PHYS 3313 – Section 001
Lecture # 24

Wednesday, April 29, 2015
Dr. Alden Stradling

- Equipartition Theorem
- Quantum Distributions
- Fermi-Dirac and Bose-Einstein Statistics
- Liquid Helium
- Laser

Announcements

- Research paper deadline is Monday, May 4
- Research presentation deadline is 8pm, Sunday, May 3
- Reminder Homework #6
  - CH7 end of chapter problems: 7, 8, 9, 12, 17 and 29
  - Due on Wednesday, Apr. 29, in class
- Reading assignments
  - CH7.6 and the entire CH8
- Final comprehensive exam 11am – 1:30pm, Monday, May 11

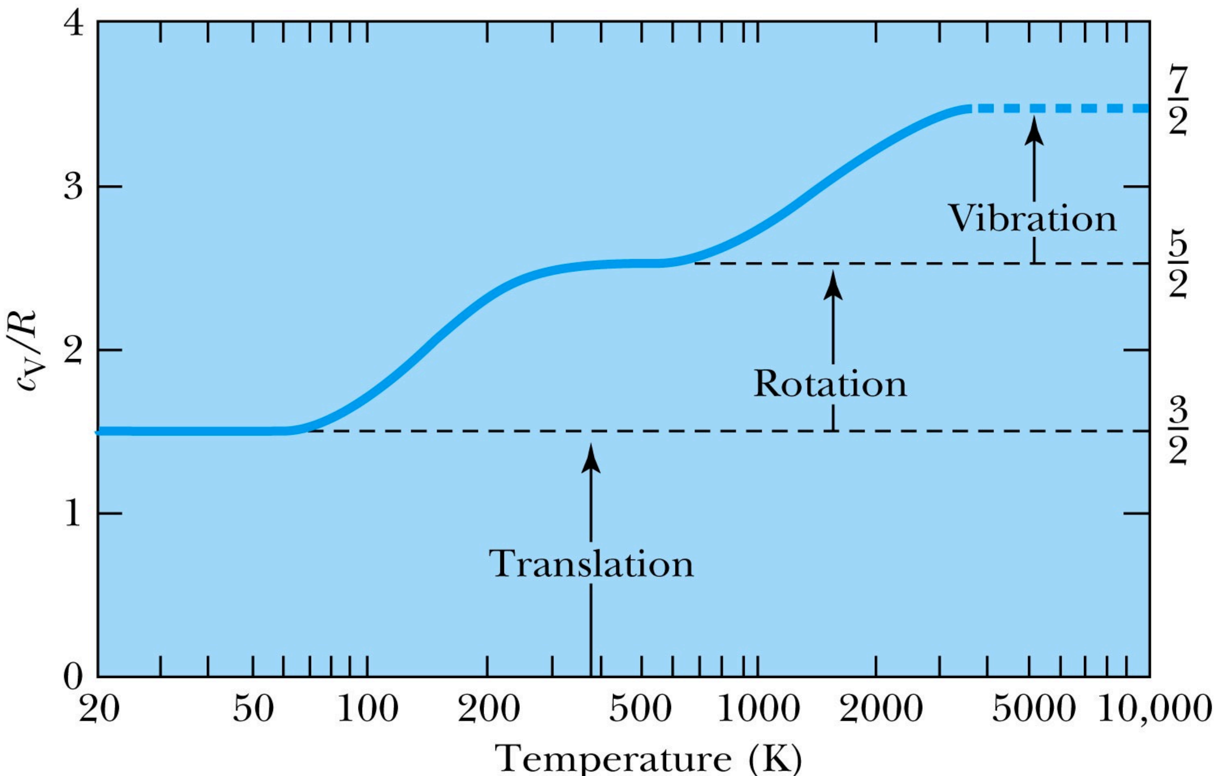
Equipartition Theorem

- Most the mass of an atom is confined to a nucleus whose magnitude is smaller than the whole atom.
  - I_z is smaller than I_x and I_y.
  - Only rotations about *x* and *y* contributes to the energy
- In some circumstances it is better to think of atoms connected to each other by a massless spring.
- The vibrational kinetic energy is $\frac{1}{2}m\left(dr/dt\right)^2$
- There are seven degrees of freedom (three translational, two rotational, and two vibrational). ➔ 7kT/2 per molecule
- While it works pretty well, the simple assumptions made for equi-partition principle, such as massless connecting rod, is not quite sufficient for detailed molecular behaviors

Molar Heat Capacity

- The heat capacities of diatomic gases are also temperature dependent, indicating that the different degrees of freedom are "turned on" at different temperatures.

Example of H_2



Classical and Quantum Statistics

- In gas, particles are so far apart, they do not interact substantially & are free→ even if they collide, they can be considered as elastic and do not affect the mean values
- If molecules, atoms, or subatomic particles are in the liquid or solid state, the **Pauli exclusion principle*** prevents two particles with identical quantum states from sharing the same space → limits available energy states in quantum systems
  - Recall there is no restriction on particle energies in classical physics.
- This affects the overall distribution of energies

*Pauli Exclusion Principle: No two electrons in an atom may have the same set of quantum numbers (n, l, m_l, m_s).

