PHYS 3313 – Section 001
Lecture #11

Monday, March 2, 2015
Dr. Jaehoon Yu

- Compton Effect
- Pair production/Pair annihilation
- Rutherford Scattering Experiment and Rutherford Atomic Model
- The Classic Atomic Model
- The Bohr Model of the Hydrogen Atom

Announcements

- Midterm Exam
  - In class this Wednesday, March. 4
  - Covers from CH1.1 through what we learn today (CH4.1) plus the math refresher in the appendices
  - Mid-term exam constitutes 20% of the total
  - **Please do NOT miss the exam! You will get an F if you miss it.**
  - BYOF: You may bring a one 8.5x11.5 sheet (front and back) of handwritten formulae and values of constants for the exam
  - No derivations, word definitions or solutions of any problems !
  - No additional formulae or values of constants will be provided!
- Colloquium at 4pm Wednesday in SH101
  - Dr. Jinyi Qi from UC Davis on Ultra Llow does PET imaging

Reminder: Special Project #3

- A total of N_i incident projectile particle of atomic number Z_1 kinetic energy KE scatter on a target of thickness t and atomic number Z_2 and has n atoms per volume. What is the total number of scattered projectile particles at an angle θ? (20 points)
- Please be sure to clearly define all the variables used in your derivation! Points will be deducted for missing variable definitions.
- This derivation must be done on your own. Please do not copy the book, internet or your friends'.
- Due is Wednesday, March 18

Prefixes, expressions and their meanings

- deca (da): 10^{1}
- hecto (h): 10^{2}
- kilo (k): 10^{3}
- mega (M): 10^{6}
- giga (G): 10^{9}
- tera (T): 10^{12}
- peta (P): 10^{15}
- exa (E): 10^{18}

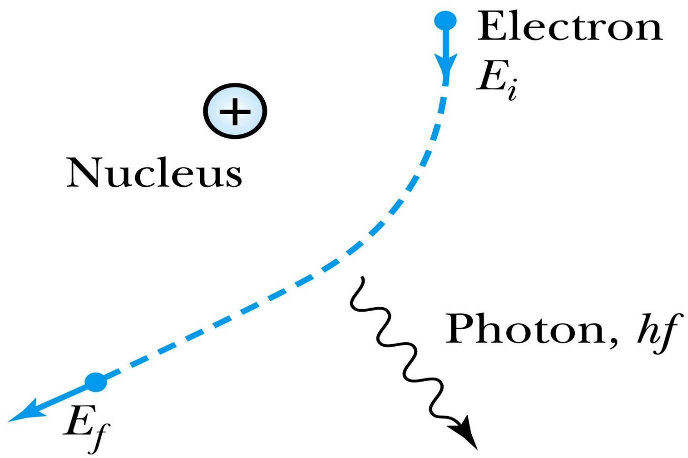
- deci (d): 10^{-1}
- centi (c): 10^{-2}
- milli (m): 10^{-3}
- micro (μ): 10^{-6}
- nano (n): 10^{-9}
- pico (p): 10^{-12}
- femto (f): 10^{-15}
- atto (a): 10^{-18}

