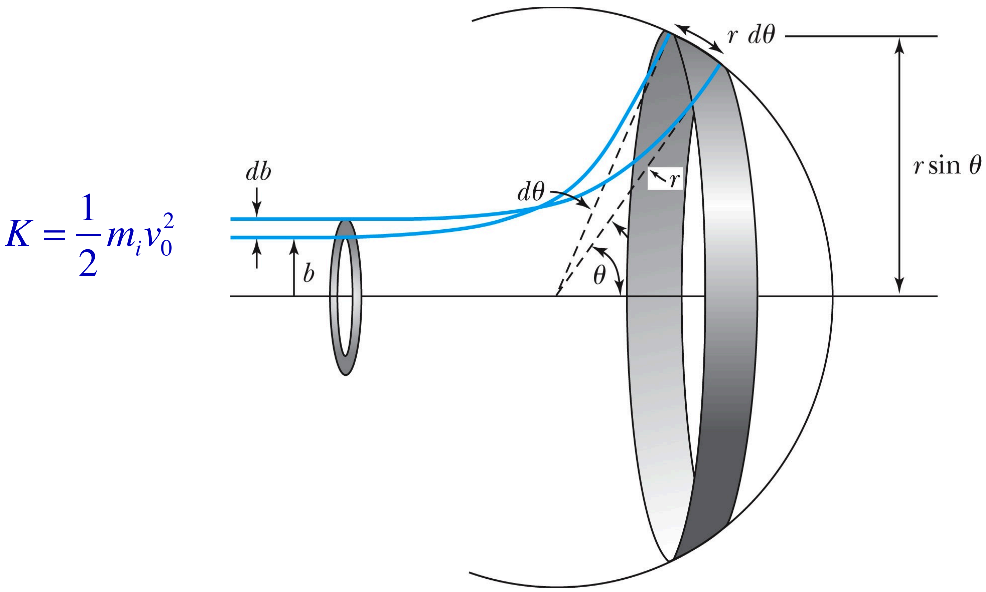
- The fraction of the incident particles scattered is

$$f = \frac{\text{target area exposed by scatterers}}{\text{total target area}}$$

- The number of scattering nuclei per unit area $nt = \frac{\rho N_A N_M t}{M_g}\cdot\frac{atoms}{cm^2}$

Rutherford Scattering Equation

- In an actual experiment, a detector is positioned from θ to $\theta + d\theta$ that corresponds to incident particles between b and $b + db$.



- The number of particles scattered into the the angular coverage per unit area is

$$N(\theta) = \frac{N_i n t}{16}\left(\frac{e^2}{4\pi\varepsilon_0}\right)^2 \frac{Z_1^2 Z_2^2}{r^2 K^2 \sin^4(\theta/2)}$$

The Important Points

1. The scattering is proportional to the **square of the atomic numbers** of *both* the incident particle (Z_1) and the target scatterer (Z_2).
2. The number of scattered particles is **inversely proportional to the square of the kinetic energy** of the incident particle.
3. For the scattering angle θ, the scattering is **proportional to 4th power of $\sin(\theta/2)$**.
4. The Scattering is **proportional to the target thickness** for thin targets.

The Classical Atomic Model

As suggested by the Rutherford Model, an atom consisted of a small, massive, positively charged nucleus surrounded by moving electrons. This then suggested consideration of a planetary model of the atom.

Let's consider atoms as a planetary model.

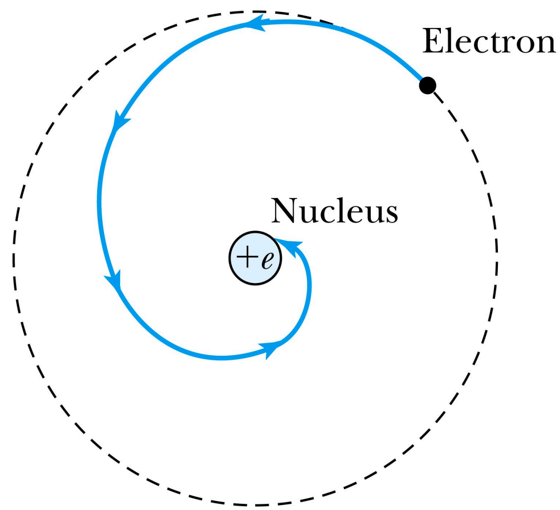
- The force of attraction on the electron by the nucleus and Newton's 2nd law give $\vec{F}_e = -\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}\hat{e}_r = \frac{mv^2}{r}\hat{e}_r$

  where *v* is the tangential speed of an electron.