Classical Distributions

- Rewrite Maxwell speed distribution in terms of energy.

$$F(v)dv = 4\pi C \exp\left(-\beta m v^2/2\right) v^2 dv = F(E)dE$$

  - Probability for finding a particle between speed v and v+dv

- For a monoatomic gas the energy is all translational kinetic energy.

$$E = \tfrac{1}{2} m v^2$$

$$dE = mv\,dv$$

$$dv = \frac{dE}{mv} = \frac{dE}{m\sqrt{2E/m}} = \frac{dE}{\sqrt{2mE}}$$

- where

$$F(E) = \frac{8\pi C}{\sqrt{2}m^{3/2}} \exp\left(-\beta E\right) E^{1/2}$$

Classical Distributions

- Boltzmann showed that the statistical factor exp($-\beta E$) is a characteristic of any classical system.
  - regardless of how quantities other than molecular speeds may affect the energy of a given state
- **Maxwell-Boltzmann factor** for classical system:

$$F_{MB} = A\exp(-\beta E)$$

- The energy distribution for classical system:

$$n(E) = g(E)F_{MB}$$

- $n(E)\,dE$: the number of particles with energies between E and $E + dE$
- $g(E)$, the **density of states**, is the number of states available per unit energy
- F_{MB}: the relative probability that an energy state is occupied at a given temperature

Quantum Distributions

- Identical particles cannot be distinguished if their wave functions overlap significantly
  - Characteristic of indistinguishability is what makes quantum statistics different from classical statistics.
- Consider two distinguishable particles in two different energy states with the same probability (0.5 each)
- The possible configurations are

E1	**E2**
A, B	
A	B
B	A
	A, B

- Since the four states are equally likely, the probability of each state is one-fourth (0.25).

Quantum Distributions

- If the two particles are indistinguishable:

State 1	State 2
XX	
X	X
	XX

- There are only three possible configurations
- Thus the probability of each is one-third (~0.33).
- Because some particles do not obey the Pauli exclusion principle, two kinds of quantum distributions are needed.
- **Fermions**: Particles with half-spins (1/2) that **obey** the Pauli principle.
  - Examples? Electron, proton, neutron, any atoms or molecules with odd number of fermions
- **Bosons**: Particles with zero or integer spins that do **NOT obey** the Pauli principle.
  - Examples? Photon, force mediators, pions, any atoms or molecules with even number of fermions

Quantum Distributions

- **Fermi-Dirac** distribution: $n(E) = g(E) F_{FD}$

  where $$F_{FD} = \frac{1}{B_{FD} \exp(\beta E) + 1}$$

- **Bose-Einstein** distribution: $n(E) = g(E) F_{BE}$

  where $$F_{BE} = \frac{1}{B_{BE} \exp(\beta E) - 1}$$

- B_i (i = FD or BE) is the normalization factor.
- Both distributions reduce to the classical Maxwell-Boltzmann distribution when $B_i \exp(\beta E)$ is much greater than 1.
  - the Maxwell-Boltzmann factor $A \exp(-\beta E)$ is much less than 1.
  - In other words, the probability that a particular energy state will be occupied is much less than 1!

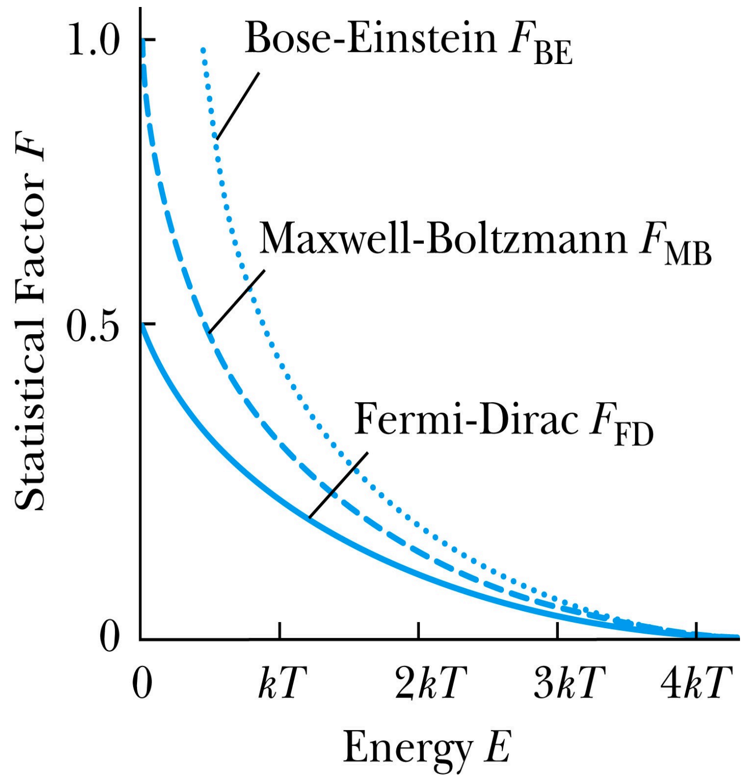
Summary of Classical and Quantum Distributions

Table 9.2 Classical and Quantum Distributions

Distributors	Properties of the Distribution	Examples	Distribution Function
Maxwell-Boltzmann	Particles are identical but distinguishable	Ideal gases	$F_{MB} = A\exp(-\beta E)$
Bose-Einstein	Particles are identical and indistinguishable with integer spin	Liquid ^{4}He, photons	$F_{BE} = \dfrac{1}{B_{BE}\exp(\beta E) - 1}$
Fermi-Dirac	Particles are identical and indistinguishable with half-integer spin	Electron gas (free electrons in a conductor)	$F_{FD} = \dfrac{1}{B_{FD}\exp(\beta E) + 1}$

Quantum Distributions

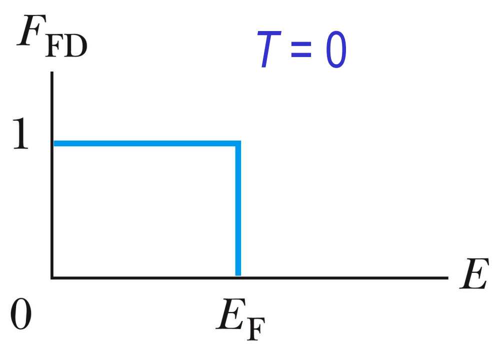
- The normalization constants for the distributions depend on the physical system being considered.
- Because bosons **do not obey the Pauli exclusion principle**, more bosons can fill lower energy states.
- Three graphs coincide at high energies – the classical limit.
- Maxwell-Boltzmann statistics may be used in the classical limit.

