

PHYS 3313 – Section 001

Lecture #14

Wednesday, March 25, 2015

Dr. Jaehoon Yu

- Characteristic X-ray Spectra
- Hydrogen Spectrum Series
- X-ray Scattering
- Bragg's Law
- de Broglie Waves
- Bohr's Quantization Conditions
- Electron Scattering

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Announcements

- Reminder: Homework #3
 - End of chapter problems on CH4: 5, 14, 17, 21, 23 and 45
 - Due: Monday, March 30
- Quiz Wednesday, April 1
 - At the beginning of the class
 - Covers CH4.1 through what we finish Monday, March 30
 - BYOF with the same rule as before
- Colloquium at 4pm today in SH101



Characteristic X-Ray Spectra and Atomic Number

- Shells have letter names:

K shell for $n = 1$

L shell for $n = 2$

⋮

- The atom is most stable in its ground state.

→ An electron from higher shells will fill the inner-shell vacancy at lower energy.

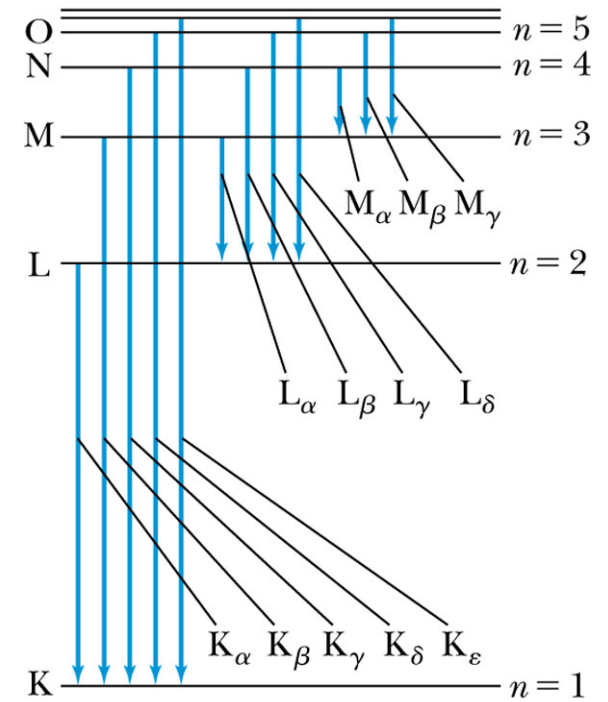
- When a transition occurs in a heavy atom, the radiation emitted is an **x ray**. ($\lambda=0.01 - 10\text{nm}$)
- It has the energy $E (\text{x ray}) = E_u - E_\ell$.



Atomic Number

L shell to K shell $\longrightarrow$ K_{α} X ray

M shell to K shell $\longrightarrow$ K_{β} X ray



- Atomic number Z = number of protons in the nucleus
- Moseley found a relationship between the frequencies of the characteristic X ray and Z .

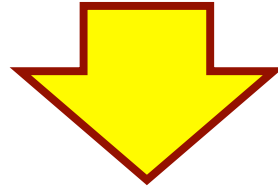
This holds for the K_{α} X ray

$$f_{K_{\alpha}} = \frac{3cR}{4} (Z - 1)^2$$

Moseley's Empirical Results

- The X ray is produced from $n = 2$ to $n = 1$ transition.
- In general, the K series of X ray wavelengths are

