PHYS 3313 – Section 001
Lecture #5

Wednesday, Feb. 4, 2015

Dr. Jaehoon Yu

- Einstein's postulates
- Lorentz Transformations
- Time Dilation
- Length Contraction
- Relativistic Velocity Addition
- The Twin Paradox

Announcements

- Reading assignments: CH 2.10 (special topic), 2.13 and 2.14
  – Please go through eq. 2.45 through eq. 2.49 and example 2.9
- Reminder: Homework #1
  – chapter 2 end of the chapter problems
  – 17, 21, 23, 24, 32, 59, 61, 66, 68, 81 and 96
  – Due is by the beginning of the class, Monday, Feb. 9
  – Work in study groups together with other students but PLEASE do write your answer in your own way!

Conclusions of Michelson Experiment

- Michelson noted that he should be able to detect a phase shift of light due to the time difference between path lengths but found none.
- He thus concluded that the hypothesis of the stationary ether must be incorrect.
- After several repeats and refinements with assistance from Edward Morley (1893-1923), again *a null result.*
- ***Thus, ether does not seem to exist!***
- Many explanations ensued afterward but none worked out!
- This experiment conclusively shattered the popular belief of light being waves

The Lorentz-FitzGerald Contraction

- Another hypothesis proposed independently by both H. A. Lorentz and G. F. FitzGerald suggested that the length ℓ_1, in the direction of the motion was *contracted* by a factor of

$$\sqrt{1-v^2/c^2}$$

- Thus making the path lengths equal to account for the zero phase shift.
  - This, however, was an ad hoc assumption that could not be experimentally tested.

Einstein's Postulates

- Fundamental assumption: Maxwell's equations must be valid in all inertial frames
- **The principle of relativity**: The laws of physics are the same in all inertial systems. There is no way to detect absolute motion, and no preferred inertial system exists
  - Published a paper in 1905 at the age 26
  - Believed the principle of relativity to be fundamental
- **The constancy of the speed of light**: Observers in all inertial systems measure the same value for the speed of light in a vacuum.

The Lorentz Transformations

General linear transformation relationship between P=(x, y, z, t) in frame S and P'=(x',y',z',t') in frame S' ➔ these assume measurements are made in S frame and transferred to S' frame

- preserve the constancy of the speed of light between inertial observers
- account for the problem of simultaneity between these observers

$$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}} \quad y' = y \quad z' = z \quad t' = \frac{t - \left(vx/c^2\right)}{\sqrt{1 - v^2/c^2}}$$

- With the definitions $\beta \equiv v/c$ and $\gamma \equiv 1/\sqrt{1-\beta^2}$

$$x' = \gamma\left(x - \beta ct\right) \quad y' = y \quad z' = z \quad t' = \gamma\left(t - \beta x/c\right)$$

Properties of the Relativistic Factor γ

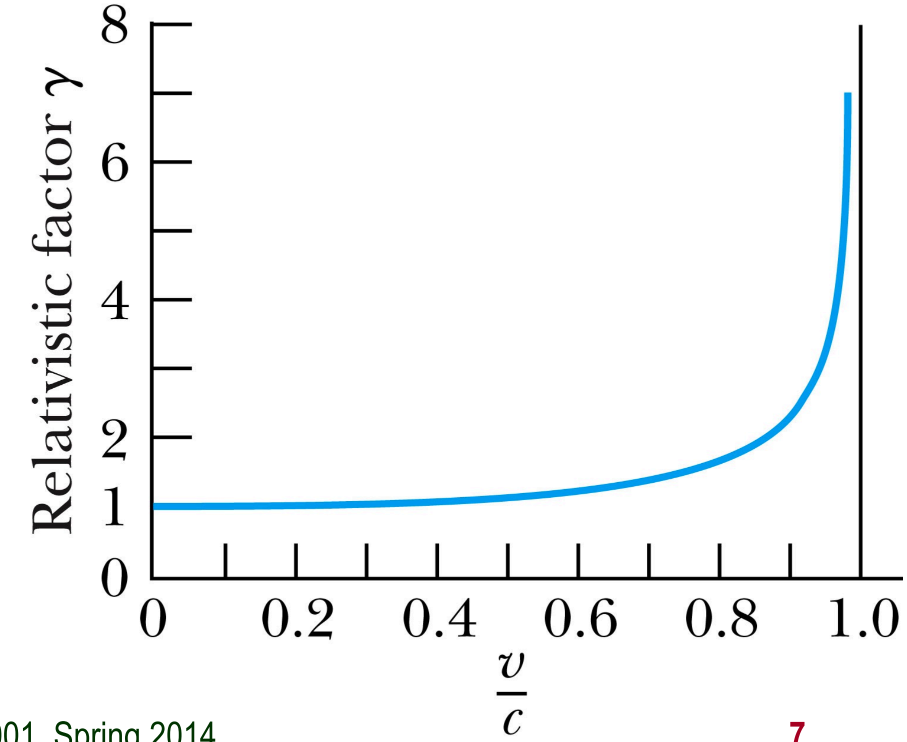
What is the property of the relativistic factor, γ?

