PHYS 3313 – Section 001
Lecture #8

Monday, Feb. 16, 2015
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- Relativistic Energy
- Relationship Between Relativistic Quantities
- Binding Energy
- Quantization
- Discovery of the X-ray and the Electron

Announcements

- Reading assignments: CH3.9
- Homework #2
  - CH3 end of the chapter problems: 2, 19, 27, 36, 41, 47 and 57
  - Due Wednesday, Feb. 25
- Quiz #2 Monday, Feb. 23
  - Beginning of the class
  - Covers CH1.1 – what we finish this this Wednesday, Feb. 18

How do we keep momentum conserved in a relativistic case?

Redefine the classical momentum in the form:

$$\vec{p} = \Gamma(u) m\vec{u} = \frac{1}{\sqrt{1-u^2/c^2}} m\vec{u}$$

This $\Gamma(u)$ is different than the γ factor since it uses the particle's speed u

➔ What? How does this make sense?

➔ Well the particle itself is moving at a relativistic speed, thus that must impact the measurements by the observer in the rest frame!!

Now, the agreed form of the momentum in all frames is (τ is the proper time):

$$\vec{p} = m\frac{d\vec{r}}{d\tau} = m\frac{d\vec{r}}{dt}\frac{dt}{d\tau} = m\vec{u}\gamma = \frac{1}{\sqrt{1-u^2/c^2}} m\vec{u}$$

Resulting in the new relativistic definition of the momentum: $\vec{p} = m\gamma\vec{u}$

When u➔0, this formula becomes that of the classical.

What can the speed **u** be to maintain the accuracy of the classical momentum to 1%?

Relativistic Energy

- Due to the new idea of relativistic mass, we must now redefine the concepts of work and energy.
  - Modify Newton's second law to include our new definition of linear momentum, and force becomes:

$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d}{dt}\left(\gamma m\vec{u}\right) = \frac{d}{dt}\left(\frac{m\vec{u}}{\sqrt{1-u^2/c^2}}\right)$$

- The work *W* done by a force **F** to move a particle from rest to a certain kinetic energy is

$$W = K = \int \frac{d}{dt}\left(\gamma m\vec{u}\right)\cdot\vec{u}dt$$

- Resulting relativistic kinetic energy becomes

$$K = \int_0^{\gamma u} um \cdot d\left(\gamma u\right) = \gamma mc^2 - mc^2 = \left(\gamma - 1\right)mc^2$$

- Why doesn't this look anything like the classical KE?

Big note on Relativistic KE

- Only $K = (\gamma - 1)mc^2$ is right!

- $K = \frac{1}{2}mu^2$ and $K = \frac{1}{2}\gamma mu^2$ are wrong!

Total Energy and Rest Energy

Rewriting the relativistic kinetic energy:

$$\gamma mc^2 = \frac{mc^2}{\sqrt{1-u^2/c^2}} = K + mc^2$$

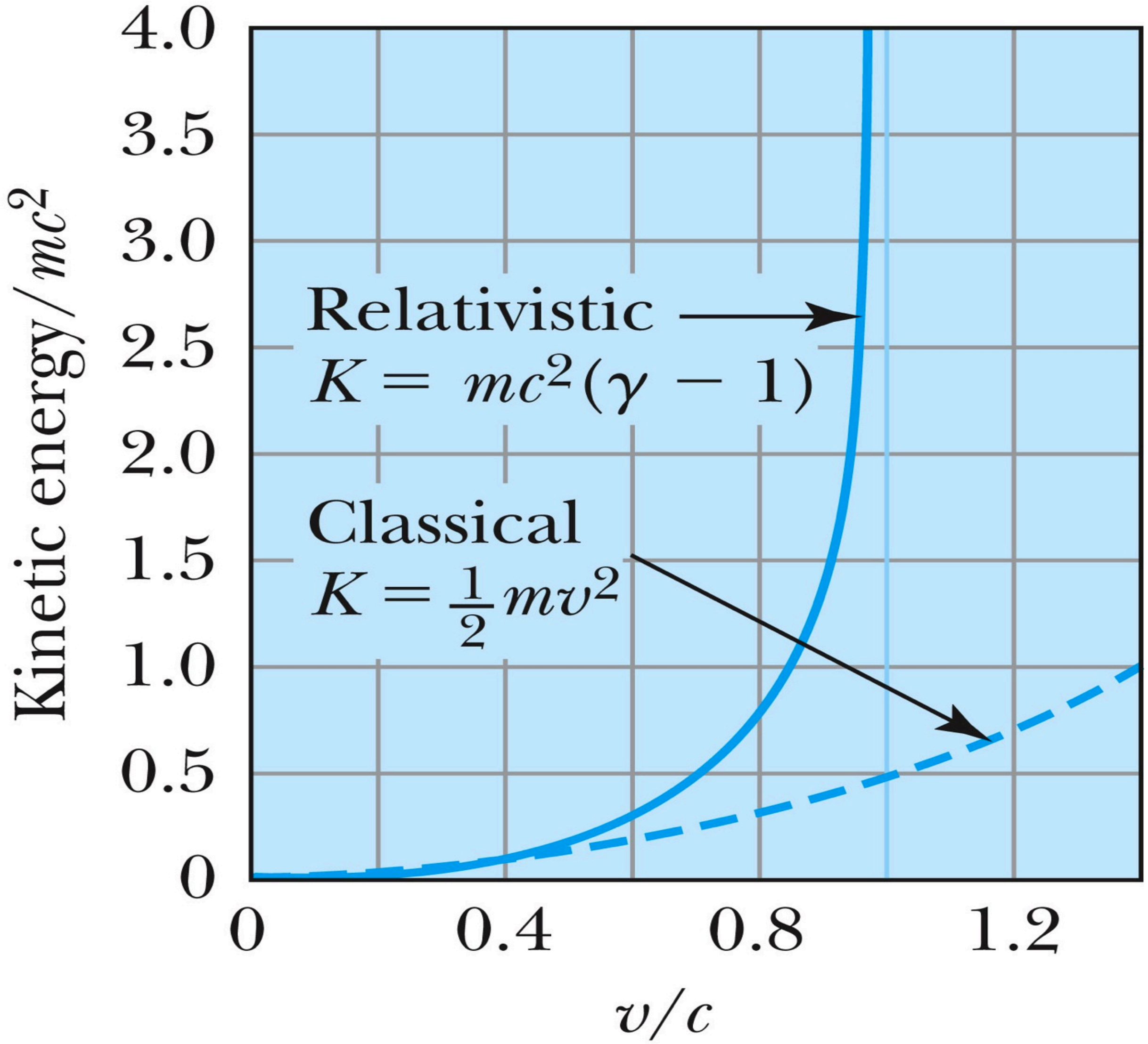
The term mc^2 is called the rest energy and is denoted by E_0.