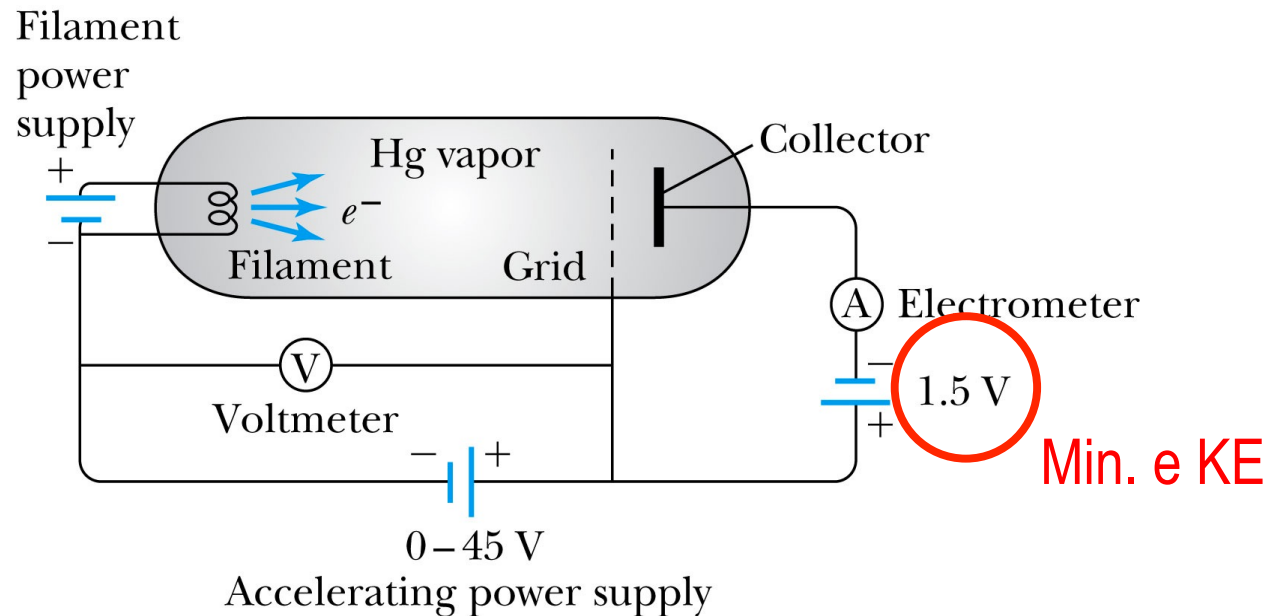
$$\frac{1}{\lambda_K} = R(Z-1)^2 \left(\frac{1}{1^2} - \frac{1}{n^2} \right) = R(Z-1)^2 \left(1 - \frac{1}{n^2} \right)$$



- Moseley's research clarified the importance of the electron shells for all the elements, not just for hydrogen
 - Concluded correctly that atomic number Z , rather than the atomic weight, is the determining factor in ordering of the periodic table

Atomic Excitation by Electrons

- Franck and Hertz studied the phenomenon of ionization KE transfer from electrons to atoms.



When the accelerating voltage is below 5 V

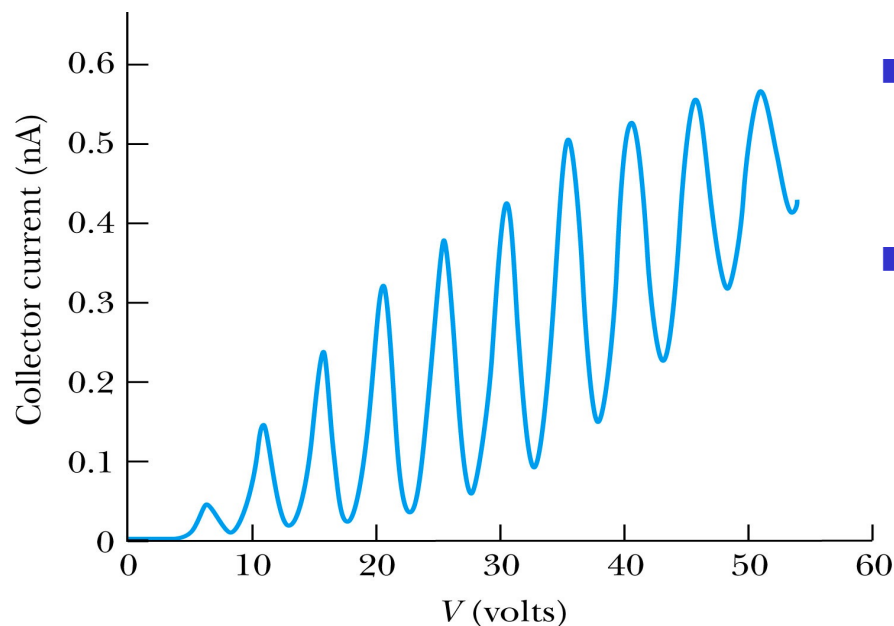
→ electrons did not lose energy going through the mercury vapor

When the accelerating voltage is above 5 V, 10V, etc..

