PHYS 3313 – Section 001 Lecture #7

Wednesday, Feb. 8, 2017 Dr. **Jae**hoon **Yu**

- Relativistic Velocity Addition
- The Twin Paradox
- Space-time Diagram
- Invariant Quantities
- The Relativistic Doppler Effect
- Relativistic Momentum and Energy



Announcements

- Submit homework #1 now!
- Reading assignments: CH 3.3 (special topic the discovery of Helium) and CH3.7
- Colloquium today
 - Dr. Cosmin Deaconu, Univ. of Chicago
 - Radio-detection of Extreme High Energy Neutrinos in Polar Ice



Physics Department The University of Texas at Arlington <u>COLLOQUIUM</u>

Radio-detection of Extremely High-Energy Neutrinos in Polar Ice

Dr. Cosmin Deaconu University of Chicago

Wednesday February 8, 2017

4:00 Room 103 Science Hall

Abstract

Interactions of the highest-energy cosmic rays with the cosmic microwave background produce a population of extremely-high-energy (~EeV) neutrinos. Detecting these cosmogenic neutrinos would help elucidate the unknown sources of ultra-high-energy cosmic rays and provide the highest-energy measurements of the neutrino-nucleon cross- section. In order to have a chance of measurement, enormous detection targets are necessary.

Radio-detection of the cascades produced by neutrino interactions in polar ice allows for economical instrumentation of vast volumes. The Antarctic Impulsive Transient Antenna (ANITA) balloon-borne interferometric payload can scan a million cubic kilometers of Antarctic ice at a time. I will discuss the recent fourth flight of ANITA as well as results from previous flights.

In-ice antenna arrays, such as the Askaryan Radio Array (ARA) at the South Pole, have smaller detector volumes but are capable of detecting lower-energy neutrinos. An in-ice phased-antenna array is expected to further reduce the detection threshold and be sensitive to the lower-energy (~PeV) astrophysical neutrino flux detected by IceCube. I will discuss efforts aimed at deploying a phased array in the 2017-2018 Antarctic season.

Refreshments will be served at 3:30 p.m. in the Physics Library

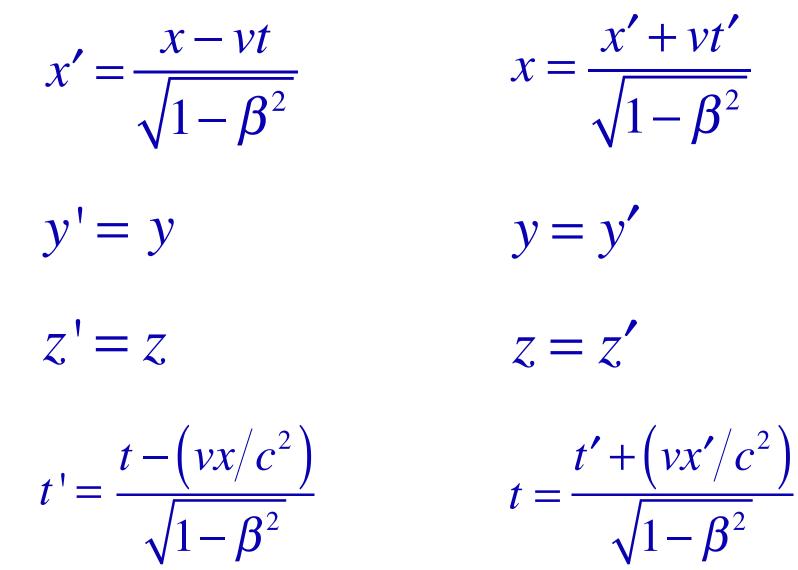
Reminder: Special Project #3

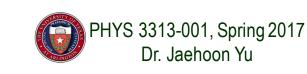
- 1. Derive the three Lorentz velocity transformation equations, starting from Lorentz coordinate transformation equations. (10 points)
- 2. Derive the three reverse Lorentz velocity transformation equations, starting from Lorentz coordinate transformation equations. (10 points)
- 3. Prove that the space-time invariant quantity $s^2=x^2-(ct)^2$ is indeed invariant, i.e. $s^2=s'^2$, in Lorentz Transformation. (5 points)
- 4. You must derive each one separately starting from the Lorentz spatial coordinate transformation equations to obtain any credit.
 - Just simply switching the signs and primes will NOT be sufficient!
 - Must take the simplest form of the equations, using β and γ .
- 5. You MUST have your own, independent handwritten answers to the above three questions even if you worked together with others. All those who share the answers will get 0 credit if copied.
- Due for the submission is this Monday, Feb. 13!