$$E_0 = mc^2$$

The sum of the kinetic energy and rest energy is interpreted as the total energy of the particle. (note that u is the speed of the particle)

$$E_{Tot} = \gamma mc^2 = \frac{mc^2}{\sqrt{1-u^2/c^2}} = \frac{E_0}{\sqrt{1-u^2/c^2}} = K + E_0$$

Relativistic and Classical Kinetic Energies



Relationship of Energy and Momentum

$$p = \gamma m u = \frac{mu}{\sqrt{1 - u^2/c^2}}$$

We square this formula, multiply by c^2, and rearrange the terms.

$$p^2c^2 = \gamma^2 m^2 u^2 c^2 = \gamma^2 m^2 c^4 \left(\frac{u^2}{c^2}\right) = \gamma^2 m^2 c^4 \beta^2$$

$$\beta^2 = 1 - \frac{1}{\gamma^2} \Rightarrow p^2c^2 = \gamma^2 m^2 c^4 \left(1 - \frac{1}{\gamma^2}\right) = \gamma^2 m^2 c^4 - m^2 c^4$$

Rewrite

$$p^2c^2 = E^2 - E_0^2$$

Rewrite

$$E^2 = p^2c^2 + E_0^2 = p^2c^2 + m^2c^4$$

Massless Particles have a speed equal to the speed of light c

- Recall that a photon has "zero" rest mass and the equation from the last slide reduces to: E = pc and we may conclude that:

$$E = \gamma mc^2 = pc = \gamma muc$$

- Thus the speed, u, of a massless particle must be c since, as $m \to 0$, $\gamma \to \infty$ and it follows that: u = c.

Units of Work, Energy and Mass

- The work done in accelerating a charge through a potential difference V is $W = qV$.
  - For a proton, with the charge $e = 1.602 \times 10^{-19}$ C being accelerated across a potential difference of 1 V, the work done is

    $$1\ \text{eV} = 1.602 \times 10^{-19}\ \text{J}$$

    $$W = (1.602 \times 10^{-19})(1\ \text{V}) = 1.602 \times 10^{-19}\ \text{J}$$

- eV is also used as a unit of energy.

Other Units

1) Rest energy of a particle:
   Example: Rest energy, E_0, of proton

$$E_0(proton) = m_p c^2 = \left(1.67\times10^{-27}\,kg\right)\cdot\left(3.00\times10^{8}\,m/s\right) = 1.50\times10^{-10}\,J$$

$$= 1.50\times10^{-10}\,J\cdot\frac{1eV}{1.602\times10^{-19}\,J} = 9.38\times10^{8}\,eV$$

2) **Atomic mass unit** (amu): Example: carbon-12

$$M\left({}^{12}C\ \text{atom}\right) = \frac{12\,g/mole}{6.02\times10^{23}\,atoms/mole}$$

$$= 1.99\times10^{-23}\,g/atom$$

$$M\left({}^{12}C\ \text{atom}\right) = 1.99\times10^{-26}\,kg/atom = 12\,u/atom$$

What is 1u in eV?

Binding Energy

- The potential energy associated with the force keeping a system together ➔ E_B.
- The difference between the rest energy of the individual particles and the rest energy of the combined bound system.

$$M_{\text{bound system}}c^2 + E_B = \sum_i m_i c^2$$

$$E_B = \sum_i m_i c^2 - M_{\text{bound system}}c^2$$

Examples 2.13 and 2.15

- Ex. 2.13: A proton with 2-GeV kinetic energy hits another proton with 2 GeV KE in a head on collision. (proton rest mass = $938 MeV/c^2$)
  - Compute v, β, p, K and E for each of the initial protons
  - What happens to the kinetic energy?
- Ex. 2.15: What is the minimum kinetic energy the protons must have in the head-on collision in the reaction $p+p \rightarrow \pi^{+}+d$, in order to produce the positively charged pion ($139.6 MeV/c^2$) and a deuteron.($1875.6 MeV/c^2$).