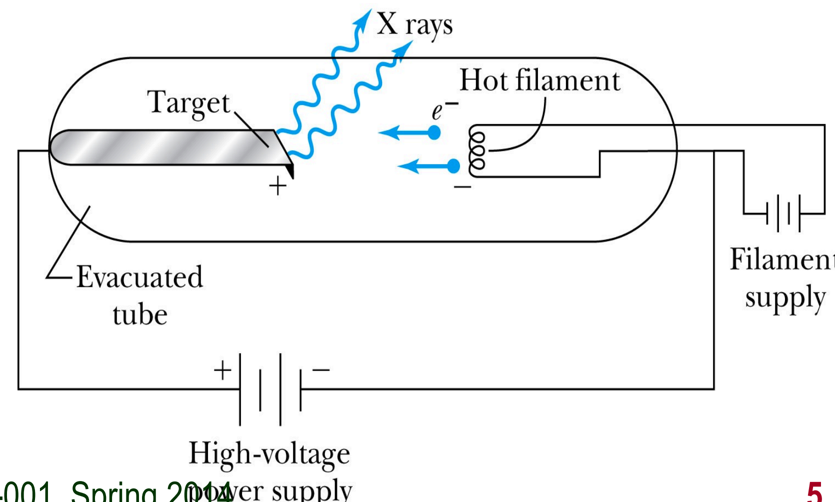
X-Ray Production

- **Bremsstrahlung** (German word for braking radiation): Radiation of a photon from an energetic electron passing through matter due to an acceleration
- Since linear momentum must be conserved, the nucleus absorbs very little energy, and it is ignored. The final energy of the electron is determined from the conservation of energy

$$E_f = E_i - hf$$

- An electron that loses a large amount of energy will produce an X-ray photon.
  - Current passing through a filament produces copious numbers of electrons by thermionic emission.
  - These electrons are focused by the cathode structure into a beam and are accelerated by potential differences of thousands of volts until they impinge on a metal anode surface, producing x rays by bremsstrahlung as they stop in the anode material
  - X-ray wavelengths range 0.01 – 10nm. What is the minimum energy of an electron to produce X-ray?



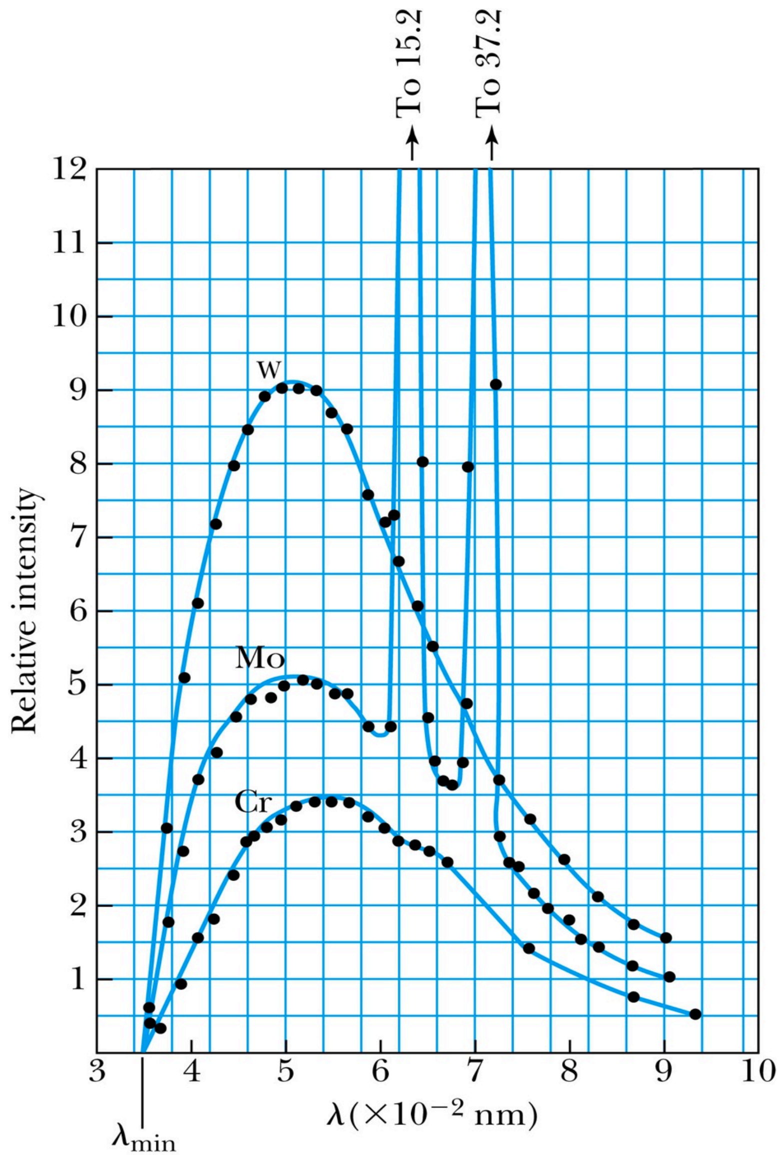


Inverse Photoelectric Effect.