→ sudden drop in the current

Atomic Excitation by Electrons

- Ground state has E_0 which can be considered as 0.
First excited state has E_1 .
The energy difference $E_1 - 0 = E_1$ is the **excitation energy**.

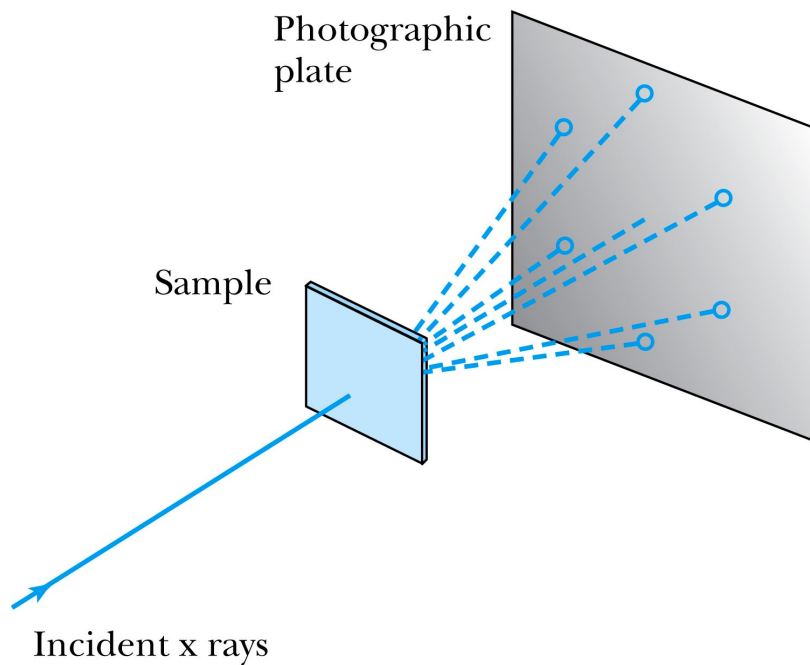


- Hg (mercury) has an excitation energy of 4.88 eV in the first excited state
- No energy can be transferred to Hg below 4.88 eV because not enough energy is available to excite an electron to the next energy level

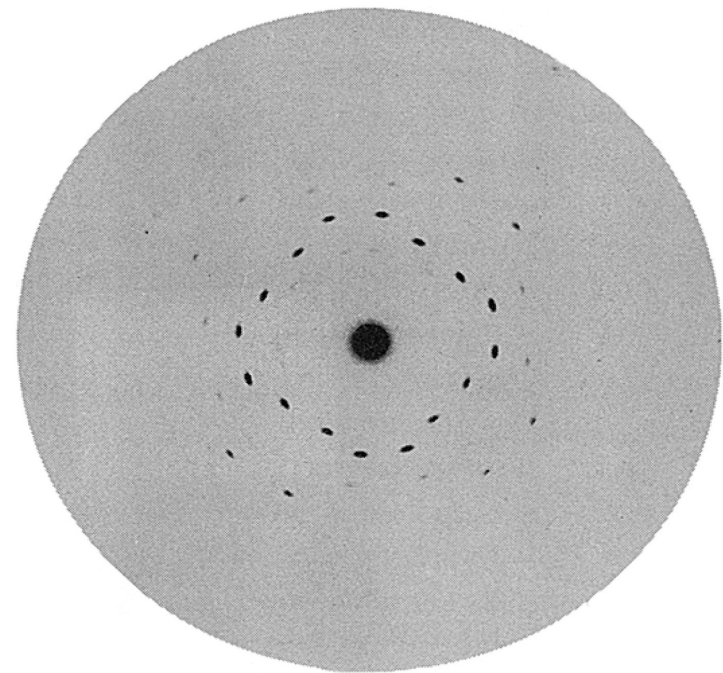
- Above 4.88 eV, the current drops because scattered electrons no longer reach the collector until the accelerating voltage reaches 9.76 eV and so on.

X-Ray Scattering

- Max von Laue suggested that if X rays were a form of electromagnetic radiation, interference effects should be observed. (Wave property!!)
- Crystals act as three-dimensional gratings, scattering the waves and producing observable interference effects.