Is it bigger or smaller than 1?

Recall Einstein's postulate, $\beta = v/c < 1$ for all observers

- $\gamma = 1$ only when $v = 0$

$$\gamma = 1\Big/\sqrt{1-\beta^2} \geq 1$$



The complete Lorentz Transformations

$$x' = \frac{x - vt}{\sqrt{1-\beta^2}} \qquad x = \frac{x' + vt'}{\sqrt{1-\beta^2}}$$

$$y' = y \qquad y = y'$$

$$z' = z \qquad z = z'$$

$$t' = \frac{t - \left(vx/c^2\right)}{\sqrt{1-\beta^2}} \qquad t = \frac{t' + \left(vx'/c^2\right)}{\sqrt{1-\beta^2}}$$

- Some things to note
  - What happens when β~0 (or v~0)?
    - The Lorentz x-formation becomes Galilean x-formation
  - Space-time are not separated
  - For non-imaginary x-formations, the frame speed cannot exceed c!

Time Dilation and Length Contraction

Direct consequences of the Lorentz Transformation:

- **Time Dilation**:

  Clocks in a moving inertial reference frame K' run slower with respect to stationary clocks in K.

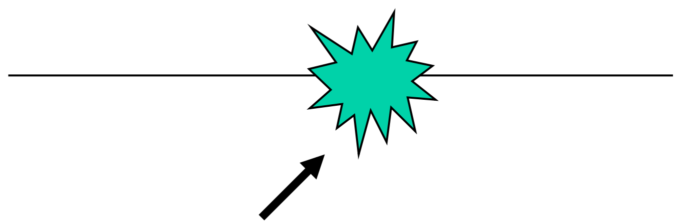
- **Length Contraction**:

  Lengths measured in a moving inertial reference frame K' are shorter with respect to the same lengths stationary in K.

Time Dilation

To understand *time dilation* the idea of **proper time** must be understood:

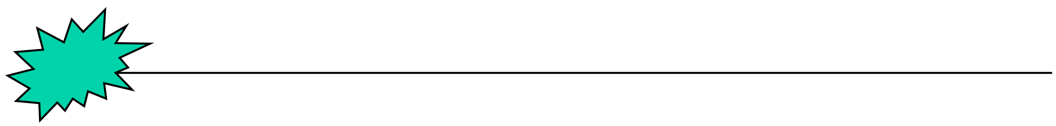
- **proper time**, T_0, is the time difference between two events occurring at the **same** position in a system as measured by **a clock at that position**.



Same location (spark "on" then off")

