# PHYS 3313 – Section 001
# Lecture #18

*Wednesday, April 8, 2015*
*Dr. Jaehoon Yu*

- Expectation Values
- Momentum Operator
- Position and Energy Operators
- Infinite Square-well Potential

# Announcements

- Quiz 3 results
  - Class average: 28.5/60
    - Equivalent to: 47.5/100
    - Previous quizzes: 23.5/100 and 46.5/100
  - Top score: 56/60
- Reminder: Homework #4
  - End of chapter problems on CH5: 8, 10, 16, 24, 26, 36 and 47
  - Due Monday, Apr. 13
- Quiz #4 at the beginning of the class Monday, Apr. 13
  - Covers CH 5.4 through what we finish today
- Colloquium 4pm tomorrow, Thursday, SH101
  - Professor Francis Halzen of U. of Wisconsin, Madison
    - Don't miss the refreshment at 3:30pm in physics lounge

# Special project #5

- Show that the Schrodinger equation becomes Newton's second law in the classical limit. (15 points)
- Deadline Wednesday, Apr. 15, 2015
- You MUST have your own answers!

# Reminder: Research Project Report

1. Must contain the following at the minimum
   – Original theory or Original observation
   – Experimental proofs or Theoretical prediction + subsequent experimental proofs
   – Importance and the impact of the theory/experiment
   – Conclusions
   – The reference to the original paper must be included!
   – Bibliography referring to web site must be minimized (<20%)
2. Each member of the group writes a 10 (max) page report, including figures
   – 10% of the total grade
   – Can share the theme and facts but you must write your own!
   – Text of the report must be your original!
   – ***Due Mon., May 4, 2015***

# Research Topic and Group Assignments

| Group Number | Reseasrch Group Members | Research Topic # & Title |
|---|---|---|
| 1 | N. Lira, C. Maxey, N Smith | Black-body Radiation |
| 2 | N. Armstrong, E. Hicks, C. Sherrill | Michelson-Morley Experiment |
| 3 | P. Clark, R. Martinez, B. Sankey | The Photo-Electric Effect |
| 4 | R. Curley, M. Escoto, N. Hergenrother, D. Pierce | The Brownian Motion |
| 5 | F. Algamdhi, M Cherry, W. Slaven | Compton Effect |
| 6 | J. Andree, K. Imam, S. Park, | Discovery of Electron |
| 7 | C. X Beary, M Fofana, S. Kernaghan, A. Singh, V. X Garza | Rutherford Scattering |
| 8 | A. Contreras, A. Crawford, B. Lam, B. Rodriguez | Super-Conductivity |
| 9 | L. De la Rosa, J. Miller, H. Venable | The Discovery of Radioactivity |

# Research Presentations

- Each of the 9 research groups makes a 10min presentation
  - 10min presentation + 5min Q&A
  - All presentations must be in power point
  - I must receive all final presentation files by 8pm, Sunday, May 4, 2015
    - No changes are allowed afterward
  - The representative of the group makes the presentation followed by all group members' participation in the Q&A session
- Date and time:
  - In class Monday, May 4 or Wednesday, May 6
- Important metrics
  - Contents of the presentation: 60%
    - Inclusion of all important points as mentioned in the report
    - The quality of the research and making the right points
  - Quality of the presentation itself: 15%
  - Presentation manner: 10%
  - Q&A handling: 10%
  - Staying in the allotted presentation time: 5%
  - Judging participation and sincerity: 5%

# Expectation Values

- The **expectation value** is the expected result of the average of many measurements of a given quantity. The expectation value of *x* is denoted by <*x*>.
- Any measurable quantity for which we can calculate the expectation value is called the **physical observable**. The expectation values of physical observables (for example, position, linear momentum, angular momentum, and energy) must be real, because the experimental results of measurements are real.
- The average value of *x* is $\bar{x} = \frac{N_1 x_1 + N_2 x_2 + N_3 x_3 + N_4 x_4 + \cdots}{N_1 + N_2 + N_3 + N_4 + \cdots} = \frac{\sum_i N_i x_i}{\sum_i N_i}$

# Continuous Expectation Values

- We can change from discrete to continuous variables by using the probability *P*(*x*,*t*) of observing the particle at the particular *x*.

$$\bar{x} = \frac{\int_{-\infty}^{+\infty} xP(x)\,dx}{\int_{-\infty}^{+\infty} P(x)\,dx}$$

