PHYS 3313 – Section 001
Lecture #13

Wednesday, Feb. 26, 2014
Dr. Jaehoon Yu

- Bohr Radius
- Bohr's Hydrogen Model and Its Limitations
- Characteristic X-ray Spectra
- Hydrogen Spectrum Series
- X-ray Scattering
- Bragg's Law

Announcements

- Mid-term exam
  - In class on next Wednesday, Mar. 5
  - Covers CH1.1 – what we finish on coming Monday, Mar. 3 + appendices
  - Mid-term exam constitutes 20% of the total
  - **Please do NOT miss the exam! You will get an F if you miss it.**
  - BYOF: You may bring a one 8.5x11.5 sheet (front and back) of handwritten formulae and values of constants for the exam
  - No derivations or solutions of any problems allowed!
  - No additional formulae or values of constants will be provided!
- Homework #3
  - End of chapter problems on CH4: 5, 14, 17, 21, 23 and 45
  - Due: Monday, March 10
- Colloquium today at 4pm in SH101
  - Dr. Billy Quarles of NASA Ames Research Center

Physics Department
The University of Texas at Arlington
COLLOQUIUM

Theia's Provenance: Regional Source of Earth's Late Impactor

Dr. Billy Quarles
NASA Ames Research Center
Space Science and Astrobiology Division
Moffett Field California

4:00 pm Wednesday February 26, 2014 room 101 SH

Abstract:

A perplexing mystery of the origin of the Moon has persisted for centuries. Many hypotheses have sought to explain the origin and the current best theory considers a late Giant Impact. However this theory begs the question on the origin of the Mars-sized impactor, *Theia*. Recent studies using smooth particle hydrodynamics (SPH) have provided a clearer picture on the composition of the impactor along with radiogenic measurements of asteroidal and lunar material. SPH simulations constrain the kinds of impacts permissible to explain the Earth-Moon system but do not give an indicator where the impactor may have originated within the early Solar System.

A five terrestrial planet model for the Solar System has been investigated and showed from a dynamical parameter space of initial semimajor axis and eccentricity that certain physical phenomena can help constrain the source region. Recent results of this type of study will be presented with a short introduction to the problems of Moon formation, the physical constraints from measurements, and possible solutions through dynamical study. As a fellow of the NASA Postdoctoral Program, a short Q&A period is allotted for inquisitive graduate students who wish to learn more about the NPP Fellowship.

Refreshments will be served at 3:30p.m in the Physics Lounge

Uncertainties

- Statistical Uncertainty: A naturally occurring uncertainty due to the number of measurements
  – Usually estimated by taking the square root of the number of measurements or samples, $\sqrt{N}$
- Systematic Uncertainty: Uncertainty occurring due to unintended biases or unknown sources
  – Biases made by personal measurement habits
  – Some sources that could impact the measurements
- In any measurement, the uncertainties provide the significance to the measurement

Bohr's Quantized Radius of Hydrogen

- The angular momentum is $\left|\vec{L}\right| = \left|\vec{r} \times \vec{p}\right| = mvr = n\hbar$
- So the speed of an orbiting e can be written $v_e = \dfrac{n\hbar}{m_e r}$
- From the Newton's law for a circular motion

$$F_e = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2} = \frac{m_e v_e^2}{r} \Rightarrow v_e = \frac{e}{\sqrt{4\pi\varepsilon_0 m_e r}}$$

- So from above two equations, we can get

$$v_e = \frac{n\hbar}{m_e r} = \frac{e}{\sqrt{4\pi\varepsilon_0 m_e r}} \Rightarrow r = \frac{4\pi\varepsilon_0 n^2\hbar^2}{m_e e^2}$$

Bohr Radius

- The radius of the hydrogen atom for stationary states is

$$r_n = \frac{4\pi\varepsilon_0 n^2 \hbar^2}{m_e e^2} = a_0 n^2$$

Where the **Bohr radius** for a given stationary state is:

$$a_0 = \frac{4\pi\varepsilon_0 \hbar^2}{m_e e^2} = \frac{\left(8.99\times 10^9\, N\cdot m^2/C^2\right)\cdot\left(1.055\times 10^{-34}\, J\cdot s\right)^2}{\left(9.11\times 10^{-31}\, kg\right)\cdot\left(1.6\times 10^{-19}\, C\right)^2} = 0.53\times 10^{-10}\, m$$

- The smallest diameter of the hydrogen atom is

$$d = 2r_1 = 2a_0 \approx 10^{-10}\, m \approx 1\,\mathring{A}$$

  - OMG!! The fundamental length!!

