PHYS 3313 – Section 001
Lecture #19

Wednesday, Apr. 2, 2014
Dr. Jaehoon Yu

- Infinite Square Well Potential
- Finite Square Well Potential
- Penetration Depth
- 3D Infinite Potential Well

Announcements

- Research paper template has been uploaded onto the class web page link to research
- Homework #4
  - End of chapter problems on CH5: 8, 10, 16, 24, 26, 36 and 47
  - Due Wednesday, Apr. 9
- Quiz #3 at the beginning of the class Monday, Apr. 7
  - Covers CH 5.1 through CH6.4
- Special colloquium at 4pm, today for triple extra credit
  - Dr. Gerald Blazey of the White House

Physics Department
The University of Texas at Arlington
COLLOQUIUM

A Primer on the Office of Science and Technology Policy

Dr. Gerald Blazey

Assistant Director for Physical Sciences

Office of Science and Technology Policy, Executive Office of the President

Washington, DC

4:00 pm Wednesday April 2, 2014 in room 101 SH

Abstract:

The Office of Science and Technology Policy (OSTP) in the Executive Office of the President has an important role establishing science policy in the US and promoting the US science and technology enterprise. After a description of the components and organization of OSTP, the main mechanisms for impacting implementing science policy are discussed. Selected examples and anecdotes involving recent policy will be discussed as well.

Gerald Blazey Biography information: Gerald Blazey received his doctoral degree in experimental particle physics from the University of Minnesota in 1986. He specializes in quantum chromondynamics (the strong theory of nuclear interactions) initially with fixed target experiments and over the past twenty years with colliding beam experiments. He has published over 300 peer reviewed articles and is a Fellow of the American Physical Society. Since joining Northern Illinois University in 1996 he has been appointed Distinguished Research Professor, Presidential Science Advisor, and Director of the Northern Illinois Center for Accelerator and Detector Development and has been principle investigator for federally funded grants from the Department of Energy, National Science Foundation, and the Department of Defense. While participating in the Fermi National Accelerator Laboratory collider program he served four years as co-Spokesperson of the DZero collaboration comprised of more than 600 physicists from over 20 countries. From 2007 to 2010 he was Program Manager for the International Linear Collider in the Office of High Energy Physics of the Department of Energy and since 2011 has been Assistant Director for Physical Sciences at the Office of Science and Technology Policy in the Executive Office of the President.

We will be serving refreshments at 3:30p.m in the physics lounge (RM 106 SH).

Reminder: Special Project #4

- Prove that the wave function $\Psi = A[\sin(kx-\omega t) + i\cos(kx-\omega t)]$ is a good solution for the time-dependent Schrödinger wave equation. Do NOT use the exponential expression of the wave function. (10 points)
- Determine whether or not the wave function $\Psi = Ae^{-\alpha|x|}$ satisfy the time-dependent Schrödinger wave equation. (10 points)
- Due for this special project is Monday, Apr. 7.
- You MUST have your own answers!

Special project #5

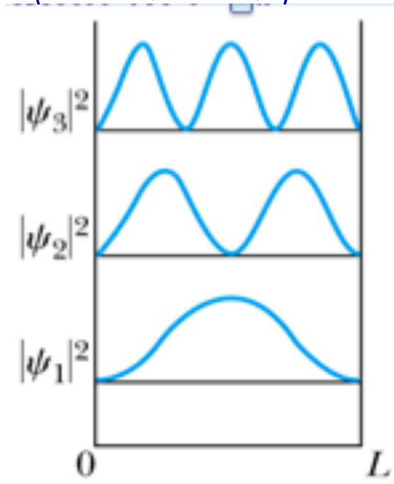
- Show that the Schrodinger equation becomes Newton's second law in the classical limit. (15 points)
- Deadline Monday, Apr. 21, 2014
- You MUST have your own answers!

How does this correspond to Classical Mech.?

- What is the probability of finding a particle in a box of length L? $\frac{1}{L}$
- Bohr's **correspondence principle** says that QM and CM must correspond to each other! When?
  - When n becomes large, the QM approaches to CM
- So when n→∞, the probability of finding a particle in a box of length L is

$$P(x)=\psi_n^*(x)\psi_n(x)=\left|\psi_n(x)\right|^2=\frac{2}{L}\lim_{n\to\infty}\sin^2\left(\frac{n\pi x}{L}\right)\approx\frac{2}{L}\left\langle\sin^2\left(\frac{n\pi x}{L}\right)\right\rangle=\frac{2}{L}\cdot\frac{1}{2}=\frac{1}{L}$$

- Which is identical to the CM probability!!
- One can also see this from the plot of P!