Time Dilation

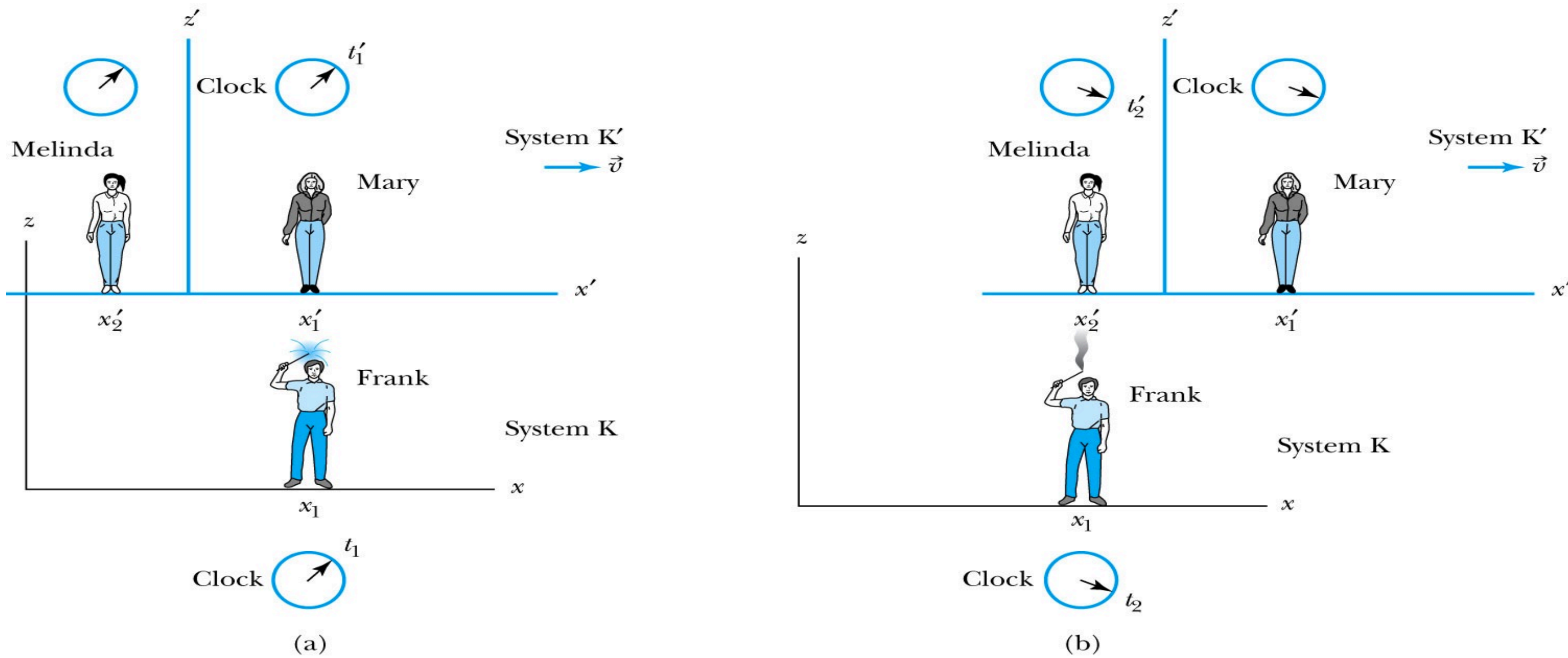
Is this a Proper Time?



spark "on" then spark "off"

Beginning and ending of the event occur at different positions

Time Dilation with Mary, Frank, and Melinda



- Frank's clock is at the same position in system K when the sparkler is lit in (a) ($t=t_1$) and when it goes out in (b) ($t=t_2$). ➔ The proper time $T_0=t_2-t_1$
- Mary, in the moving system K', is beside the sparkler when it was lit ($t=t_1'$)
- Melinda then moves into the position where and when the sparkler extinguishes ($t=t_2'$)
- Thus, Melinda, at the new position, measures the time in system K' when the sparkler goes out in (b).

According to Mary and Melinda…

- Mary and Melinda measure the two times for the sparkler to be lit and to go out in system K' as times $t_1'$and t_2' so that by the Lorentz transformation:

$$t'_2 - t'_1 = \frac{(t_1 - t_2) - (v/c^2)(x_1 - x_2)}{\sqrt{1-\beta^2}}$$

  - Note here that Frank records $x_2 - x_1 = 0$ in K with a proper time: $T_0 = t_2 - t_1$ or

$$T' = t'_2 - t'_1 = \frac{T_0}{\sqrt{1-\beta^2}} = \gamma T_0$$

Time Dilation: Moving Clocks Run Slow

1) $T' > T_0$ or the time measured between two events at *different positions* is greater than the time between the same events at *one position:* ***time dilation.***

The proper time is always the shortest time!!

2) The events do not occur at the same space and time coordinates in the two systems

3) System K requires 1 clock and K' requires 2 clocks.