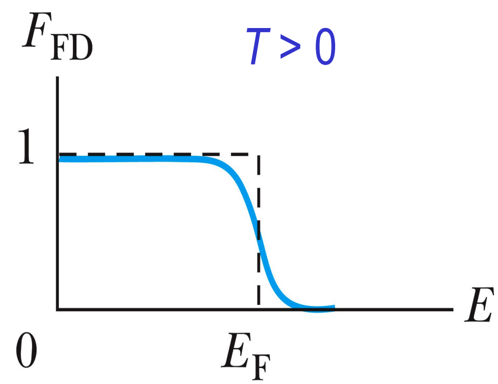


Fermi-Dirac Statistics

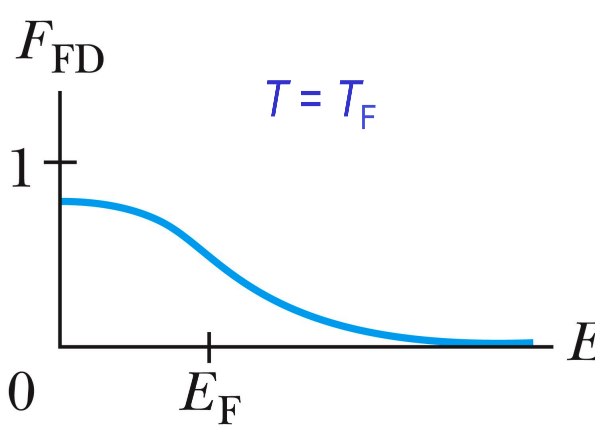
- This is most useful for electrical conduction
- The normalization factor B_{FD} $B_{FD} = \exp(-\beta E_F)$
  - *Where* E_F is called the **Fermi energy**.
- The Fermi-Dirac Factor becomes $$F_{FD} = \frac{1}{\exp\left[\beta(E - E_F)\right] + 1}$$
- When $E = E_F$, the exponential term is 1. ➔ F_{FD} =1/2
- In the limit as T $\rightarrow$ 0, $$F_{FD} = \begin{cases} 1 \text{ for } E < E_F \\ 0 \text{ for } E > E_F \end{cases}$$
- At T = 0, fermions occupy the lowest energy levels available to them
  - Since they cannot all fill the same energy due to Pauli Exclusion principle, they will fill the energy states up to Fermi Energy
- Near T = 0, there is little a chance that the thermal agitation will kick a fermion to an energy greater than E_F.

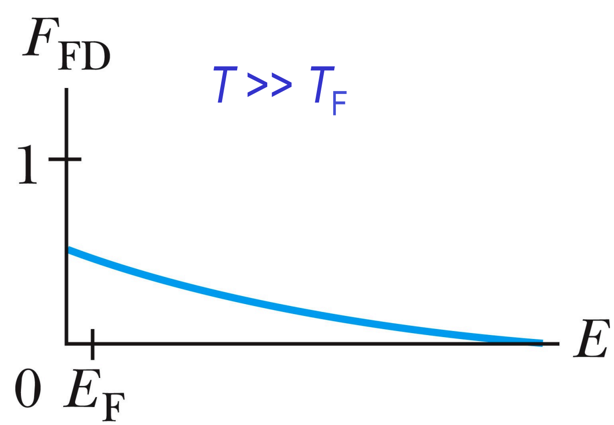
Fermi-Dirac Statistics





- As the temperature increases from $T = 0$, the Fermi-Dirac factor "smears out", and more fermions jump to higher energy level above Fermi energy
- We can define **Fermi temperature**, defined as $T_F \equiv E_F / k$