(a)



(b)

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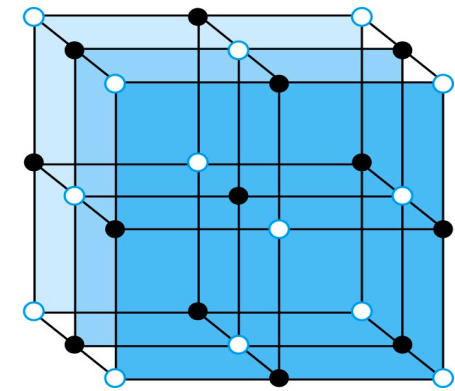
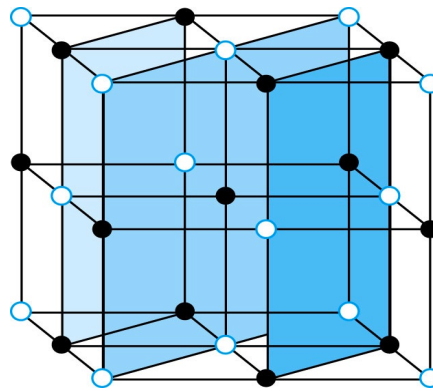


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Bragg's Law

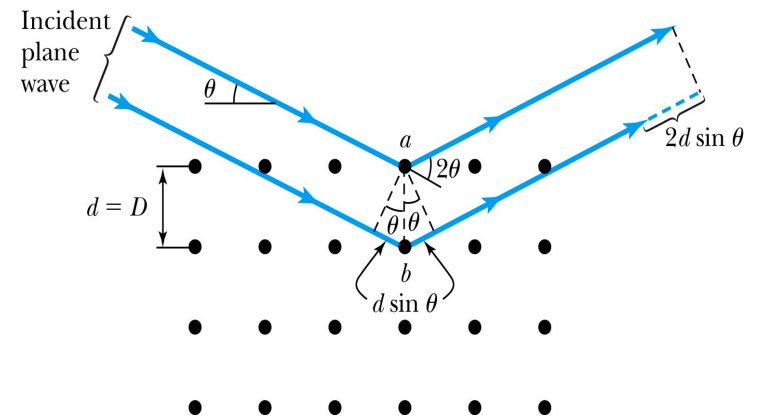
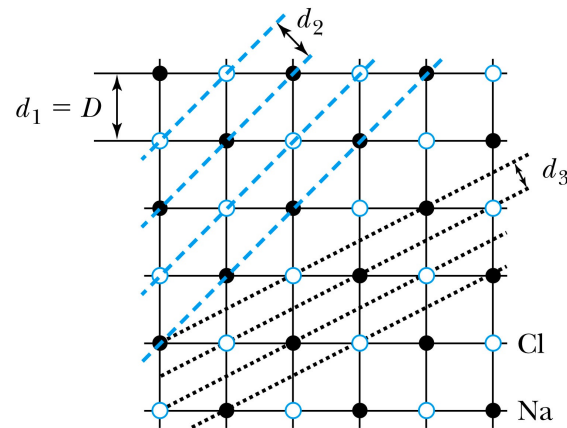
- William Lawrence Bragg interpreted the x-ray scattering as the reflection of the incident X-ray beam from a unique set of planes of atoms within the crystal.
- There are two conditions for constructive interference of the scattered X rays:

- The angle of incidence must equal the angle of reflection of the outgoing wave. (total reflection)
- The difference in path lengths between two rays must be an integral number of wavelengths.



