PHYS 3313 – Section 001
Lecture #6

Monday, Feb. 6, 2017
Dr. Jaehoon Yu

- Lorentz Transformations
- Time Dilation
- Length Contraction
- Length Contraction
- Relativistic Velocity Addition
- The Twin Paradox

Announcements

- Reminder: Homework #1
  - chapter 2 end of the chapter problems
  - 17, 21, 23, 24, 32, 59, 61, 66, 68, 81 and 96
  - Due is by the beginning of the class, Wednesday, Feb. 8
  - Work in study groups together with other students but PLEASE do write your answer in your own way!

Reminder: Special Project #3

1. Derive the three Lorentz velocity transformation equations, starting from Lorentz coordinate transformation equations. (10 points)
2. Derive the three reverse Lorentz velocity transformation equations, starting from Lorentz coordinate transformation equations. (10 points)
3. Prove that the space-time invariant quantity $s^2=x^2-(ct)^2$ is indeed invariant, i.e. $s^2=s'^2$, in Lorentz Transformation. (5 points)
4. You must derive each one separately starting from the Lorentz spatial coordinate transformation equations to obtain any credit.
   – Just simply switching the signs and primes will NOT be sufficient!
   – Must take the simplest form of the equations, using β and γ.
5. You MUST have your own, independent handwritten answers to the above three questions even if you worked together with others. All those who share the answers will get 0 credit if copied.

- Due for the submission is this Monday, Feb. 13!

Properties of the Relativistic Factor γ

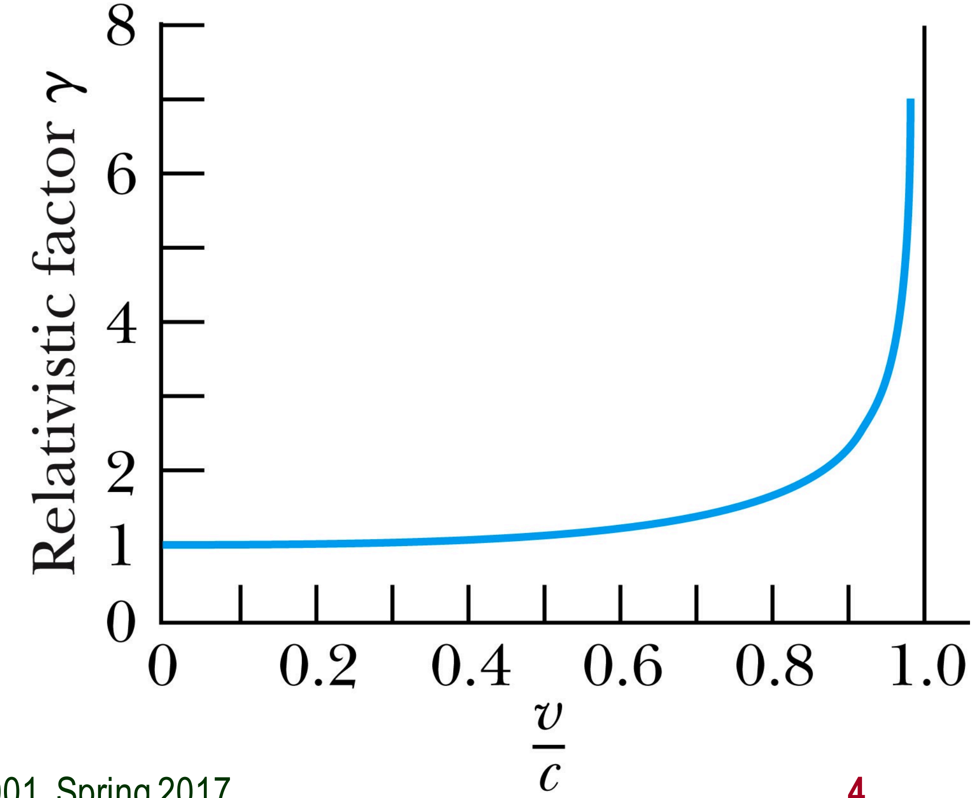
What is the property of the relativistic factor, γ?

Is it bigger or smaller than 1?

