PHYS 3313 – Section 001
Lecture # 20

Wednesday, April 15, 2015
Dr. Jaehoon Yu

- Finite Square Well Potential
- Penetration Depth
- Degeneracy
- Simple Harmonic Oscillator
- Barriers and Tunneling
- Alpha Particle Decay

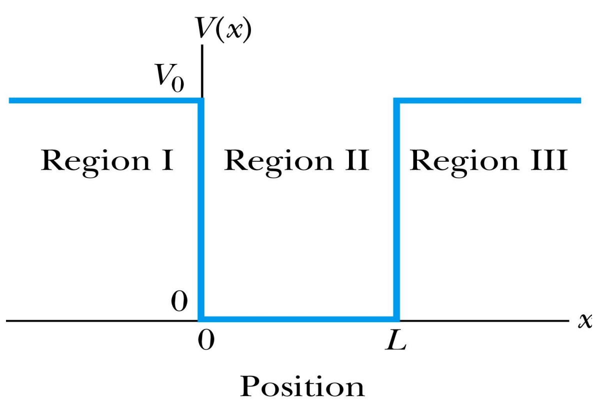
Announcements

- Research paper deadline is Monday, May 4
- Research presentation deadline is Sunday, May 3
- Homework #5
  - CH6 end of chapter problems: 34, 39, 46, 62 and 65
  - Due Wednesday, Apr. 22
- Reading assignments
  - CH7.6 and the entire CH8
- Bring out the special project #5
- No Colloquium today

Group NUMBER	Reseasrch Group Members	Research Topic # & Title	Presentation Date and ORDER
1	N. Lira, C. Maxey, N Smith	Black-body Radiation	May 6 - 1
2	N. Armstrong, E. Hicks, C. Sherrill	Michelson-Morley Experiment	May 6 - 2
3	P. Clark, R. Martinez, B. Sankey	The Photo-Electric Effect	May 6 - 3
4	R. Curley, M. Escoto, N. Hergenrother, D. Pierce	The Brownian Motion	May 4 - 2
5	F. Algamdhi, M Cherry, W. Slaven	Compton Effect	May 4 - 1
6	J. Andree, K. Imam, S. Park,	Discovery of ELECTRON	May 4 - 5
7	C. Beary, M Fofana, S. Kernaghan, A. Singh, V. Garza	Rutherford Scattering	May 6 - 4
8	A. Contreras, A. Crawford, B. Lam, B. Rodriguez	Super-Conductivity	May 4 - 4
9	L. De la Rosa, J. Miller, H. Venable	The Discovery of Radioactivity	May 4 - 3

Finite Square-Well Potential

- The finite square-well potential is
$$V(x) = \begin{cases} V_0 & x \le 0, \\ 0 & 0 < x < L \\ V_0 & x \ge L \end{cases}$$
- The Schrödinger equation outside the finite well in regions I and III is
$-\frac{\hbar^2}{2m}\frac{1}{\psi}\frac{d^2\psi}{dx^2} = (E - V_0)$ for regions I and III, or using $\alpha^2 = 2m(V_0 - E)/\hbar^2$
yields $\frac{d^2\psi}{dx^2} = \alpha^2\psi$. The solution to this differential has exponentials of the form $e^{\alpha x}$ and $e^{-\alpha x}$. In the region x > L, we reject the positive exponential and in the region x < 0, we reject the negative exponential. Why?



$$\psi_I(x) = Ae^{\alpha x} \quad \text{region I}, x < 0$$

$$\psi_{III}(x) = Ae^{-\alpha x} \quad \text{region III}, x > L$$

This is because the wave function should be 0 as x→±infinity.

Finite Square-Well Solution

- Inside the square well, where the potential *V* is zero and the particle is free, the wave equation becomes $\frac{d^2\psi}{dx^2} = -k^2\psi$ where $k = \sqrt{2mE/\hbar^2}$
- Instead of a sinusoidal solution we can write

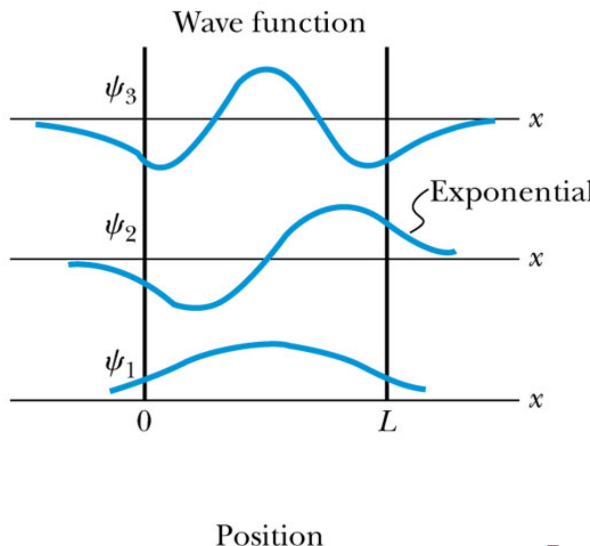
$$\psi_{II}(x) = Ce^{ikx} + De^{-ikx} \quad \text{region II, } 0<x<L$$

- The boundary conditions require that

$$\psi_I = \psi_{II} \text{ at } x = 0 \text{ and } \psi_{II} = \psi_{III} \text{ at } x = L$$

and the wave function must be smooth where the regions meet.