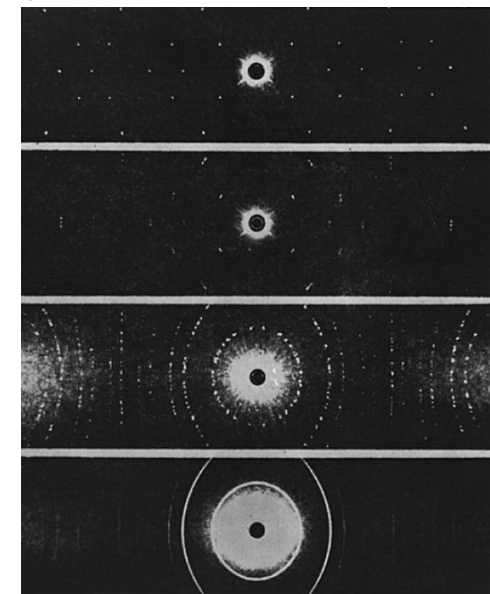
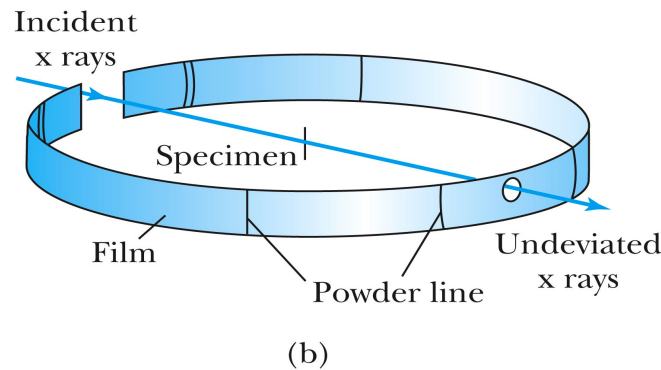
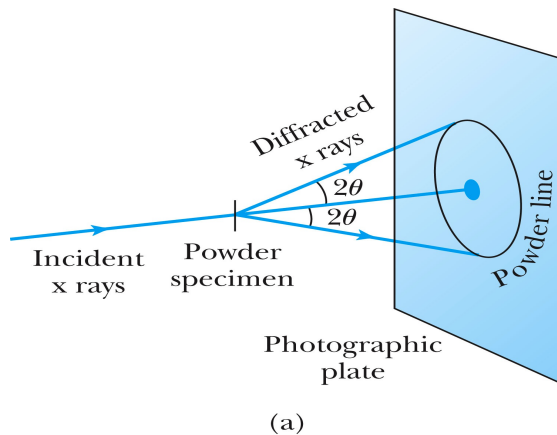
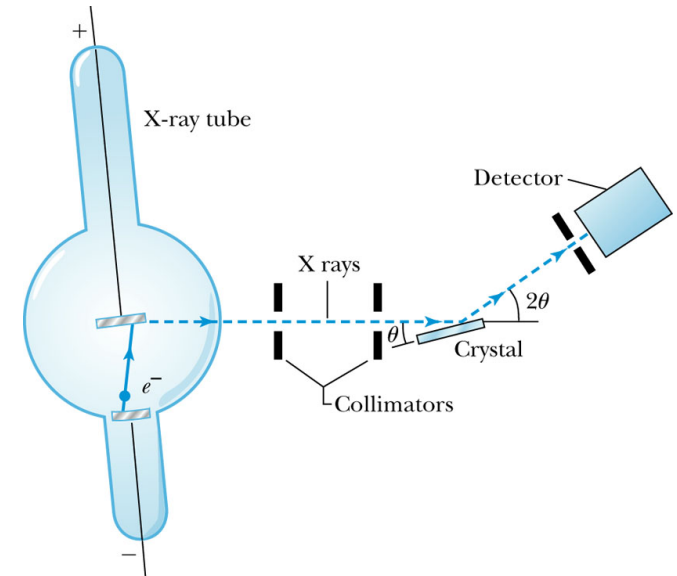
- Bragg's Law:**

$$n\lambda = 2d \sin \theta$$
 $(n = \text{integer})$



The Bragg Spectrometer

- A Bragg spectrometer scatters X rays from several crystals. The intensity of the diffracted beam is determined as a function of scattering angle by rotating the crystal and the detector.
- When a beam of X rays passes through the powdered crystal, the dots become a series of rings due to random arrangement.



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(c)

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Ex 5.1: Bragg's Law

X rays scattered from rock salt (NaCl) are observed to have an intense maximum at an angle of 20° from the incident direction. Assuming $n=1$ (from the intensity), what must be the wavelength of the incident radiation?