Recall Einstein's postulate, $\beta = v/c < 1$ for all observers

- $\gamma = 1$ only when $v = 0$

$$\gamma = 1\Big/\sqrt{1-\beta^2} \geq 1$$



The complete Lorentz Transformations

$$x' = \frac{x - vt}{\sqrt{1-\beta^2}} \qquad x = \frac{x' + vt'}{\sqrt{1-\beta^2}}$$

$$y' = y \qquad y = y'$$

$$z' = z \qquad z = z'$$

$$t' = \frac{t - \left(vx/c^2\right)}{\sqrt{1-\beta^2}} \qquad t = \frac{t' + \left(vx'/c^2\right)}{\sqrt{1-\beta^2}}$$

- Some things to note
  - What happens when β~0 (or v~0)?
    - The Lorentz x-formation becomes Galilean x-formation
  - Space-time are not separated
  - For non-imaginary transformations, the frame speed cannot exceed c!

Time Dilation and Length Contraction

Direct consequences of the Lorentz Transformation:

- **Time Dilation**:

  The clock in a moving inertial reference frame K' run slower with respect to the stationary clock in K.

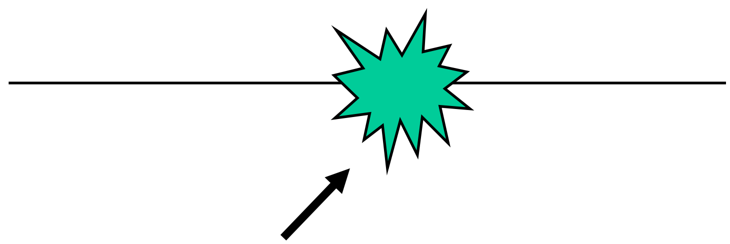
- **Length Contraction**:

  Lengths measured in a moving inertial reference frame K' are shorter with respect to the same lengths stationary in K.

Time Dilation

To understand *time dilation* the idea of **proper time** must be understood:

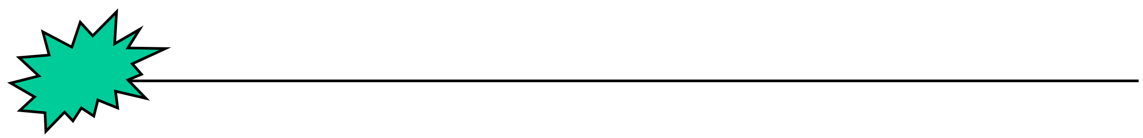
- **proper time**, T_0, is the time difference between two events occurring at the **same** position in a system as measured by **a clock at that position**.



Same location (spark "on" then off")

Time Dilation

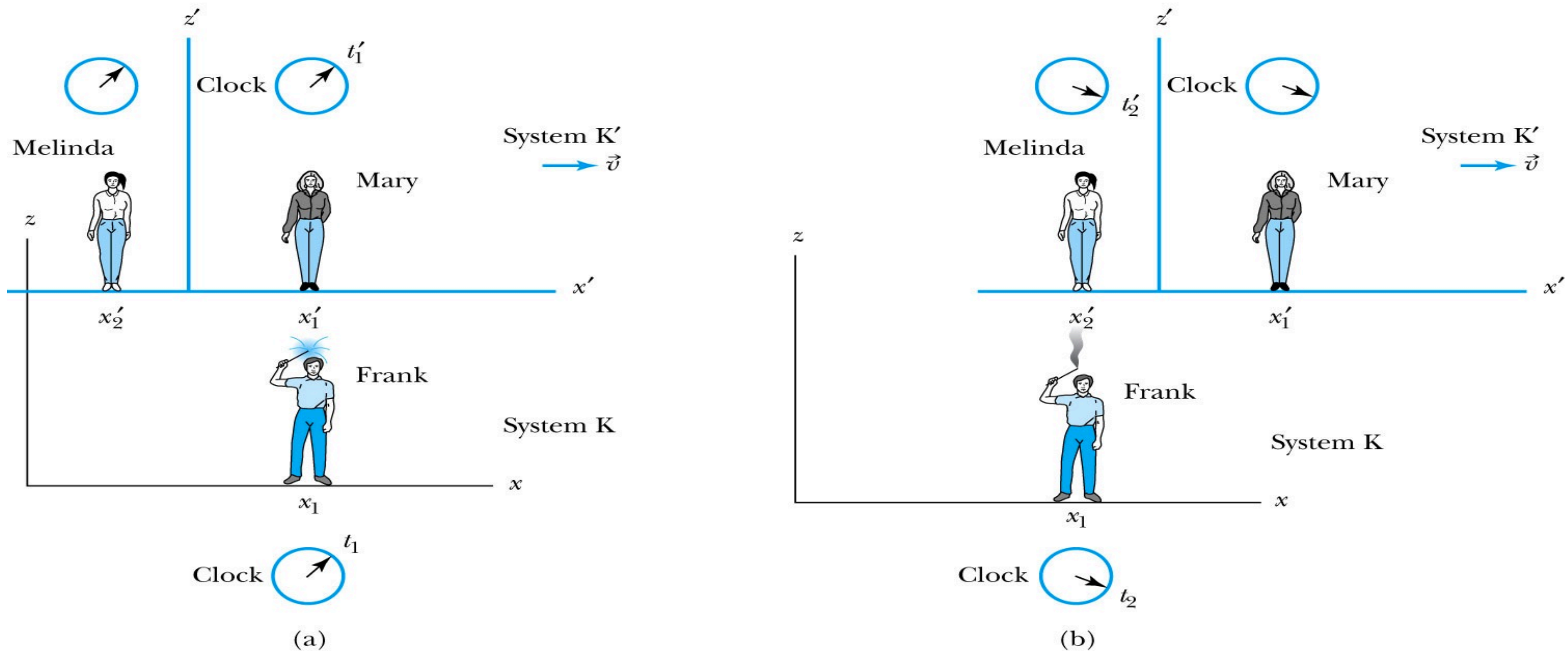
Is this a Proper Time?