- Conservation of energy requires that the electron KE equals the maximum photon energy
  - Work function neglected since it's small compared to the potential energy of the electron.
- This is the **Duane-Hunt limit**
  - The photon wavelength depends only on the accelerating voltage
  - The same for all targets.

$$eV_0 = hf_{\max} = \frac{hc}{\lambda_{\min}}$$

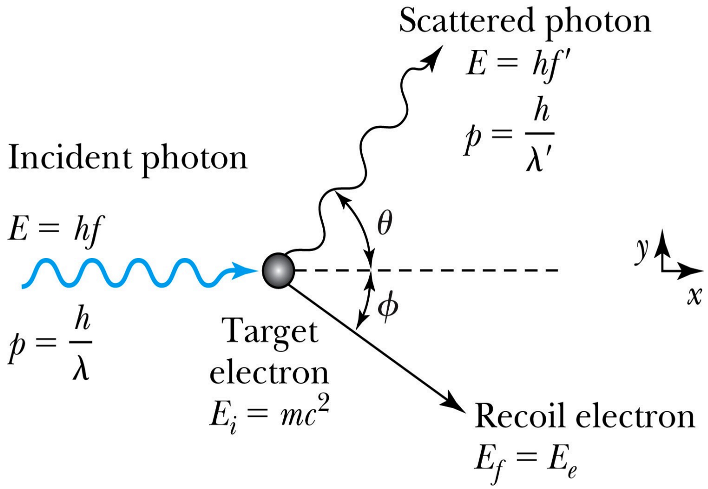
$$\lambda_{\min} = \frac{hc}{eV_0} = \frac{1.24\times10^{-6}\,V\cdot m}{V_0}$$



Compton Effect

- When a photon enters matter, it is likely to interact with one of the atomic electrons.
- The photon is scattered from only one electron
- The laws of conservation of energy and momentum apply as in any elastic collision between two particles. The momentum of a particle moving at the speed of light is

$$p = \frac{E}{c} = \frac{hf}{c} = \frac{h}{\lambda}$$



- The electron energy can be written as

$$E_e^2 = \left(m_e c^2\right)^2 + p_e^2 c^2$$

- Change of the scattered photon wavelength is known as the **Compton effect**:

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_e c}\left(1 - \cos\theta\right)$$

Pair Production and Annihilation

- If a photon can create an electron, it must also create a positive charge to balance charge conservation.
- In 1932, C. D. Anderson observed a positively charged electron (e^+) in cosmic radiation. This particle, called the positron, had been predicted to exist several years earlier by P. A. M. Dirac.
- A photon's energy can be converted entirely into an electron and a positron in a process called the **pair production**.
  - Can only happen inside a material
  - How much energy do you think is needed?

$$\gamma \rightarrow e^- + e^+$$

Pair Production in Empty Space?

- Energy conservation for pair production in empty space

$$hf = E_+ + E_- + K.E.$$

- Momentum conservation yields

$$hf = p_- c\cos\theta + p_+ c\cos\theta$$

- Thus max momentum exchange $hf_{\max} = p_- c + p_+ c$
- Recall that the total energy for a particle can be written as

$$E_\pm^2 = p_\pm^2 c^2 + m_e^2 c^4$$

However this yields a contradiction: $hf > p_- c + p_+ c$

Hence the conversion of energy in empty space is impossible and thus pair production cannot happen in empty space

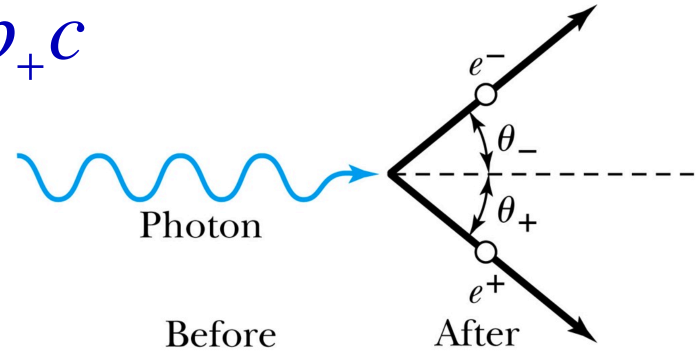
Pair Production in Matter

- Since the relations $hf_{max} = p_-c + p_+c$ and $hf > p_-c + p_+c$ contradict each other, a photon can not produce an electron and a positron in empty space.
- In the presence of matter, however, the nucleus absorbs some energy and momentum.