- Bragg's law: $n\lambda = 2d \sin \theta$
- *What do we need to know to use this? The lattice spacing d !*
- *We know $n=1$ and $2\theta=20^\circ$.*
- *We use the density of NaCl to find out what d is.*

$$\frac{N_{\text{molecules}}}{V} = \frac{N_A \rho_{\text{NaCl}}}{M} = \frac{(6.02 \times 10^{23} \text{ molecules/mol}) \cdot (2.16 \text{ g/cm}^3)}{58.5 \text{ g/mol}} =$$

$$= 2.22 \times 10^{22} \frac{\text{molecules}}{\text{cm}^3} = 4.45 \times 10^{22} \frac{\text{atoms}}{\text{cm}^3} = 4.45 \times 10^{28} \frac{\text{atoms}}{\text{m}^3}$$

$$\frac{1}{d^3} = 4.45 \times 10^{28} \frac{\text{atoms}}{\text{m}^3} \quad \Rightarrow \quad d = \frac{1}{\sqrt[3]{4.45 \times 10^{28}}} = 0.282 \text{ nm}$$

$$\lambda = 2d \sin \theta = 2 \cdot 0.282 \cdot \sin 10^\circ = 0.098 \text{ nm}$$

De Broglie Waves

- Louis V. de Broglie suggested that mass particles should have wave properties similar to electromagnetic radiation → many experiments supported this!
- Thus the wavelength of a matter wave is called the de Broglie wavelength:

$$\lambda = \frac{h}{p}$$

- This can be considered as the probing beam length scale
- Since for a photon, $E = pc$ and $E = hf$, the energy can be written as

$$E = hf = pc = p\lambda f$$

Ex 5.2: De Broglie Wavelength

Calculate the De Broglie wavelength of (a) a tennis ball of mass 57g traveling 25m/s (about 56mph) and (b) an electron with kinetic energy 50eV.

- What is the formula for De Broglie Wavelength? $\lambda = \frac{h}{p}$
- (a) for a tennis ball, $m=0.057\text{kg}$.

$$\lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34}}{0.057 \cdot 25} = 4.7 \times 10^{-34} \text{ m}$$

- (b) for electron with 50eV KE, since KE is small, we can use non-relativistic expression of electron momentum!

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2m_e K}} = \frac{hc}{\sqrt{2m_e c^2 K}} = \frac{1240 \text{ eV} \cdot \text{nm}}{\sqrt{2 \cdot 0.511 \text{ MeV} \cdot 50 \text{ eV}}} = 0.17 \text{ nm}$$

- What are the wavelengths of you running at the speed of 2m/s? What about your car of 2 metric tons at 100mph? How about the proton with 14TeV kinetic energy?
- What is the momentum of the photon from a green laser ($\lambda=532\text{nm}$)?



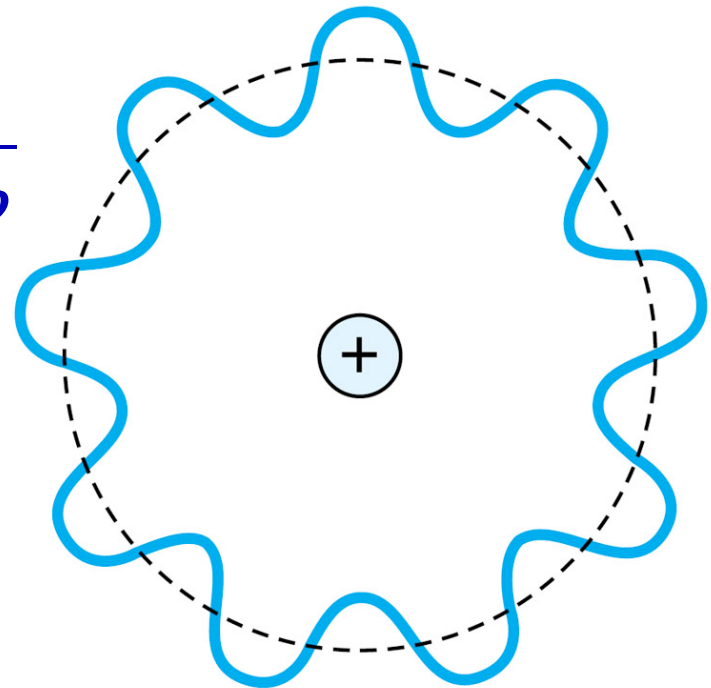
Bohr's Quantization Condition

- One of Bohr's assumptions concerning his hydrogen atom model was that the angular momentum of the electron-nucleus system in a stationary state is an integral multiple of $h/2\pi$.
- The electron is a standing wave in an orbit around the proton. This standing wave will have nodes and be an integral number of wavelengths.