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The Complete Lorentz Transformations





Addition of Velocities

How do we add velocities in a relativistic case? Taking differentials of the Lorentz transformation, relative velocities may be calculated:

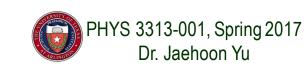
$$dx = \gamma (dx' + vdt')$$

$$dy = dy'$$

$$dz = dz'$$

$$dt = \gamma \left[dt' + (v/c^2) dx' \right]$$

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So that...

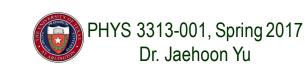
defining velocities as: $v_x = dx/dt$, $v_y = dy/dt$, $v'_x = dx'/dt'$, etc. it can be shown that:

$$\mathbf{v}_{x} = \frac{dx}{dt} = \frac{\gamma \left[dx' + v dt' \right]}{\gamma \left[dt' + \frac{v}{c^{2}} dx' \right]} = \frac{v'_{x} + v}{1 + \left(v/c^{2} \right) v'_{x}}$$

With similar relations for v_y and v_{z} :

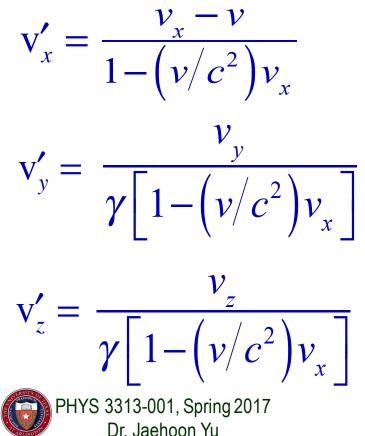
$$\mathbf{v}_{y} = \frac{dy}{dt} = \frac{v'_{y}}{\gamma \left[1 + \left(v/c^{2}\right)v'_{x}\right]} \quad \mathbf{v}_{z} = \frac{dz}{dt} = \frac{v'_{z}}{\gamma \left[1 + \left(v/c^{2}\right)v'_{x}\right]}$$

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The Lorentz Velocity Transformations In addition to the previous relations, the Lorentz velocity transformations for v'_x , v'_y , and v'_z can be obtained by switching primed and unprimed and changing v to -v.(the velocity of the moving frame!!)



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Velocity Addition Summary

- Galilean Velocity addition $v_x = v'_x + v$ where $v_x = \frac{dx}{dt}$ and $v'_x = \frac{dx'}{dt'}$
- From inverse Lorentz transform $dx = \gamma(dx' + vdt')$ and $dt = \gamma(dt' + \frac{v}{c^2}dx')$

• So
$$v_x = \frac{dx}{dt} = \frac{\gamma(dx' + vdt')}{\gamma(dt' + \frac{v}{c^2}dx')} \div \frac{dt'}{dt'} = \frac{\frac{dx'}{dt'} + v}{1 + \frac{v}{c^2}\frac{dx'}{dt'}} = \frac{\frac{v_x' + v}{1 + \frac{vv_x'}{c^2}}}{1 + \frac{vv_x'}{c^2}}$$

• Thus $v_x = \frac{v_x' + v}{v_x'}$

• What would be the measured speed of light in S frame?

- Since
$$v'_{x} = c$$
 we get $v_{x} = \frac{c+v}{1+\frac{v^{2}}{c^{2}}} = \frac{c^{2}(c+v)}{c(c+v)} = c$

Observer in S frame measures c too! Strange but true!

 $1 + \frac{VV_x}{c^2}$

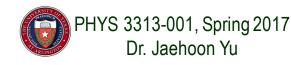


Velocity Addition Example

• Tom Brady is riding his bus at 0.8c relative to the observer. He throws a ball at 0.7c in the direction of his motion. What speed does the observer see?

$$v_{x} = \frac{v_{x}^{'} + v}{1 + \frac{v_{x}^{'}}{c^{2}}} \qquad v_{x} = \frac{.7c + .8c}{1 + \frac{.7 \times .8c^{2}}{c^{2}}} = 0.962c$$

- What if he threw it just a bit harder?
- Doesn't help—asymptotically approach c, can't exceed (it's not just a postulate it's the law)



A test of Lorentz velocity addition: π^0 decay

- How can one test experimentally the correctness of the Lorentz velocity transformation vs Galilean one?
- In 1964, T. Alvager and company performed a measurements of the arrival time of two photons resulting from the decay of a π^0 in two detectors separated by 30m.
- Each photon has a speed of 0.99975c. What are the speed predicted by Galilean and Lorentz x-mation?

$$- v_{G} = c + 0.99975c = 1.99975c$$

$$v_L = \frac{c + 0.99975c}{1 + 0.99975c^2/c^2} = \approx c$$

• How much time does the photon take to arrive at the detector?