Details of the computation

$$\frac{2}{L}\left\langle \sin^2\left(\frac{n\pi x}{L}\right)\right\rangle = \frac{2}{L}\cdot\frac{1}{2\pi}\int_0^{2\pi}\left(\sin^2 y\right)dy$$

$$\text{Let } y = \frac{n\pi x}{L}$$

$$= \frac{2}{L}\cdot\frac{1}{2\pi}\int_0^{2\pi}\frac{1}{2}\left(1-\cos 2y\right)dy$$

$$= \frac{2}{L}\cdot\frac{1}{2\pi}\cdot\frac{1}{2}\left[y-\frac{1}{2}\sin 2y\right]_0^{2\pi} = \frac{2}{L}\cdot\frac{1}{2\pi}\cdot\frac{1}{2}\left[2\pi-0-0\right]$$

$$= \frac{2}{L}\cdot\frac{1}{\cancel{2}\cancel{\pi}}\cdot\frac{1}{2}\cdot\cancel{2}\cancel{\pi} = \frac{\cancel{2}}{L}\cdot\frac{1}{\cancel{2}} = \frac{1}{L}$$

Ex 6.8: Expectation values inside a box

Determine the expectation values for x, x^2, p and p^2 of a particle in an infinite square well for the first excited state.

What is the wave function of the first excited state? n=? 2

$$\psi_{n=2}(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{2\pi x}{L}\right)$$

$$\langle x \rangle_{n=2} = \int_{-\infty}^{+\infty} \psi^*_{n=2}(x)\, x \psi_{n=2}(x) = \frac{2}{L}\int_0^L x \sin^2\left(\frac{2\pi x}{L}\right) dx = \frac{L}{2}$$

$$\langle x^2 \rangle_{n=2} = \frac{2}{L}\int_0^L x^2 \sin^2\left(\frac{2\pi x}{L}\right) dx = 0.32L^2$$

$$\langle p \rangle_{n=2} = \frac{2}{L}\int_0^L \sin\left(\frac{2\pi x}{L}\right)(-i\hbar)\frac{\partial}{\partial x}\left[\sin\left(\frac{n\pi x}{L}\right)\right] dx = -i\hbar \frac{2}{L}\frac{2\pi}{L}\int_0^L \sin\left(\frac{2\pi x}{L}\right)\cos\left(\frac{2\pi x}{L}\right) dx = 0$$

$$\langle p^2 \rangle_{n=2} = \frac{2}{L}\int_0^L \sin\left(\frac{2\pi x}{L}\right)(-i\hbar)^2\frac{\partial^2}{\partial x^2}\left[\sin\left(\frac{2\pi x}{L}\right)\right] dx = \hbar^2 \frac{2}{L}\left(\frac{2\pi}{L}\right)^2 \int_0^L \sin^2\left(\frac{2\pi x}{L}\right) dx = \frac{4\pi^2\hbar^2}{L^2}$$

$$E_2 = \frac{4\pi^2\hbar^2}{2mL^2} = \frac{\langle p^2 \rangle_{n=2}}{2m}$$

Ex 6.9: Proton Transition Energy

A typical diameter of a nucleus is about 10^{-14}m. Use the infinite square-well potential to calculate the transition energy from the first excited state to the ground state for a proton confined to the nucleus.