$$2\pi r = n\lambda = n \frac{h}{p} \Rightarrow r = \frac{nh}{2\pi p}$$

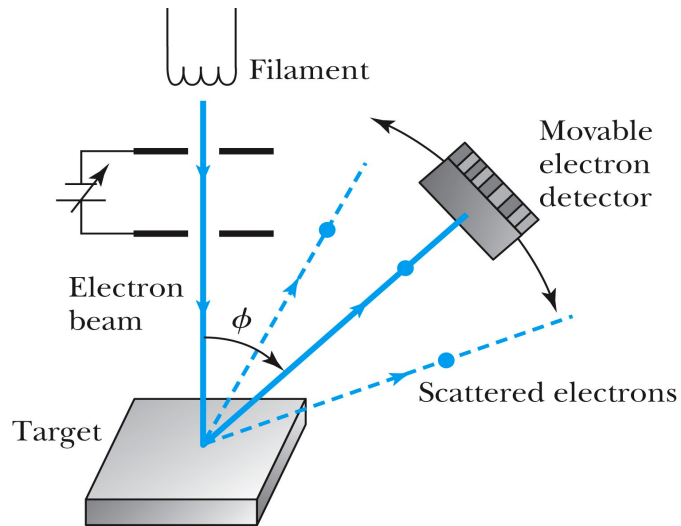
- The angular momentum becomes:

$$L = rp = \frac{nh}{2\pi p} p = n \frac{h}{2\pi} = n\hbar$$

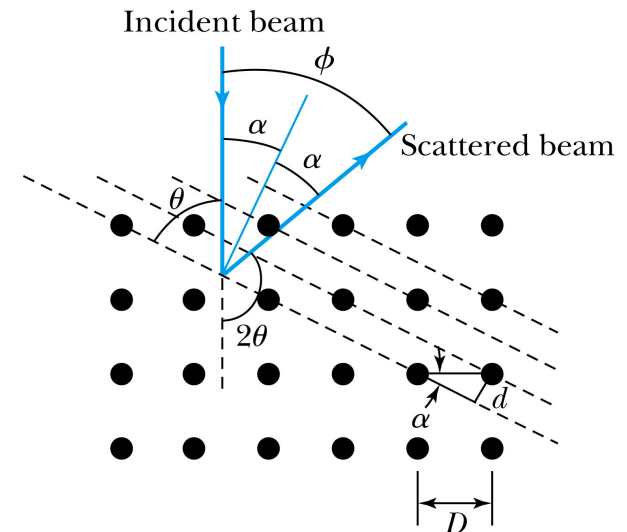


Electron Scattering

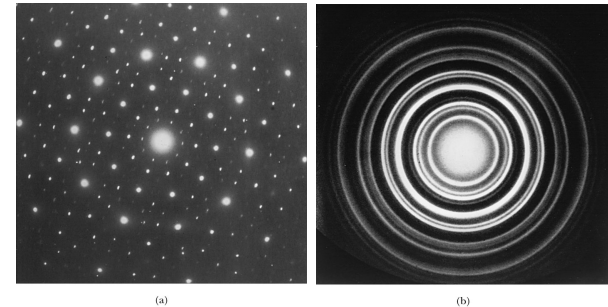
- Davisson and Germer experimentally observed that electrons were diffracted much like X rays in nickel crystals. → direct proof of de Broglie wave!



$$\lambda = \frac{D \sin \phi}{n}$$



- George P. Thomson (1892–1975), son of J. J. Thomson, reported seeing the effects of electron diffraction in transmission experiments. The first target was celluloid, and soon after that gold, aluminum, and platinum were used. The randomly oriented polycrystalline sample of SnO_2 produces rings as shown in the figure at right.



- Photons, which we thought were waves, act particle like (eg Photoelectric effect or Compton Scattering)
- Electrons, which we thought were particles, act wave particle like (eg electron scattering)
- de Broglie: All matter has intrinsic wavelength.
 - Wavelength is inversely proportional to momentum
 - The more massive... the smaller the wavelength... the harder to observe the wavelike properties
 - So while photons appear mostly wavelike, electrons (next lightest particle!) appear mostly particle like.
- How can we reconcile the wave/particle views?