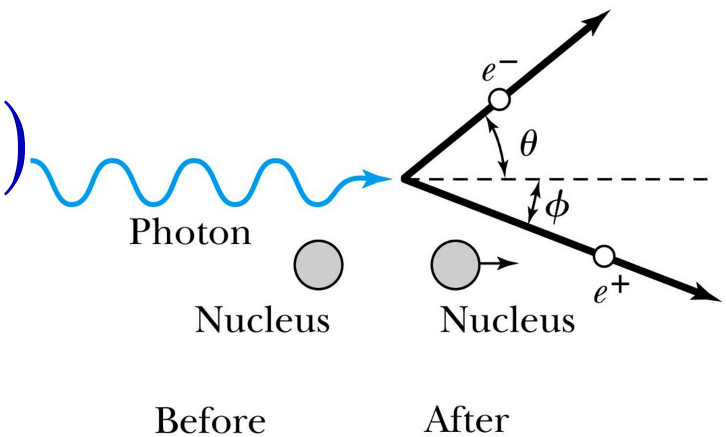
$$hf = E_- + E_+ + K.E.(\text{nucleus})$$

- The photon energy required for pair production in the presence of matter is

$$hf > 2m_ec^2$$



(a) Free space (**cannot occur**)



(b) Beside nucleus

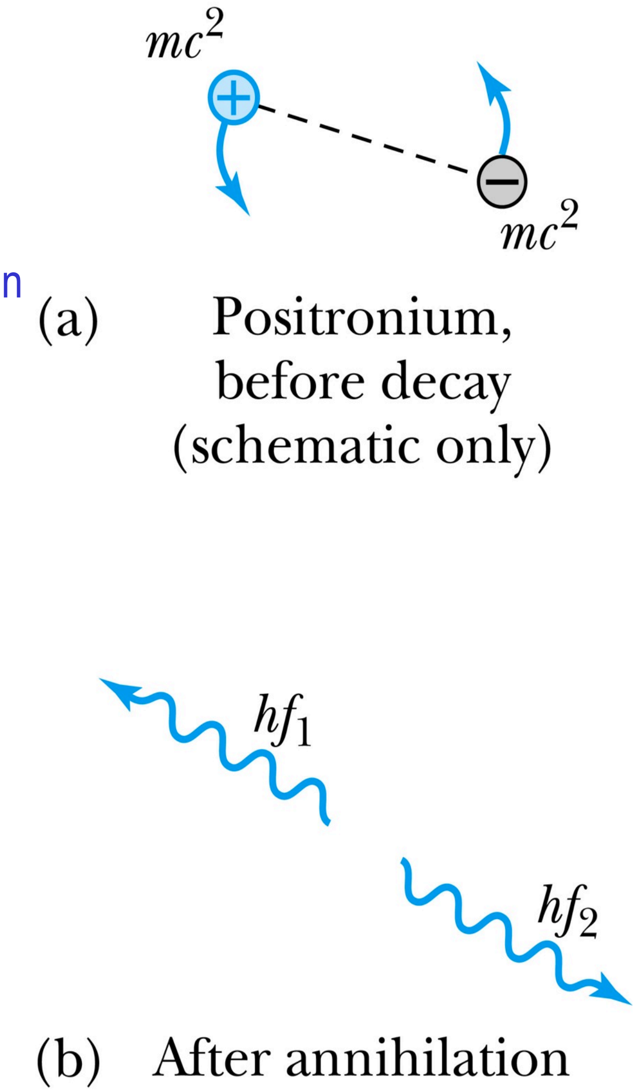
Pair Annihilation

- A positron going through matter will likely **annihilate** with an electron.
- A positron is drawn to an electron and form an atom-like configuration called **positronium**.
- Pair annihilation in empty space will produce two photons to conserve momentum. Annihilation near a nucleus can result in a single photon.
- Conservation of energy: $2m_e c^2 \approx hf_1 + hf_2$
- Conservation of momentum: $0 = \frac{hf_1}{c} - \frac{hf_2}{c}$
- The two photons will be almost identical, so that