- The total energy is $E = K + V = \frac{e^2}{8\pi\varepsilon_0 r} - \frac{e^2}{4\pi\varepsilon_0 r} = -\frac{e^2}{8\pi\varepsilon_0 r}$

The Planetary Model is Doomed

- From classical E&M theory, an accelerated electric charge radiates energy (electromagnetic radiation) which means total energy must decrease. → *Radius r must decrease!!*



Electron crashes into the nucleus!?

- Physics had reached a turning point in 1900 with Planck's hypothesis of the quantum behavior of radiation.

The Bohr Model of the Hydrogen Atom – The assumptions

- "Stationary" states or orbits must exist in atoms, i.e., orbiting electrons ***do not radiate*** energy in these orbits. These orbits or stationary states are of a fixed definite energy E.
- The emission or absorption of electromagnetic radiation can occur only in conjunction with a transition between two stationary states. The frequency, f, of this radiation is proportional to the *difference* in energy of the two stationary states:
- $$E = E_1 - E_2 = hf$$
- *where h is Planck's Constant*
  - *Bohr thought this has to do with fundamental length of order ~10^{-10}m*
- Classical laws of physics do not apply to transitions between stationary states.
- The mean kinetic energy of the electron-nucleus system is quantized as $K = nhf_{orb}/2$, where f_{orb} is the frequency of rotation. This is equivalent to the angular momentum of a stationary state to be an integral multiple of $h/2\pi$

How did Bohr Arrived at the angular momentum quantization?

- The mean kinetic energy of the electron-nucleus system is quantized as $K = nhf_{orb}/2$, where f_{orb} is the frequency of rotation. This is equivalent to the angular momentum of a stationary state to be an integral multiple of $h/2\pi$.
- Kinetic energy can be written $K = \frac{nhf}{2} = \frac{1}{2}mv^2$
- Angular momentum is defined as $\left|\vec{L}\right| = \left|\vec{r} \times \vec{p}\right| = mvr$
- The relationship between linear and angular quantifies $v = r\omega;\ \omega = 2\pi f$
- Thus, we can rewrite $K = \frac{1}{2}mvr\omega = \frac{1}{2}L\omega = \frac{1}{2}2\pi Lf = \frac{nhf}{2}$

$$2\pi L = nh \Rightarrow \quad L = n\frac{h}{2\pi} = n\hbar \quad \text{, where } \hbar = \frac{h}{2\pi}$$

Bohr's Quantized Radius of Hydrogen

- The angular momentum is $\left|\vec{L}\right| = \left|\vec{r} \times \vec{p}\right| = mvr = n\hbar$
- So the speed of an orbiting e can be written $v_e = \frac{n\hbar}{m_e r}$
- From the Newton's law for a circular motion

$$F_e = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2} = \frac{m_e v_e^2}{r} \Rightarrow v_e = \frac{e}{\sqrt{4\pi\varepsilon_0 m_e r}}$$

- So from above two equations, we can get

$$v_e = \frac{n\hbar}{m_e r} = \frac{e}{\sqrt{4\pi\varepsilon_0 m_e r}} \Rightarrow r = \frac{4\pi\varepsilon_0 n^2\hbar^2}{m_e e^2}$$

Bohr Radius

- The radius of the hydrogen atom for stationary states is

$$r_n = \frac{4\pi\varepsilon_0 n^2 \hbar^2}{m_e e^2} = a_0 n^2$$

Where the **Bohr radius** for a given stationary state is:

$$a_0 = \frac{4\pi\varepsilon_0 \hbar^2}{m_e e^2} = \frac{\left(8.99\times10^{9}\,N\cdot m^2/C^2\right)\cdot\left(1.055\times10^{-34}\,J\cdot s\right)^2}{\left(9.11\times10^{-31}\,kg\right)\cdot\left(1.6\times10^{-19}\,C\right)^2} = 0.53\times10^{-10}\,m$$

- The smallest diameter of the hydrogen atom is

$$d = 2r_1 = 2a_0 \approx 10^{-10}\,m \approx 1\mathring{A}$$

  - OMG!! The fundamental length!!

- $n = 1$ gives its lowest energy state (called the "ground" state)