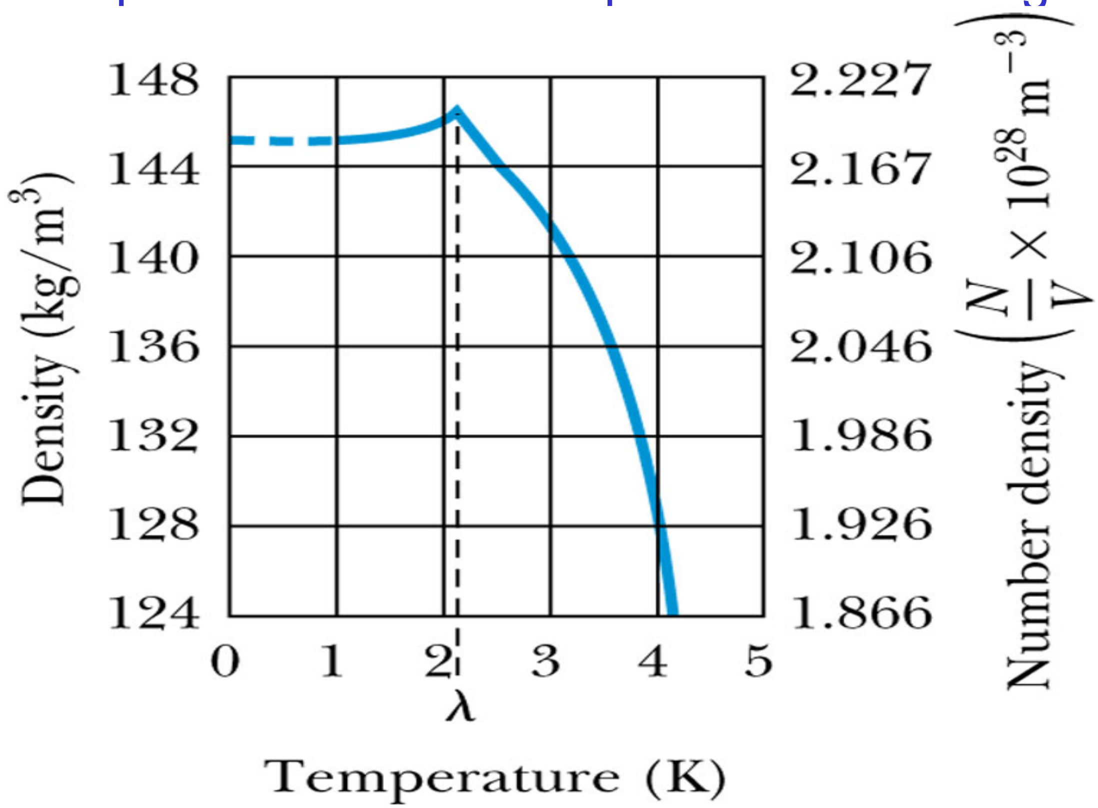




- When $T >> T_F$, F_{FD} approaches a simple decaying exponential

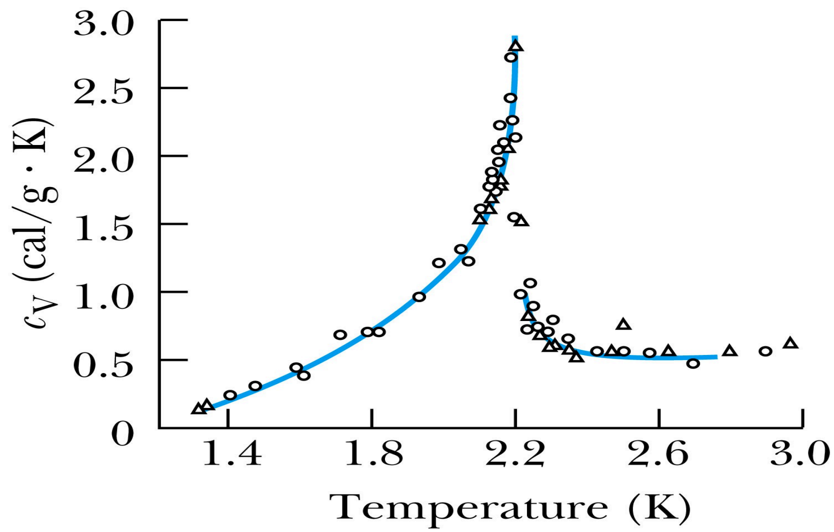
Liquid Helium

- Has the lowest boiling point of any element (4.2 K at 1 atmosphere pressure) and has no solid phase at normal pressure
- Helium is so light and has high speed and so escapes outside of the Earth atmosphere ➔ Must be captured from underground