- n = 1 gives its lowest energy state (called the **"ground" state**)

Ex. 4.6 Justification for non-relativistic treatment of orbital e

- Are we justified for non-relativistic treatment of the orbital electrons?
  - When do we apply relativistic treatment?
    - When v/c>0.1
- Orbital speed: $v_e = \frac{e}{\sqrt{4\pi\varepsilon_0 m_e r}}$
- Thus

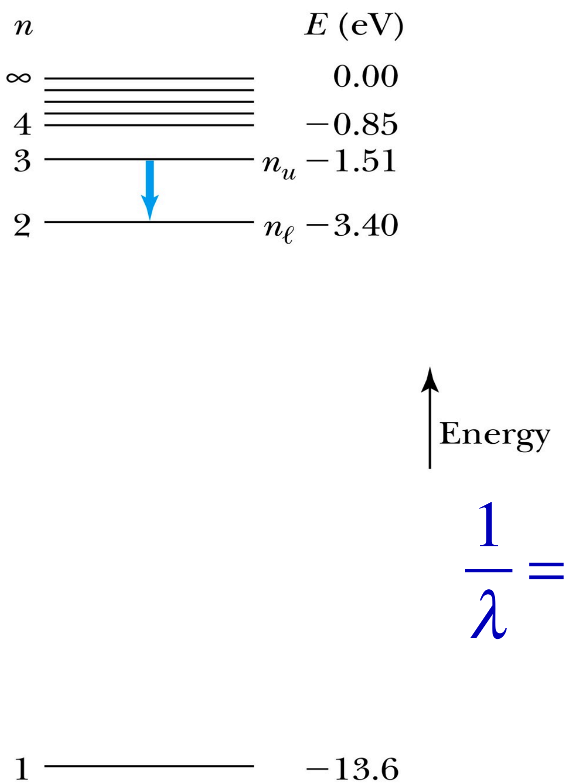
$$v_e = \frac{\left(1.6\times10^{-16}\right)\cdot\left(9\times10^{9}\right)}{\sqrt{\left(9.1\times10^{-31}\right)\cdot\left(0.5\times10^{-10}\right)}} \approx 2.2\times10^{6}\left(m/s\right) < 0.01c$$

The Hydrogen Atom

- The energies of the stationary states

$$E_n = -\frac{e^2}{8\pi\varepsilon_0 r_n} = -\frac{e^2}{8\pi\varepsilon_0 a_0 n^2} = -\frac{E_0}{n^2} \qquad E_0 = -\frac{e^2}{8\pi\varepsilon_0 a_0} = \frac{-\left(1.6\times10^{-19}C\right)^2}{8\pi\left(8.99\times10^9 N\cdot m^2/C^2\right)\cdot\left(0.53\times10^{-10}m\right)} = -13.6eV$$

where E_0 is the ground state energy



- Emission of light occurs when the atom is in an excited state and decays to a lower energy state ($n_u \rightarrow n_\ell$).