spark "on"   then spark "off"

Beginning and ending of the event occur at different positions

Time Dilation with Mary, Frank, and Melinda



- Frank's clock is at the same position in system K when the sparkler is lit in (a) ($t=t_1$) and when it goes out in (b) ($t=t_2$). ➔ The proper time $T_0=t_2-t_1$
- Mary, in the moving system K', is beside the sparkler when it was lit ($t=t_1'$)
- Melinda then moves into the position where and when the sparkler extinguishes ($t=t_2'$)
- Thus, Melinda, at the new position, measures the time in system K' when the sparkler goes out in (b).

According to Mary and Melinda…

- Mary and Melinda measure the two times for the sparkler to be lit and to go out in system K' as times t_1'and t_2' so that by the Lorentz transformation:

$$t'_2 - t'_1 = \frac{(t_2 - t_1) - (v/c^2)(x_2 - x_1)}{\sqrt{1-\beta^2}}$$

  - Note here that Frank records $x_2 - x_1 = 0$ in K with a proper time: $T_0 = t_2 - t_1$ or

$$T' = t'_2 - t'_1 = \frac{T_0}{\sqrt{1-\beta^2}} = \gamma T_0$$

Time Dilation: Moving Clocks Run Slow

1) $T' > T_0$ or the time measured between two events at *different positions* is greater than the time between the same events at *one position:* ***time dilation.***

   The proper time is always the shortest time!!

2) The events do not occur at the same space and time coordinates in the two systems

3) System K requires 1 clock and K' requires 2 clocks.