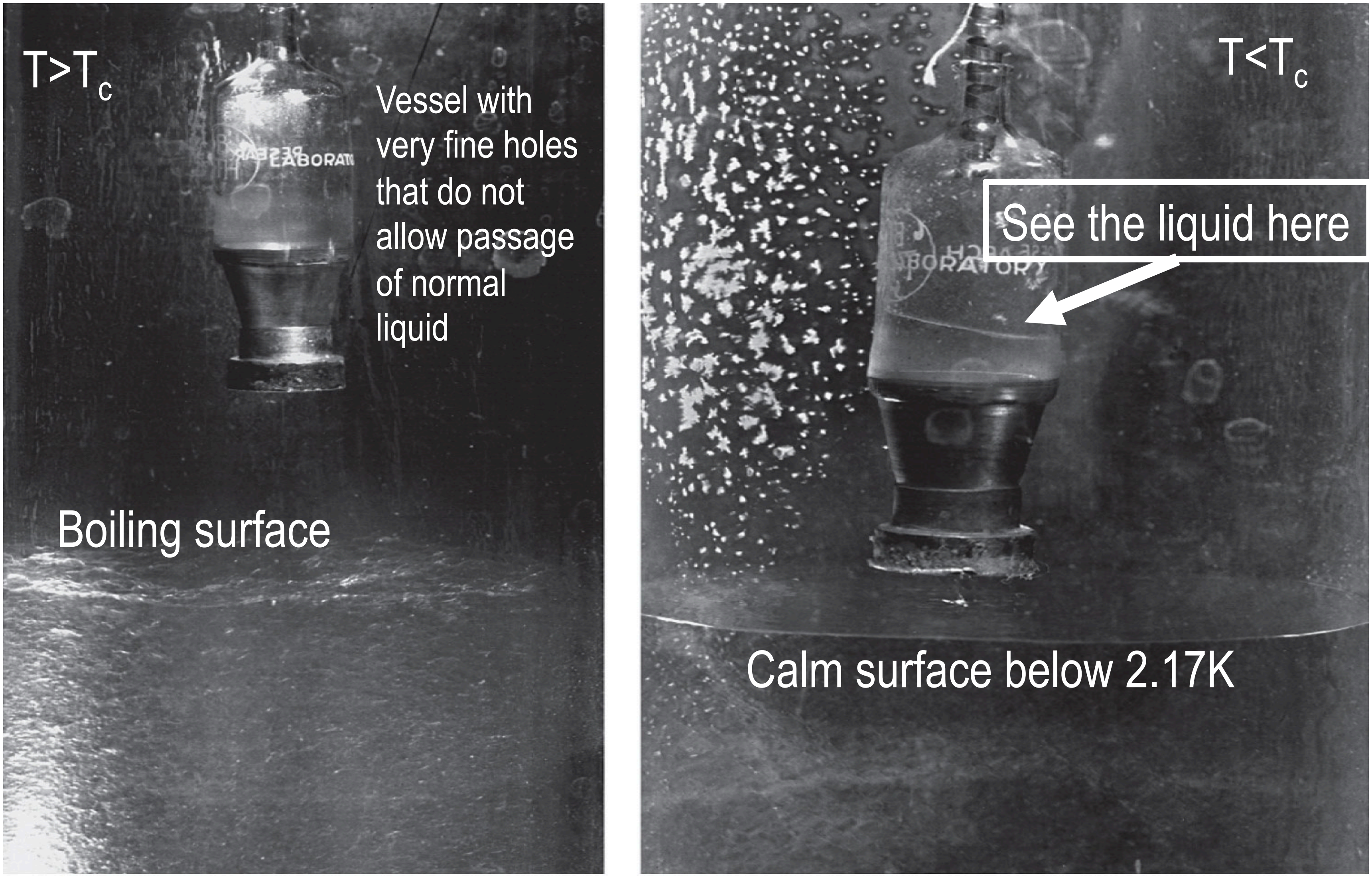
Liquid Helium

The specific heat of liquid helium as a function of temperature



- The temperature at about 2.17 K is referred to as the ***critical temperature (T_c), transition temperature*, or the *lambda point*.**
- As the temperature is reduced from 4.2 K toward the lambda point, the liquid boils vigorously. At 2.17 K the boiling suddenly stops.
- What happens at 2.17 K is a transition from the **normal phase** to the **superfluid phase**.

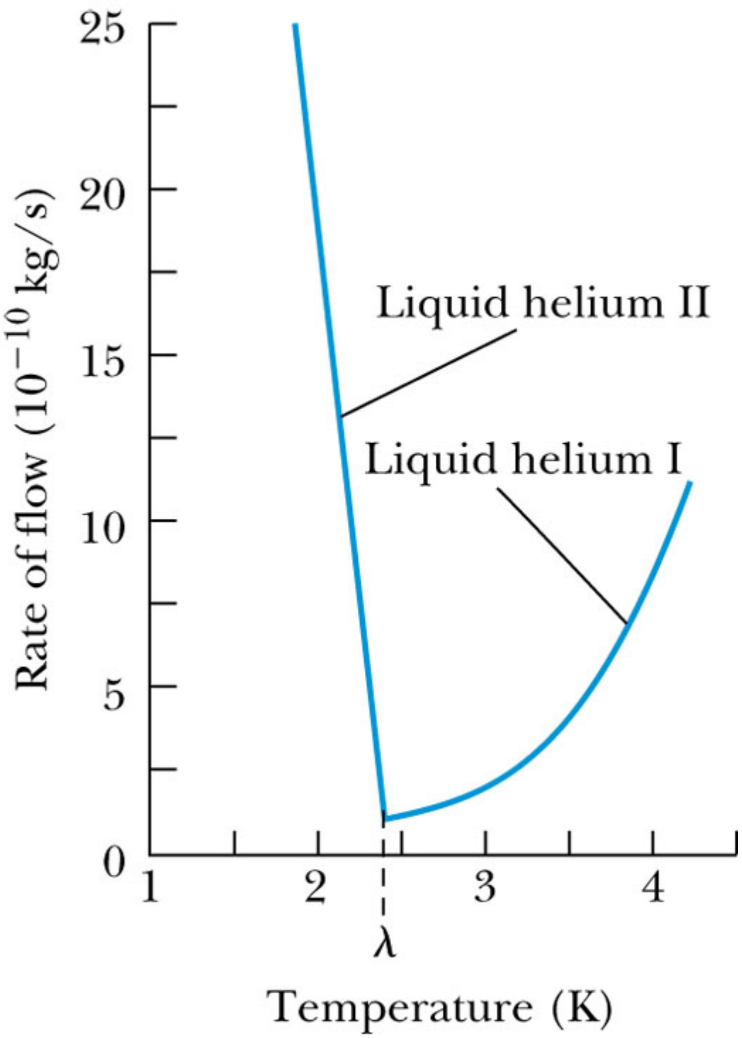
He Transition to Superfluid State



(a) (b)

Liquid Helium

- The rate of flow increases dramatically as the temperature is reduced because the superfluid has a low viscosity.
- **Creeping film** – formed when the viscosity is very low
- But when the viscosity is measured through the drag on a metal surface, He behaves like a normal fluid
  ➔ Contradiction!!