Time Dilation Example: muon lifetime

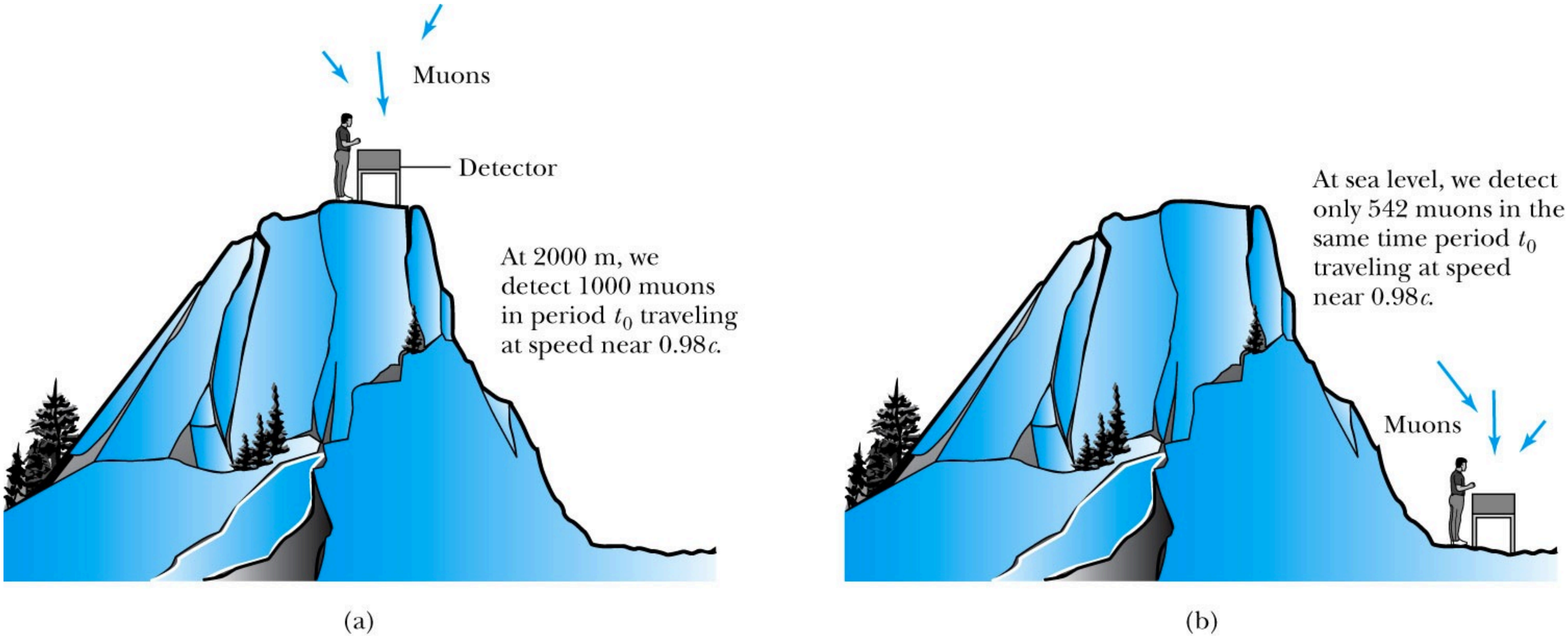
- Muons are essentially heavy electrons (~200 times heavier)
- Muons are typically generated in collisions of cosmic rays in upper atmosphere and, unlike electrons, decay ($t_0 = 2.2$ μsec)
- For a muon incident on Earth with v=0.998c, an observer on Earth would see what lifetime of the muon?
- 2.2 μsec?

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \approx 16$$

- t=35 μsec
- Moving clocks run slow so when an outside observer measures, they see a longer lifetime than the muon itself sees.

Experimental Verification of Time Dilation

Arrival of Muons on the Earth's Surface



The number of muons detected with speeds near 0.98*c* is much different (a) on top of a mountain than

(b) at sea level, because of the muon's decay. The experimental result agrees with our time dilation equation.

Length Contraction

To understand *length contraction* the idea of **proper length** must be understood:

- Let an observer in each system K and K' have a meter stick at rest in ***their own system*** such that each measures the same length at rest.
- The length as measured at rest at the same time is called the **proper length**.

Length Contraction cont'd

Each observer lays the stick down along his or her respective *x* axis, putting the left end at x_ℓ (or x'_ℓ) and the right end at x_r (or x'_r).

- Thus, in the rest frame K, Frank measures his stick to be:

$$L_0 = x_r - x_l$$

- Similarly, in the moving frame K', Mary measures her stick at rest to be:

$$L'_0 = x'_r - x'_l$$

- Frank in his rest frame measures the moving length in Mary's frame which is moving with velocity v.
- Thus using the Lorentz transformations Frank measures the length of the stick in K' as:

$$x'_r - x'_l = \frac{(x_r - x_l) - v(t_r - t_l)}{\sqrt{1-\beta^2}}$$

Where both ends of the stick must be measured simultaneously, i.e, $t_r = t_\ell$