Time Dilation Example: muon lifetime

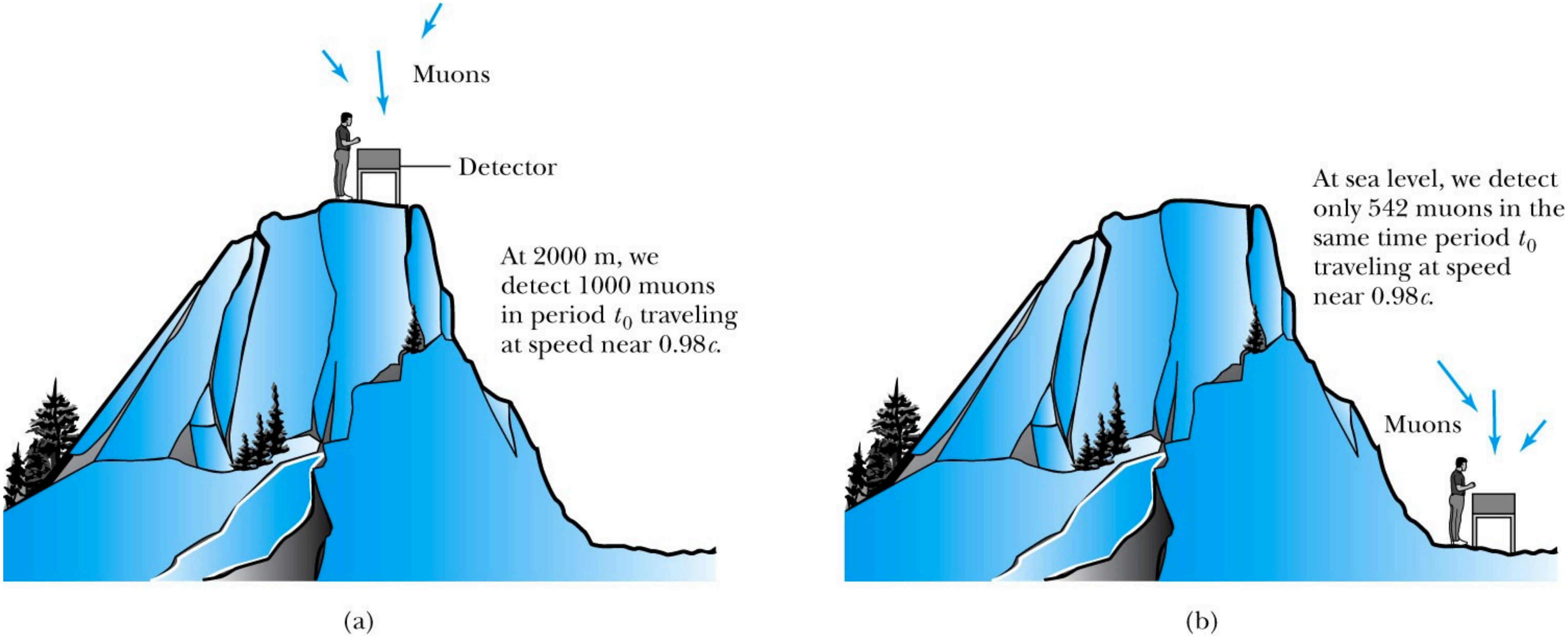
- Muons are essentially heavy electrons (~200 times heavier)
- Muons are typically generated in collisions of cosmic rays in upper atmosphere and, unlike electrons, decay ($t_0 = 2.2$ μsec)
- For a muon incident on Earth with v=0.998c, an observer on Earth would see what lifetime of the muon?
- 2.2 μsec?

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \approx 16$$

- t=35 μsec
- Moving clocks run slow so when an outside observer measures, they see a longer lifetime than the muon itself sees.

Experimental Verification of Time Dilation

Arrival of Muons on the Earth's Surface



The number of muons detected with speeds near $0.98c$ is much different (a) on top of a mountain than

(b) at sea level, because of the muon's decay. The experimental result agrees with our time dilation equation.

Length Contraction

To understand *length contraction* the idea of **proper length** must be understood:

- Let an observer in each system K and K' have a meter stick at rest in ***their own system*** such that each measures the same length at rest.
- The length as measured at rest <u>**at the same time**</u> is called the **proper length**.

Length Contraction cont'd

Each observer lays a stick down along his or her respective *x* axis, putting the left end at x_ℓ (or x'_ℓ) and the right end at x_r (or x'_r).

- Thus, in the rest frame K, Frank measures his stick to be:

$$L_0 = x_r - x_l$$

- Similarly, in the moving frame K', Mary measures her stick at rest to be:

$$L'_0 = x'_r - x'_l$$

- Frank in his rest frame measures the length of the stick in Mary's frame which is moving with speed v.
- Thus using the Lorentz transformations Frank measures the length of the stick in K' as:

$$x'_r - x'_l = \frac{(x_r - x_l) - v(t_r - t_l)}{\sqrt{1-\beta^2}}$$

Where both ends of the stick must be measured simultaneously, i.e, $t_r = t_\ell$