Liquid Helium

- Fritz London claimed (1938) that liquid helium below the lambda point is a mixture of superfluid and normal fluid.
  - As the temperature approaches absolute zero, the superfluid approaches 100% superfluid.
- The fraction of helium atoms in the superfluid state:

$$F = 1 - \left(\frac{T}{T_c}\right)^{3/2}$$

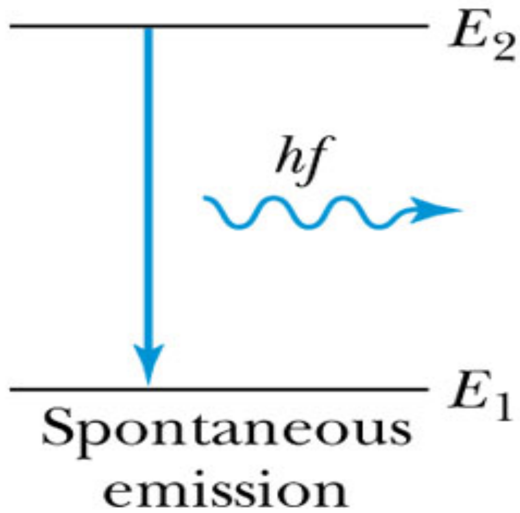
- Superfluid liquid helium (^{4}He) is referred to as a **Bose-Einstein condensation**.
  - ^{4}He is a boson thus it is not subject to the Pauli exclusion principle
  - all particles are in the same quantum state

Bose-Einstein Condensation in Gases

- BE condensation in liquid has been accomplished but gas condensation state hadn't been until 1995
- The strong Coulomb interactions among gas particles made it difficult to obtain the low temperatures and high densities needed to produce the BE condensate.
- Finally success was achieved by E. Cornell and C. Weiman in Boulder, CO, with Rb (at 20nK) and W. Kettle at MIT on Sodium (at 20μK) ➔ Awarded of Nobel prize in 2001
- The procedure
  - Laser cool their gas of ^{87}Rb atoms to about 1 mK.
  - Used a magnetic trap to cool the gas to about 20 nK, driving away atoms with higher speeds and keeping only the low speed ones
  - At about 170 nK, Rb gas went through a transition, resulting in very cold and dense state of gas
- Possible application of BEC is an atomic laser but it will take long time..

Stimulated Emission and Lasers

- **Spontaneous emission**: A molecule in an excited state will decay to a lower energy state and emit a photon, without any stimulus from the outside.

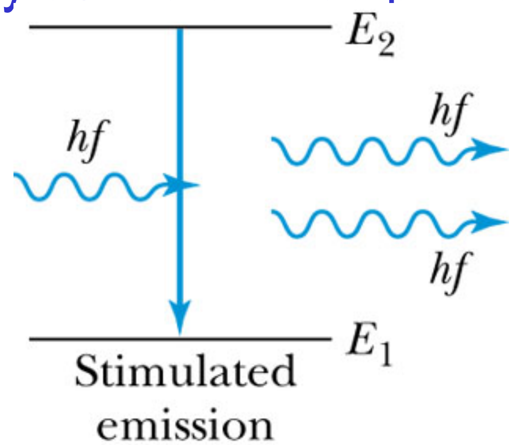


- Due to the uncertainty principle, the best we can do is to calculate the probability that a spontaneous transition will occur.
  - And the phases of the photons emitted from this process are random
- If a spectral line has a width ΔE, then a lower-bound estimate of the lifetime is $\Delta t = \hbar / (2\,\Delta E)$.

Stimulated Emission and Lasers

Stimulated emission: A photon incident upon a molecule in an excited state causes the unstable system to decay to a lower state.

- The photon emitted tends to have the same phase and direction as the stimulated radiation.
- If the incoming photon has the same energy as the emitted photon:
  - The result is two photons of the same wavelength and phase traveling in the same direction.
  - Because the incoming photon just triggers emission of the second photon.
- These two photons are said to be coherent!

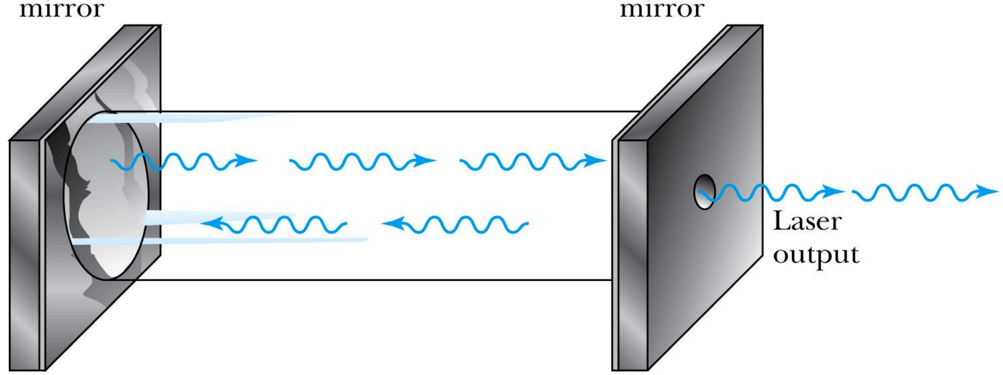


- Einstein explained this stimulated emission in his 1917 paper “On the Quantum Theory of Radiation

Stimulated Emission and Lasers

- **Laser** stands for "light amplification by the stimulated emission of radiation"
  - The first working laser was by Theodore H. Maiman in 1960
- **Masers**: Microwaves are used instead of visible light.
  - The first working maser was by C.H. Townse in 1954

helium-neon laser



Stimulated Emission and Lasers

- The body of the laser is a closed tube, filled with about a 9/1 ratio of helium and neon.
- Photons bouncing back and forth between two mirrors are used to stimulate the transitions in neon.
- Photons produced by stimulated emission will be coherent, and the photons that escape through the silvered mirror will be a coherent beam.

How are atoms put into the excited state?