The energy of the state n is $E_n = n^2 \frac{\pi^2 \hbar^2}{2mL^2}$

What is n for the ground state? n=1

$$E_1 = \frac{\pi^2 \hbar^2}{2mL^2} = \frac{\pi^2 \hbar^2 c^2}{2mc^2 L^2} = \frac{1}{mc^2} \frac{\pi^2 \cdot (197.3 eV \cdot nm)^2}{2 \cdot (10^{-5} nm)} = \frac{1.92 \times 10^{15} eV^2}{938.3 \times 10^6 eV} = 2.0 MeV$$

What is n for the 1st excited state? n=2

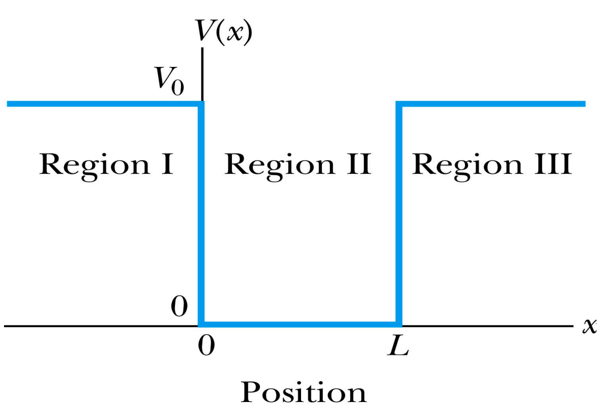
$$E_2 = 2^2 \frac{\pi^2 \hbar^2}{2mL^2} = 8.0 MeV$$

So the proton transition energy is

$$\Delta E = E_2 - E_1 = 6.0 MeV$$

Finite Square-Well Potential

- The finite square-well potential is $$V(x)=\begin{cases} V_0 & x\le 0, \\ 0 & 0<x<L \\ V_0 & x\ge L \end{cases}$$
- The Schrödinger equation outside the finite well in regions I and III is
$-\frac{\hbar^2}{2m}\frac{1}{\psi}\frac{d^2\psi}{dx^2}=E-V_0$ for regions I and III, or using $\alpha^2=2m(V_0-E)/\hbar^2$ yields $\frac{d^2\psi}{dx^2}=\alpha^2\psi$. The solution to this differential has exponentials of the form $e^{\alpha x}$ and $e^{-\alpha x}$. In the region x > L, we reject the positive exponential and in the region x < 0, we reject the negative exponential. Why?



$$\psi_I(x)=Ae^{\alpha x}\quad \text{region I}, x<0$$
$$\psi_{III}(x)=Ae^{-\alpha x}\quad \text{region III}, x>L$$

This is because the wave function should be 0 as x→infinity.

Finite Square-Well Solution

- Inside the square well, where the potential *V* is zero and the particle is free, the wave equation becomes $\frac{d^2\psi}{dx^2} = -k^2\psi$ where $k = \sqrt{2mE/\hbar^2}$
- Instead of a sinusoidal solution we can write

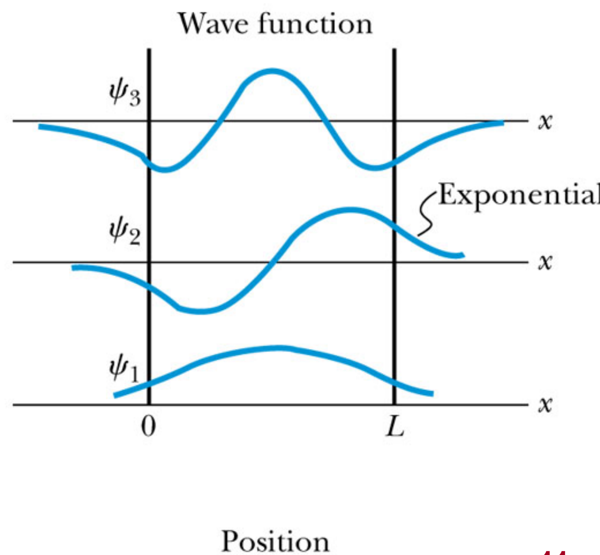
$$\psi_{II}(x) = Ce^{ikx} + De^{-ikx} \quad \text{region II, } 0 < x < L$$

- The boundary conditions require that

$$\psi_I = \psi_{II} \text{ at } x = 0 \text{ and } \psi_{II} = \psi_{III} \text{ at } x = L$$

and the wave function must be smooth where the regions meet.

- Note that the wave function is nonzero outside of the box.
- Non-zero at the boundary either..
- What would the energy look like?



Penetration Depth

- The penetration depth is the distance outside the potential well where the probability significantly decreases. It is given by

$$\delta x \approx \frac{1}{\alpha} = \frac{\hbar}{\sqrt{2m\left(V_0 - E\right)}}$$

- It should not be surprising to find that the penetration distance that violates classical physics is proportional to Planck's constant.