$$f_1 = f_2 = f$$

- The two photons from a positronium annihilation will move in the opposite directions with an energy of:

$$hf = m_e c^2 = 0.511\text{MeV}$$

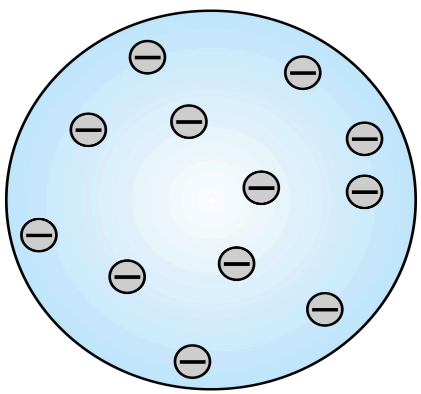


The Atomic Models of Thomson and Rutherford

- Without seeing it, 19th century scientists believed atoms have structure.
- Pieces of evidence that scientists had in 1900 to indicate that the atom was not a fundamental unit
- There are simply too many kinds of atoms (~70 known at that time), belonging to a distinct chemical element
  - Too many to be fundamental!!
- Atoms and electromagnetic phenomena seem to be intimately related
- The issue of **valence** → Why certain elements combine with some elements but not with others?
  - Is there a characteristic internal atomic structure?
- The discoveries of radioactivity, x rays, and the electron

Thomson's Atomic Model

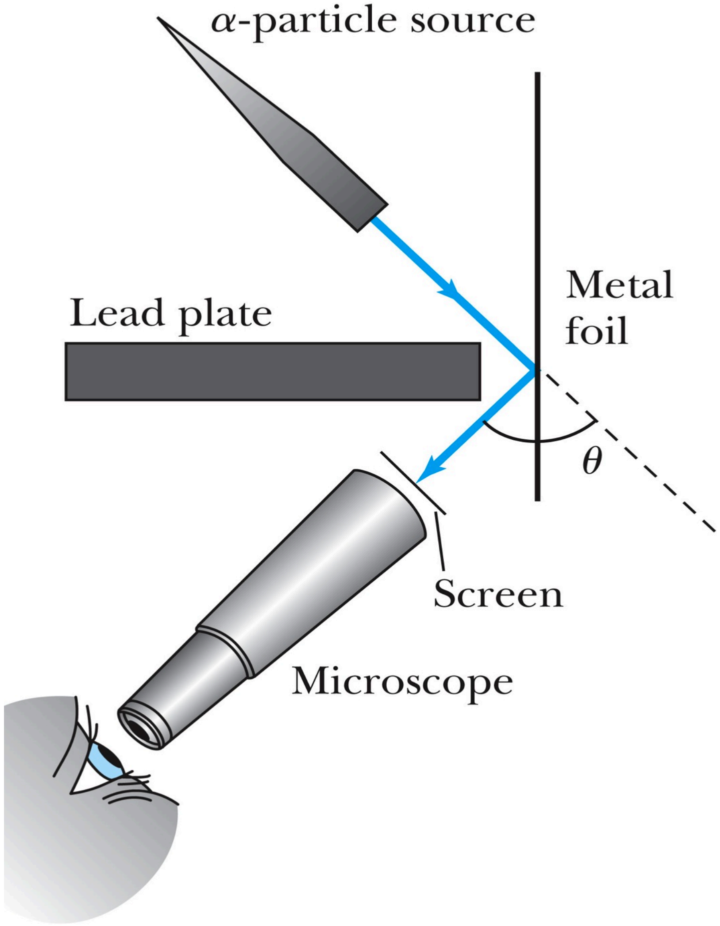
- Thomson's "plum-pudding" model
  - Atoms are electrically neutral and have electrons in them
  - Atoms must have an equal amount of positive charges in it to balance electron negative charges
  - So how about positive charges spread uniformly throughout a sphere the size of the atom with the newly discovered "negative" electrons embedded in a uniform background.