- Using the wave function, the expectation value is:

$$\bar{x} = \frac{\int_{-\infty}^{+\infty} x\Psi(x,t)^{*}\,\Psi(x,t)\,dx}{\int_{-\infty}^{+\infty} \Psi(x,t)^{*}\,\Psi(x,t)\,dx}$$

- The expectation value of any function *g*(*x*) for a normalized wave function:

$$\langle g(x)\rangle = \int_{-\infty}^{+\infty} \Psi(x,t)^{*}\,g(x)\Psi(x,t)\,dx$$

# Momentum Operator

- To find the expectation value of *p*, we first need to represent *p* in terms of *x* and *t*. Consider the derivative of the wave function of a free particle with respect to *x*:

$$\frac{\partial \Psi}{\partial x} = \frac{\partial}{\partial x}\left[e^{i(kx-\omega t)}\right] = ike^{i(kx-\omega t)} = ik\Psi$$

With $k = p/\hbar$ we have $\frac{\partial \Psi}{\partial x} = i\frac{p}{\hbar}\Psi$

This yields $p\left[\Psi(x,t)\right] = -i\hbar\frac{\partial \Psi(x,t)}{\partial x}$

- This suggests we define the momentum operator as $\hat{p} = -i\hbar\frac{\partial}{\partial x}$
- The expectation value of the momentum is

$$\langle p\rangle = \int_{-\infty}^{+\infty}\Psi^*(x,t)\hat{p}\Psi(x,t)\,dx = -i\hbar\int_{-\infty}^{+\infty}\Psi^*(x,t)\frac{\partial \Psi(x,t)}{\partial x}dx$$

# Position and Energy Operators

- The position *x* is its own operator as seen above.
- The time derivative of the free-particle wave function is
$$\frac{\partial \Psi}{\partial t} = \frac{\partial}{\partial t}\left[e^{i(kx-\omega t)}\right] = -i\omega e^{i(kx-\omega t)} = -i\omega\Psi$$
Substituting $\omega = E/\hbar$ yields $E\left[\Psi(x,t)\right] = i\hbar\frac{\partial \Psi(x,t)}{\partial t}$

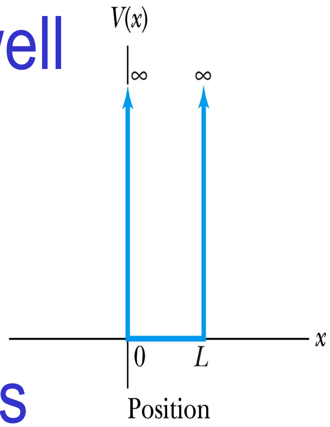
- So the energy operator is $\hat{E} = i\hbar\frac{\partial}{\partial t}$
- The expectation value of the energy is

$$\langle E\rangle = \int_{-\infty}^{+\infty}\Psi^*(x,t)\hat{E}\Psi(x,t)dx = i\hbar\int_{-\infty}^{+\infty}\Psi^*(x,t)\frac{\partial \Psi(x,t)}{\partial t}dx$$

# Infinite Square-Well Potential

- The simplest such system is that of a particle trapped in a box with infinitely hard walls that the particle cannot penetrate. This potential is called an infinite square well and is given by

$$V(x)=\begin{cases}\infty & x\le 0, x\ge L\\ 0 & 0<x<L\end{cases}$$




- The wave function must be zero where the potential is infinite.
- Where the potential is zero inside the box, the time independent Schrödinger wave equation $-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2}+V(x)\psi(x)=E\psi(x)$ becomes $\frac{d^2\psi}{dx^2}=-\frac{2mE}{\hbar^2}\psi \quad =-k^2\psi$ where $k=\sqrt{2mE/\hbar^2}$.
- The general solution is $\psi(x)=A\sin kx+B\cos kx$.

# Quantization

- Since the wave function must be continuous, the boundary conditions of the potential dictate that the wave function must be zero at $x = 0$ and $x = L$. These yield valid solutions for B=0, and for **integer values** of $n$ such that $kL = n\pi$ ➔ $k=n\pi/L$
- The wave function is now $$\psi_n(x) = A\sin\left(\frac{n\pi x}{L}\right)$$
- We normalize the wave function $\int_{-\infty}^{+\infty} \psi_n^*(x)\psi_n(x)dx = 1$

$$A^2\int_{-\infty}^{+\infty} \sin^2\left(\frac{n\pi x}{L}\right)dx = A^2\int_0^L \sin^2\left(\frac{n\pi x}{L}\right)dx = 1$$

- The normalized wave function becomes

$$\psi_n(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi x}{L}\right)$$

- These functions are identical to those obtained for a vibrating string with fixed ends.