Here Mary's proper length is $L'_0 = x'_r - x'_\ell$

and Frank's measured length is $L = x_r - x_\ell$

Measurement in Rest Frame

The observer in the rest frame measures the moving length as L given by

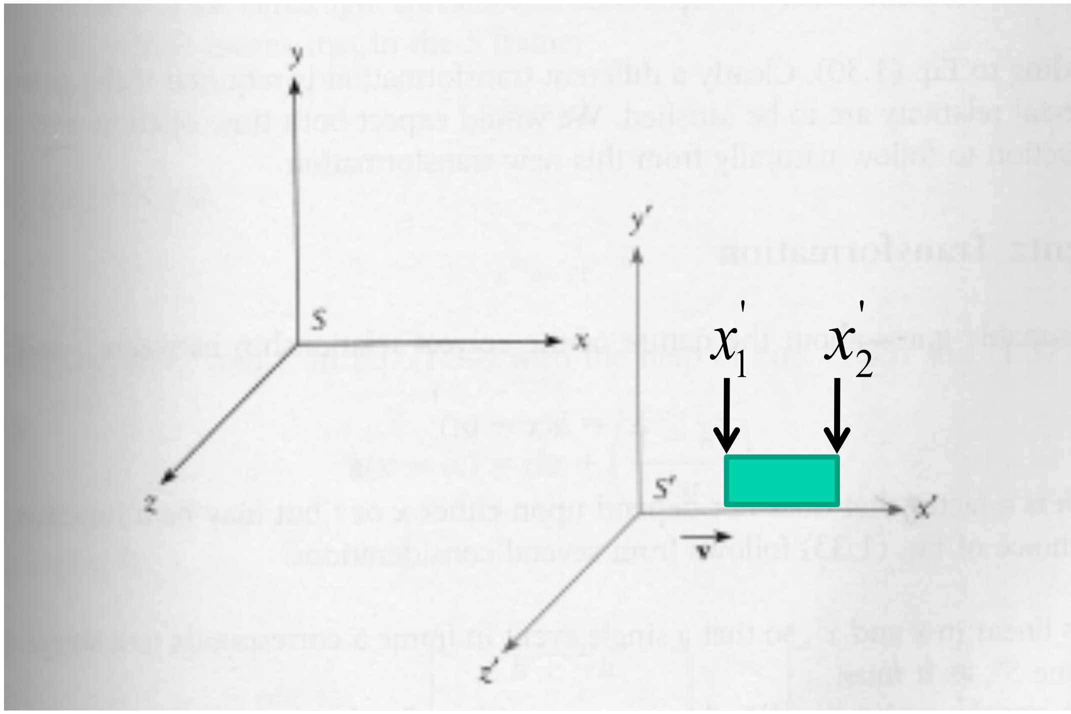
$$L'_0 = \frac{L}{\sqrt{1-\beta^2}} = \gamma L$$

but since both Mary and Frank in their respective frames measure $L'_0 = L_0$

$$L = L_0\sqrt{1-\beta^2} = \frac{L_0}{\gamma}$$

and $L_0 > L$, i.e. the moving stick shrinks

Length Contraction Summary



- Proper length (length of object in its own frame:

  $$L_0 = x_2' - x_1'$$

- Length of the object in observer's frame:

  $$L = x_2 - x_1$$

$$L_0' = L_0 = x_2' - x_1' = \gamma(x_2 - vt) - \gamma(x_1 - vt) = \gamma(x_2 - x_1)$$

$L_0 = \gamma L$ $\quad L = L_0 / \gamma$ $\quad$ Since $\gamma>1$, the length is shorter in the direction of motion (length contraction!)

More about Muons

- Rate: 1/cm^2/minute at Earth's surface (so for a person with 600 cm^2 surface area, the rate would be 600/60=10 muons/sec passing through the body!)
- They are typically produced in atmosphere about 6 km above surface of Earth and often have velocities that are a substantial fraction of speed of light, v=.998 c for example and lifetime of 2.2 μsec $vt_0 = 2.994\times 10^8 \frac{m}{\sec}\cdot 2.2\times 10^{-6}\sec = 0.66km$
- How do they reach the Earth if they only go 660 m and not 6000 m?
- The time dilation stretches life time to t=35 μsec not 2.2 μsec, thus they can travel 16 times further, or about 10 km, implying they easily reach the ground
- But riding on a muon, the trip takes only 2.2 μsec, so how do they reach the ground???
- Muon-rider sees the ground moving towards him, so the length he has to travel contracts and is only $L_0/\gamma = 6/16 = 0.38km$
- At 1000 km/sec, it would take 5 seconds to cross U.S., pretty fast, but does it give length contraction? $L = .999994L_0$ {not much contraction} (for v=0.9c, the length is reduced by 44%)