PHYS 3313 – Section 001
Lecture #7

Wednesday, Feb. 8, 2017
Dr. Jaehoon Yu

- Relativistic Velocity Addition
- The Twin Paradox
- Space-time Diagram
- Invariant Quantities
- The Relativistic Doppler Effect
- Relativistic Momentum and Energy

Announcements

- Submit homework #1 now!
- Reading assignments: CH 3.3 (special topic – the discovery of Helium) and CH3.7
- Colloquium today
  - Dr. Cosmin Deaconu, Univ. of Chicago
  - Radio-detection of Extreme High Energy Neutrinos in Polar Ice

Physics Department
The University of Texas at Arlington
COLLOQUIUM

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Radio-detection of Extremely High-Energy Neutrinos in Polar Ice

Dr. Cosmin Deaconu
University of Chicago

Wednesday February 8, 2017
4:00 Room 103 Science Hall

Abstract
Interactions of the highest-energy cosmic rays with the cosmic microwave background produce a population of extremely-high-energy (~EeV) neutrinos. Detecting these cosmogenic neutrinos would help elucidate the unknown sources of ultra-high-energy cosmic rays and provide the highest-energy measurements of the neutrino-nucleon cross- section. In order to have a chance of measurement, enormous detection targets are necessary.

Radio-detection of the cascades produced by neutrino interactions in polar ice allows for economical instrumentation of vast volumes. The Antarctic Impulsive Transient Antenna (ANITA) balloon-borne interferometric payload can scan a million cubic kilometers of Antarctic ice at a time. I will discuss the recent fourth flight of ANITA as well as results from previous flights.

In-ice antenna arrays, such as the Askaryan Radio Array (ARA) at the South Pole, have smaller detector volumes but are capable of detecting lower-energy neutrinos. An in-ice phased-antenna array is expected to further reduce the detection threshold and be sensitive to the lower-energy (~PeV) astrophysical neutrino flux detected by IceCube. I will discuss efforts aimed at deploying a phased array in the 2017-2018 Antarctic season.

Refreshments will be served at 3:30 p.m. in the Physics Library

Reminder: Special Project #3

1. Derive the three Lorentz velocity transformation equations, starting from Lorentz coordinate transformation equations. (10 points)
2. Derive the three reverse Lorentz velocity transformation equations, starting from Lorentz coordinate transformation equations. (10 points)
3. Prove that the space-time invariant quantity $s^2=x^2-(ct)^2$ is indeed invariant, i.e. $s^2=s'^2$, in Lorentz Transformation. (5 points)
4. You must derive each one separately starting from the Lorentz spatial coordinate transformation equations to obtain any credit.
   – Just simply switching the signs and primes will NOT be sufficient!
   – Must take the simplest form of the equations, using β and γ.
5. You MUST have your own, independent handwritten answers to the above three questions even if you worked together with others. All those who share the answers will get 0 credit if copied.

- Due for the submission is this Monday, Feb. 13!

The Complete Lorentz Transformations

$$x' = \frac{x - vt}{\sqrt{1-\beta^2}}$$

$$x = \frac{x' + vt'}{\sqrt{1-\beta^2}}$$

$$y' = y$$

$$y = y'$$

$$z' = z$$

$$z = z'$$

$$t' = \frac{t - \left(vx/c^2\right)}{\sqrt{1-\beta^2}}$$

$$t = \frac{t' + \left(vx'/c^2\right)}{\sqrt{1-\beta^2}}$$

Addition of Velocities

How do we add velocities in a relativistic case?

Taking differentials of the Lorentz transformation, relative velocities may be calculated:

$$dx = \gamma\left(dx' + v\,dt'\right)$$

$$dy = dy'$$

$$dz = dz'$$

$$dt = \gamma\left[dt' + \left(v/c^2\right)dx'\right]$$

So that…

defining velocities as: $v_x = dx/dt$, $v_y = dy/dt$, $v'_x = dx'/dt'$, etc. it can be shown that:

$$\mathrm{v}_x = \frac{dx}{dt} = \frac{\gamma\left[dx' + v\,dt'\right]}{\gamma\left[dt' + \frac{v}{c^2}dx'\right]} = \frac{v'_x + v}{1 + \left(v/c^2\right)v'_x}$$

With similar relations for v_y and v_z:

$$\mathrm{v}_y = \frac{dy}{dt} = \frac{v'_y}{\gamma\left[1 + \left(v/c^2\right)v'_x\right]} \qquad \mathrm{v}_z = \frac{dz}{dt} = \frac{v'_z}{\gamma\left[1 + \left(v/c^2\right)v'_x\right]}$$

The Lorentz Velocity Transformations

In addition to the previous relations, the **Lorentz velocity transformations** for v'_x, v'_y, and v'_z can be obtained by switching primed and unprimed and changing v to $-v$.(the velocity of the moving frame!!)

$$v'_x = \frac{v_x - v}{1 - \left(v/c^2\right)v_x}$$

$$v'_y = \frac{v_y}{\gamma\left[1 - \left(v/c^2\right)v_x\right]}$$

$$v'_z = \frac{v_z}{\gamma\left[1 - \left(v/c^2\right)v_x\right]}$$

Velocity Addition Summary

- Galilean Velocity addition $v_x = v_x' + v$ where $v_x = \frac{dx}{dt}$ and $v_x' = \frac{dx'}{dt'}$
- From inverse Lorentz transform $dx = \gamma(dx' + vdt')$ and $dt = \gamma(dt' + \frac{v}{c^2}dx')$
- So $$v_x = \frac{dx}{dt} = \frac{\gamma(dx' + vdt')}{\gamma(dt' + \frac{v}{c^2}dx')} \div \frac{dt'}{dt'} = \frac{\frac{dx'}{dt'} + v}{1 + \frac{v}{c^2}\frac{dx'}{dt'}} = \frac{v_x' + v}{1 + \frac{vv_x'}{c^2}}$$
- Thus $$v_x = \frac{v_x' + v}{1 + \frac{vv_x'}{c^2}}$$
- What would be the measured speed of light in S frame?
  - Since $v_x' = c$ we get $$v_x = \frac{c + v}{1 + \frac{v^2}{c^2}} = \frac{c^2(c+v)}{c(c+v)} = c$$

Observer in S frame measures c too! Strange but true!

Velocity Addition Example

- Tom Brady is riding his bus at 0.8c relative to the observer. He throws a ball at 0.7c in the direction of his motion. What speed does the observer see?

$$v_x = \frac{v_x' + v}{1 + \frac{v v_x'}{c^2}}$$

$$v_x = \frac{.7c + .8c}{1 + \frac{.7 \times .8c^2}{c^2}} = 0.962c$$

- What if he threw it just a bit harder?
- Doesn't help—asymptotically approach c, can't exceed (it's not just a postulate it's the law)

A test of Lorentz velocity addition: π^0 decay

- How can one test experimentally the correctness of the Lorentz velocity transformation vs Galilean one?
- In 1964, T. Alvager and company performed a measurements of the arrival time of two photons resulting from the decay of a π^0 in two detectors separated by 30m.
- Each photon has a speed of 0.99975c. What are the speed predicted by Galilean and Lorentz x-mation?
  - $v_G = c + 0.99975c = 1.99975c$
  - $v_L = \frac{c + 0.99975c}{1 + 0.99975c^2/c^2} = \quad \approx c$
- How much time does the photon take to arrive at the detector?