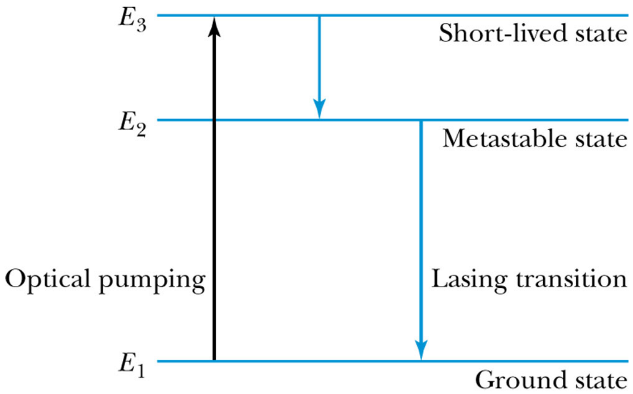
We cannot rely on the photons in the tube; if we did:

1) Any photon produced by stimulated emission would have to be "used up" to excite another atom.
2) There may be nothing to prevent spontaneous emission from atoms in the excited state. ➔ The beam would not be coherent.

Stimulated Emission and Lasers

Use a multilevel atomic system to see those problems.

- **Three-level system**



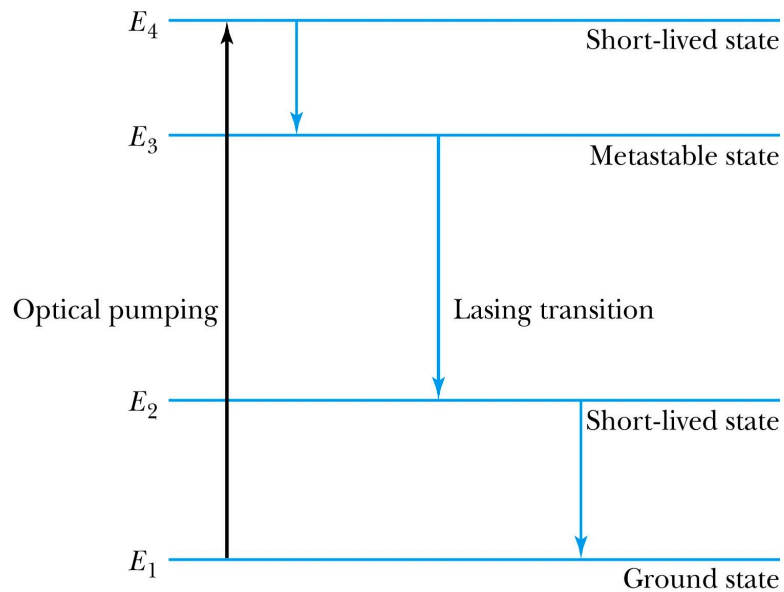
1) Atoms in the ground state are *pumped* to a higher state by some external energy source (power supply)
2) The atom decays quickly from E_3 to E_2.
   The spontaneous transition from E_2 to E_1 is forbidden by a $\Delta\ell = \pm 1$ selection rule. ➔ E_2 is said to be *metastable*.
3) *Population inversion*: more atoms are in the metastable than in the ground state

Stimulated Emission and Lasers

- After an atom has been returned to the ground state from E_2, we want the external power supply to return it immediately to E_3, but it may take some time for this to happen.
- A photon with energy $E_2 - E_1$ can be absorbed by the atom ➔ resulting a much weaker beam
- This is undesirable because the absorbed photon is unavailable for stimulating another transition.
- A four level system can help avoiding this system