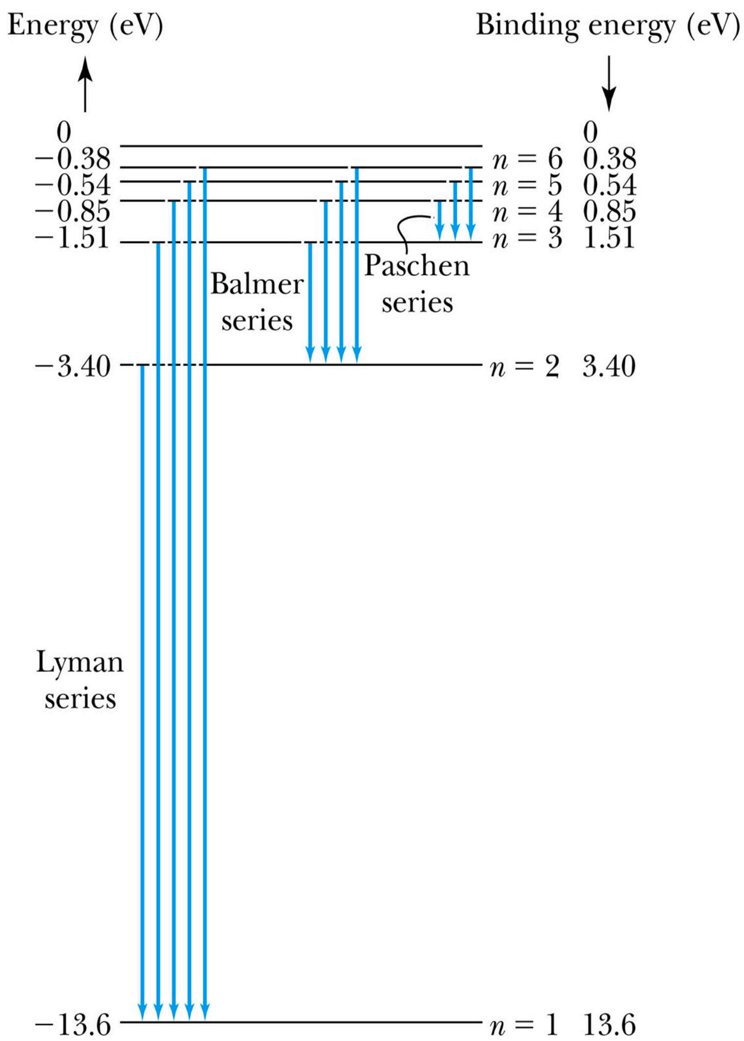
$$hf = E_u - E_l$$

where f is the frequency of a photon.

$$\frac{1}{\lambda} = \frac{f}{c} = \frac{E_u - E_l}{hc} = \frac{E_0}{hc}\left(\frac{1}{n_l^2} - \frac{1}{n_u^2}\right) = R_\infty\left(\frac{1}{n_l^2} - \frac{1}{n_u^2}\right)$$

R_∞ is the **Rydberg constant**. $R_\infty = E_0/hc$

Transitions in the Hydrogen Atom



- **Lyman series:** The atom will remain in the excited state for a short time before emitting a photon and returning to a lower stationary state. All hydrogen atoms exist in $n = 1$ (invisible).
- **Balmer series:** When sunlight passes through the atmosphere, hydrogen atoms in water vapor absorb the wavelengths (visible).

Fine Structure Constant

- The electron's speed on an orbit in the Bohr model:

$$v_e = \frac{n\hbar}{m_e r_n} = \frac{n\hbar}{m_e \dfrac{4\pi\varepsilon_0 n^2\hbar^2}{m_e e^2}} = \frac{1}{n}\frac{e^2}{4\pi\varepsilon_0\hbar}$$

- On the ground state, $v_1 = 2.2 \times 10^6$ m/s ~ less than 1% of the speed of light
- The ratio of v_1 to c is the **fine structure constant, α.**

$$\alpha \equiv \frac{v_1}{c} = \frac{\hbar}{ma_0 c} = \frac{e^2}{4\pi\varepsilon_0\hbar c} =$$

$$\frac{\left(1.6\times10^{-19}C\right)^2}{\left(8.99\times10^9\,N\cdot m^2/C^2\right)\cdot\left(1.055\times10^{-34}\,J\cdot s\right)\cdot\left(3\times10^8\,m/s\right)} \approx \frac{1}{137}$$

The Correspondence Principle

Classical electrodynamics + Bohr's atomic model

Determine the properties of radiation

Need a principle to relate the new modern results with classical ones.

Bohr's correspondence principle

In the limits where classical and quantum theories should agree, the quantum theory must produce the classical results.