- Note that the wave function is nonzero outside of the box.
- Non-zero at the boundary either..
- What would the energy look like?



Penetration Depth

- The penetration depth is the distance outside the potential well where the probability significantly decreases. It is given by

$$\delta x \approx \frac{1}{\alpha} = \frac{\hbar}{\sqrt{2m\left(V_0 - E\right)}}$$

- It should not be surprising to find that the penetration distance that violates classical physics is proportional to Planck's constant.

Three-Dimensional Infinite-Potential Well

- The wave function must be a function of all three spatial coordinates.
- We begin with the conservation of energy $E = K + V = \frac{p^2}{2m} + V$
- Multiply this by the wave function to get

$$E\psi = \left(\frac{p^2}{2m} + V\right)\psi = \frac{p^2}{2m}\psi + V\psi$$

- Now consider momentum as an operator acting on the wave function. In this case, the operator must act twice on each dimension. Given:

$$p^2 = p_x^2 + p_y^2 + p_z^2 \qquad \hat{p}_x\psi = -i\hbar\frac{\partial\psi}{\partial x} \quad \hat{p}_y\psi = -i\hbar\frac{\partial\psi}{\partial y} \quad \hat{p}_z\psi = -i\hbar\frac{\partial\psi}{\partial z}$$

- The three dimensional Schrödinger wave equation is

$$-\frac{\hbar^2}{2m}\left(\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2}\right) + V\psi = E\psi \xrightarrow{\text{Rewrite}} -\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = E\psi$$

Ex 6.10: Expectation values inside a box

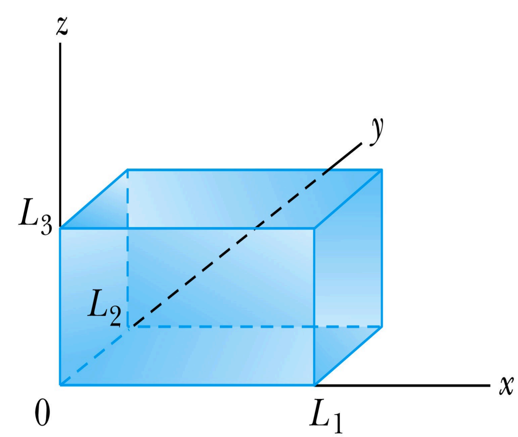
Consider a free particle inside a box with lengths L_1, L_2 and L_3 along the x, y, and z axes, respectively, as shown in the figure. The particle is constrained to be inside the box. Find the wave functions and energies. Then find the ground energy and wave function and the energy of the first excited state for a cube of sides L.

What are the boundary conditions for this situation?

Particle is free, so x, y and z wave functions are independent from each other!

Each wave function must be 0 at the wall! Inside the box, potential V is 0.

$$-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = E\psi \Rightarrow -\frac{\hbar^2}{2m}\nabla^2\psi = E\psi$$



A reasonable solution is

$$\psi(x,y,z) = A\sin(k_1 x)\sin(k_2 y)\sin(k_3 z)$$

Using the boundary condition

$$\psi = 0 \text{ at } x = L_1 \Rightarrow k_1 L_1 = n_1\pi \Rightarrow k_1 = n_1\pi / L_1$$

So the wave numbers are $k_1 = \frac{n_1\pi}{L_1}$ $\quad k_2 = \frac{n_2\pi}{L_2}$ $\quad k_3 = \frac{n_3\pi}{L_3}$

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The energy can be obtained through the Schrödinger equation

$$-\frac{\hbar^2}{2m}\nabla^2\psi = -\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\psi = E\psi$$

$$\frac{\partial\psi}{\partial x} = \frac{\partial}{\partial x}\left(A\sin(k_1x)\sin(k_2y)\sin(k_3z)\right) = k_1A\cos(k_1x)\sin(k_2y)\sin(k_3z)$$

$$\frac{\partial^2\psi}{\partial x^2} = \frac{\partial^2}{\partial x^2}\left(A\sin(k_1x)\sin(k_2y)\sin(k_3z)\right) = -k_1^2A\sin(k_1x)\sin(k_2y)\sin(k_3z) = -k_1^2\psi$$

$$-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\psi = \frac{\hbar^2}{2m}\left(k_1^2+k_2^2+k_3^2\right)\psi = E\psi$$

$$E = \frac{\hbar^2}{2m}\left(k_1^2+k_2^2+k_3^2\right) = \frac{\pi^2\hbar^2}{2m}\left(\frac{n_1^2}{L_1^2}+\frac{n_2^2}{L_2^2}+\frac{n_3^2}{L_3^2}\right)$$

What is the ground state energy?

$E_{1,1,1}$ when $n_1=n_2=n_3=1$, how much?

When are the energies the same for different combinations of n_i?

Degeneracy*

- Analysis of the Schrödinger wave equation in three dimensions introduces three quantum numbers that quantize the energy.
- A quantum state is degenerate when there is more than one wave function for a given energy.
- Degeneracy results from particular properties of the potential energy function that describes the system. A perturbation of the potential energy, such as the spin under a B field, can remove the degeneracy.

***Mirriam-webster: having two or more states or subdivisions**