Stimulated Emission and Lasers

- **Four-level system**

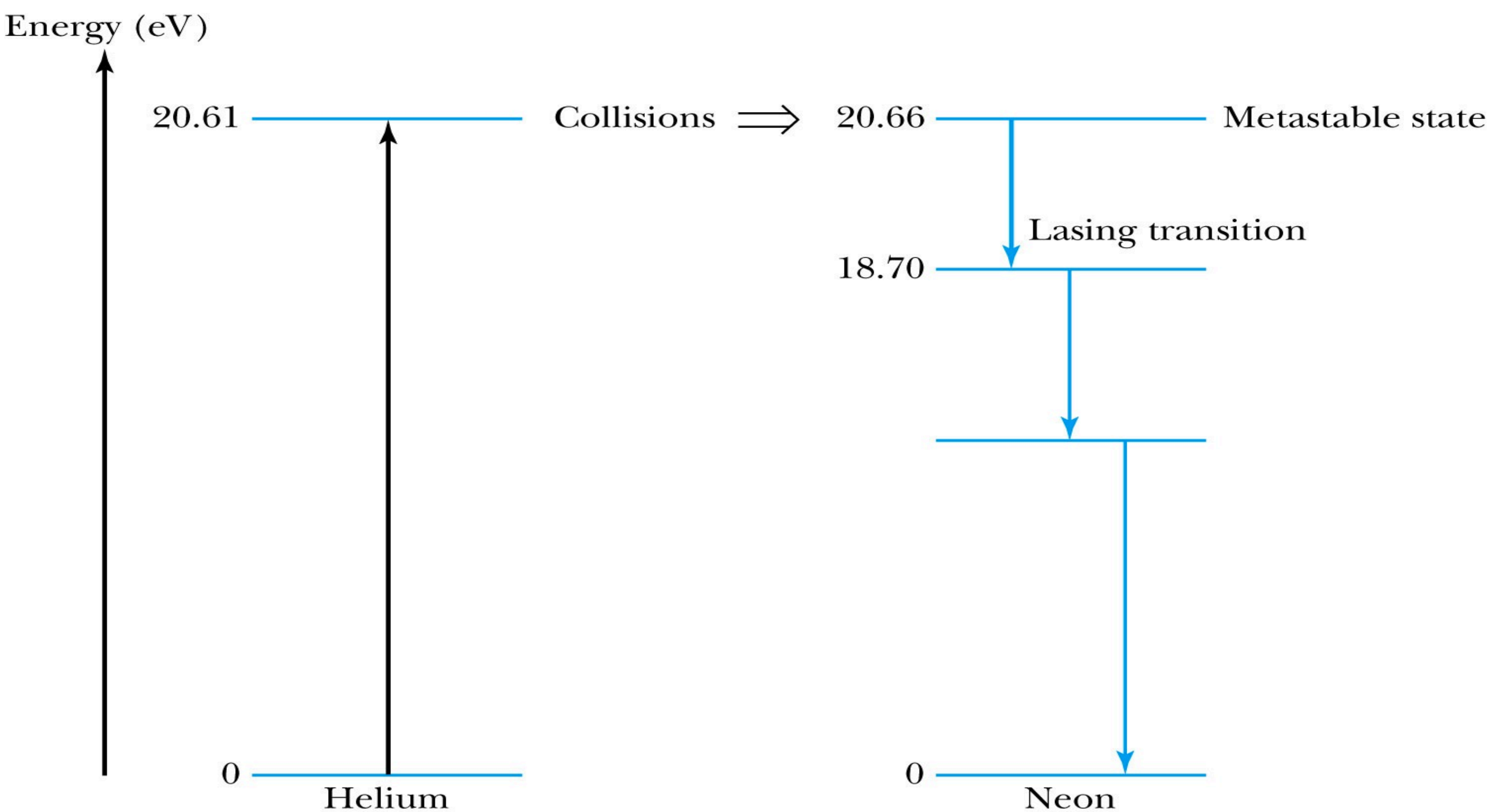


1) Atoms are pumped from the ground state to E_4.
2) They decay quickly to the metastable state E_3.
3) The stimulated emission takes atoms from E_3 to E_2.
4) The spontaneous transition from E_2 to E_1 is not forbidden, so E_2 will not exist long enough for a photon to be kicked from E_2 to E_3.

   → Lasing process can proceed efficiently.

Stimulated Emission and Lasers

- The red helium-neon laser uses transitions between energy levels in both helium and neon via their collisions

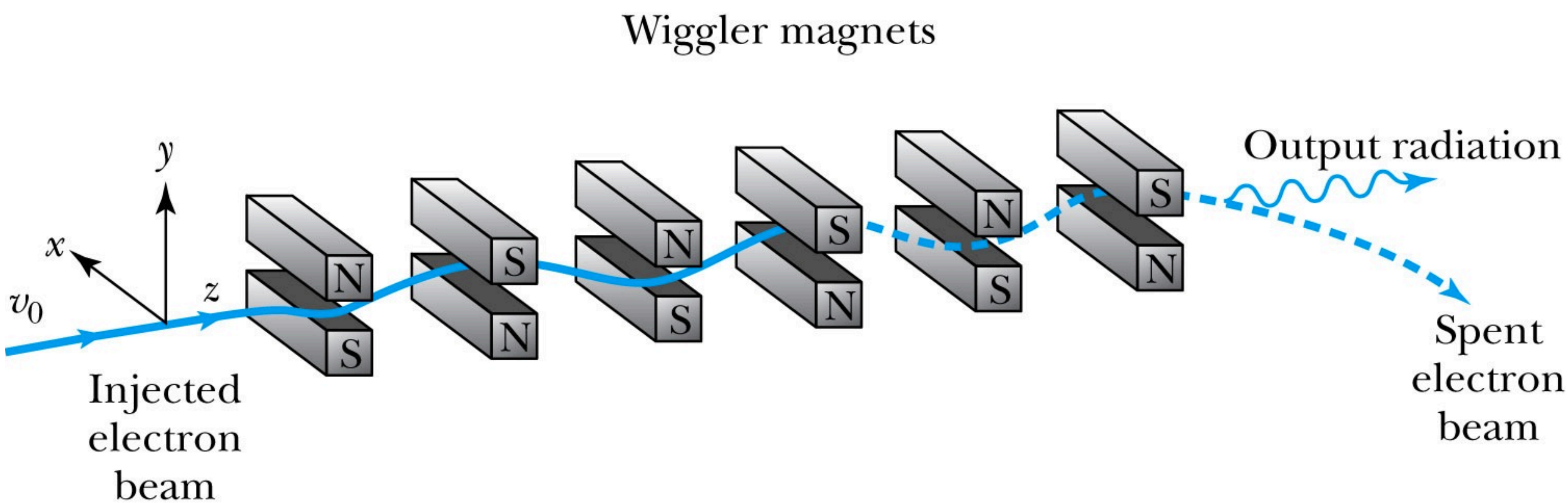


Tunable & Free Electron Lasers

Tunable laser:

- The emitted radiation wavelength can be adjusted as wide as 200 nm adjusting the mixture of organic dyes
- Semi conductor lasers are replacing dye lasers.

Free-electron laser:



Free Electron Lasers

- This laser relies on charged particles.
- A series of magnets called *wigglers* is used to accelerate a beam of electrons.
- Free electrons are not tied to atoms; they aren't dependent upon atomic energy levels and can be tuned to wavelengths well into the UV part of the spectrum.
- Went down to 0.15nm wavelength at SLAC light source in 2009

Scientific Applications of Lasers

- An extremely coherent and nondivergent beam is used in making precise determination of large and small distances.
  - Precise determination of the speed of light resulted from precision laser measurement → redefinition of 1m
  - Precise (to 10cm) determination of the distance between the Earth and the moon
- Pulsed lasers are used in thin-film deposition to study the electronic properties of different materials.
- The use of lasers in fusion research for containing enough nuclei in a confined space for fusion to occur
  - **Inertial confinement**: A pellet of deuterium and tritium would be induced into fusion by an intense burst of laser light coming simultaneously from many directions.