- Thomson thought when the atom was heated the electrons could vibrate about their equilibrium positions and thus produce electromagnetic radiation.

Experiments of Geiger and Marsden

- Rutherford, Geiger, and Marsden conceived a new technique for investigating the structure of matter by scattering a particle off atoms.
- Geiger showed that many particles were scattered from thin gold-leaf targets at backward angles greater than 90°.
- Time to do some calculations!



Ex 4.1: Maximum Scattering Angle

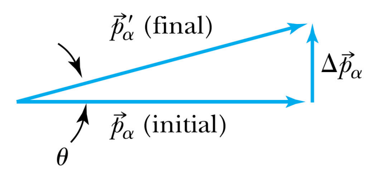
Geiger and Marsden (1909) observed backward-scattered (θ>=90°) α particles when a beam of energetic α particles was directed at a piece of gold foil as thin as 6.0×10^{-7}m. Assuming an α particle scatters from an electron in the foil, what is the maximum scattering angle?

- The maximum scattering angle corresponds to the maximum momentum change
- Using the momentum conservation and the KE conservation for an elastic collision, the maximum momentum change of the α particle is in a head-on collision

$$M_\alpha \vec{v}_\alpha = M_\alpha \vec{v}'_\alpha + m_e \vec{v}'_e$$
$$\frac{1}{2}M_\alpha v_\alpha^2 = \frac{1}{2}M_\alpha v_\alpha'^2 + \frac{1}{2}m_2 v_e^2$$

$$\Rightarrow \Delta\vec{p}_\alpha = M_\alpha \vec{v}_\alpha - M_\alpha \vec{v}'_\alpha = m_e \vec{v}'_e \Rightarrow \Delta p_{\alpha-\max} = 2m_e v_\alpha$$

- Determine θ by letting $\Delta p_{\max}$ be perpendicular to the direction of motion.



$$\theta_{\max} = \frac{\Delta p_{\alpha-\max}}{p_\alpha} = \frac{2m_e v_\alpha}{m_\alpha v_\alpha} = \frac{2m_e}{m_\alpha} = 2.7\times10^{-4}\,rad = 0.016^\circ$$

Multiple Scattering from Electrons

- If an α particle were scattered by many electrons, then *N* electrons results in $\langle\theta\rangle_{total} \sim \sqrt{N}*\theta$
- The number of atoms across a thin gold layer of 6×10^{-7} m:

$$\frac{N_{Molecules}}{cm^3} = N_{Avogadro}\left(molecules/mol\right)\times\left[\frac{1}{g-molecular-weight}\left(\frac{mol}{g}\right)\right]\cdot\left[\rho\left(\frac{g}{cm^3}\right)\right]$$

$$= 6.02\times10^{23}\left(\frac{molecules}{mol}\right)\cdot\left(\frac{1mol}{197g}\right)\cdot\left(19.3\frac{g}{cm^3}\right)$$

$$= 5.9\times10^{22}\frac{molecules}{cm^3} = 5.9\times10^{28}\frac{atoms}{m^3}$$

- Assume the distance between atoms is $d = \left(5.9\times10^{28}\right)^{-1/3} = 2.6\times10^{-10}\left(m\right)$

  and there are $N = \dfrac{6\times10^{-7}m}{2.6\times10^{-10}m} = 2300\left(atoms\right)$

  This gives $\langle\theta\rangle_{total} = \sqrt{2300}\left(0.016^\circ\right) = 0.8^\circ$

Rutherford's Atomic Model

- $\langle\theta\rangle_{total} \sim 0.8*79=6.8^{o}$ even if the α particle scattered from all 79 electrons in each atom of gold

- The experimental results were inconsistent with Thomson's atomic model.
- Rutherford proposed that an atom has a positively charged core (nucleus) surrounded by the negative electrons.