Here Mary's proper length is $L'_0 = x'_r - x'_\ell$

and Frank's measured length of Mary's stick is $L = x_r - x_\ell$

Measurement in Rest Frame

The observer in the rest frame measures the moving length as L given by

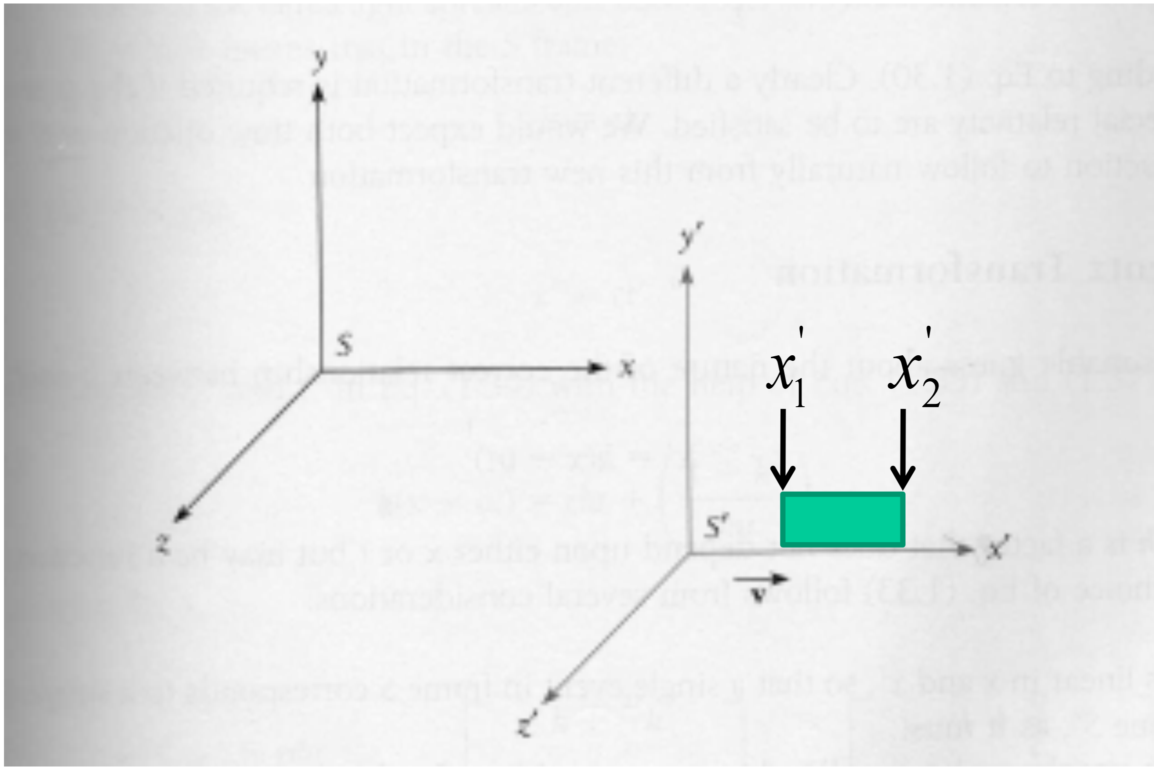
$$L'_0 = \frac{L}{\sqrt{1-\beta^2}} = \gamma L$$

but since both Mary and Frank in their respective frames measure $L'_0 = L_0$

$$L = L_0\sqrt{1-\beta^2} = \frac{L_0}{\gamma}$$

and $L_0 > L$, i.e. the moving stick shrinks

Length Contraction Summary



- Proper length (length of object in its own frame:

$$L_0 = x_2' - x_1'$$

- Length of the object in observer's frame:

$$L = x_2 - x_1$$

$$L_0' = L_0 = x_2' - x_1' = \gamma(x_2 - vt) - \gamma(x_1 - vt) = \gamma(x_2 - x_1)$$

$L_0 = \gamma L$ $\quad$ $L = L_0 / \gamma$ $\quad$ Since $\gamma > 1$, the length is shorter in the direction of motion (length contraction!)

More about Muons

- Rate: 1/cm^2/minute at Earth's surface (so for a person with 600 cm^2 surface area, the rate would be 600/60=10 muons/sec passing through the body!)
- They are typically produced in atmosphere about 6 km above surface of Earth and often have velocities that are a substantial fraction of speed of light, v=.998 c for example and lifetime of 2.2 μsec $vt_0 = 2.994\times 10^8 \frac{m}{\sec}\cdot 2.2\times 10^{-6}\sec = 0.66km$
- How do they reach the Earth if they only go 660 m and not 6000 m?
- The time dilation stretches life time to t=35 μsec not 2.2 μsec, thus they can travel 16 times further, or about 10 km, implying they easily reach the ground
- But riding on a muon, the trip takes only 2.2 μsec, so how do they reach the ground???
- Muon-rider sees the ground moving towards him, so the length he has to travel contracts and is only $L_0/\gamma = 6/16 = 0.38km$
- At 1000 km/sec, it would take 5 seconds to cross U.S., pretty fast, but does it give length contraction? $L = .999994L_0$ {not much contraction}
(for v=0.9c, the length is reduced by 66%)

The Complete Lorentz Transformations

$$x' = \frac{x - vt}{\sqrt{1-\beta^2}} \qquad x = \frac{x' + vt'}{\sqrt{1-\beta^2}}$$

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Addition of Velocities

How do we add velocities in a relativistic case?

Taking differentials of the Lorentz transformation, relative velocities may be calculated:

$$dx = \gamma\left(dx' + v\,dt'\right)$$

$$dy = dy'$$

$$dz = dz'$$

$$dt = \gamma\left[dt' + \left(v/c^2\right)dx'\right]$$