PHYS 3313 – Section 001
Lecture #9

Wednesday, Feb. 15, 2017
Dr. Jaehoon Yu

- The Relativistic Doppler Effect
- Relativistic Momentum and Energy
- Relationship Between Relativistic Quantities
- Binding Energy
- Quantization
- Discovery of the X-ray and the Electron

Announcements

- Homework #2
  - CH3 end of the chapter problems: 2, 19, 27, 36, 41, 47 and 57
  - Due Wednesday, Feb. 22
- Reminder: Quiz #2 Monday, Feb. 20
  - Beginning of the class
  - Covers CH1.1 – what we finish today
  - You can bring your calculator but it must not have any relevant formula pre-input
  - BYOF: You may bring a one 8.5x11.5 sheet (front and back) of handwritten formulae and values of constants for the exam
  - No derivations, word definitions, or solutions of any problems !
    - Lorentz velocity addition NOT allowed!!
  - No additional formulae or values of constants will be provided!
- Colloquium today
  - Dr. P. Onyisi of UT Austin

Physics Department
The University of Texas at Arlington
COLLOQUIUM

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Studies and Searches with the 13 TeV LHC

Peter Onyisi
University of Texas Austin

Wednesday February 15, 2017
4:00 Room 103 Science Hall

Abstract
The Large Hadron Collider at CERN is accumulating its first large dataset near its design energy. After the discovery of the Higgs boson, attention is turning to other new features of nature the LHC may be able to illuminate. These include searches for new types of matter (including dark matter) and new forces, leading to tests of grand organizing principles that guide theoretical progress in the field. I will review the problems with our current models that lead us to pursue these searches for new particles and interactions and some of the diverse studies that have been performed with the latest LHC data.

Refreshments will be served at 3:30 p.m. in the Physics Library

Results of Relativistic Doppler Effect

When source/receiver is approaching with $\beta = v/c$ the resulting frequency is

$$f = \sqrt{\frac{1+\beta}{1-\beta}}\, f_0$$

Higher than the actual source's frequency, blue shift!!

When source/receiver is receding with $\beta = v/c$ the resulting frequency is

$$f = \sqrt{\frac{1-\beta}{1+\beta}}\, f_0$$

Lower than the actual source's frequency, red shift!!

If we use $+\beta$ for approaching source/receiver and $-\beta$ for receding source/receiver, relativistic Doppler Effect can be expressed

$$f = \sqrt{\frac{1+\beta}{1-\beta}}\, f_0$$

For more generalized case

$$f = \frac{1+\beta\cos\theta'}{\sqrt{1-\beta^2}}\, f_0$$

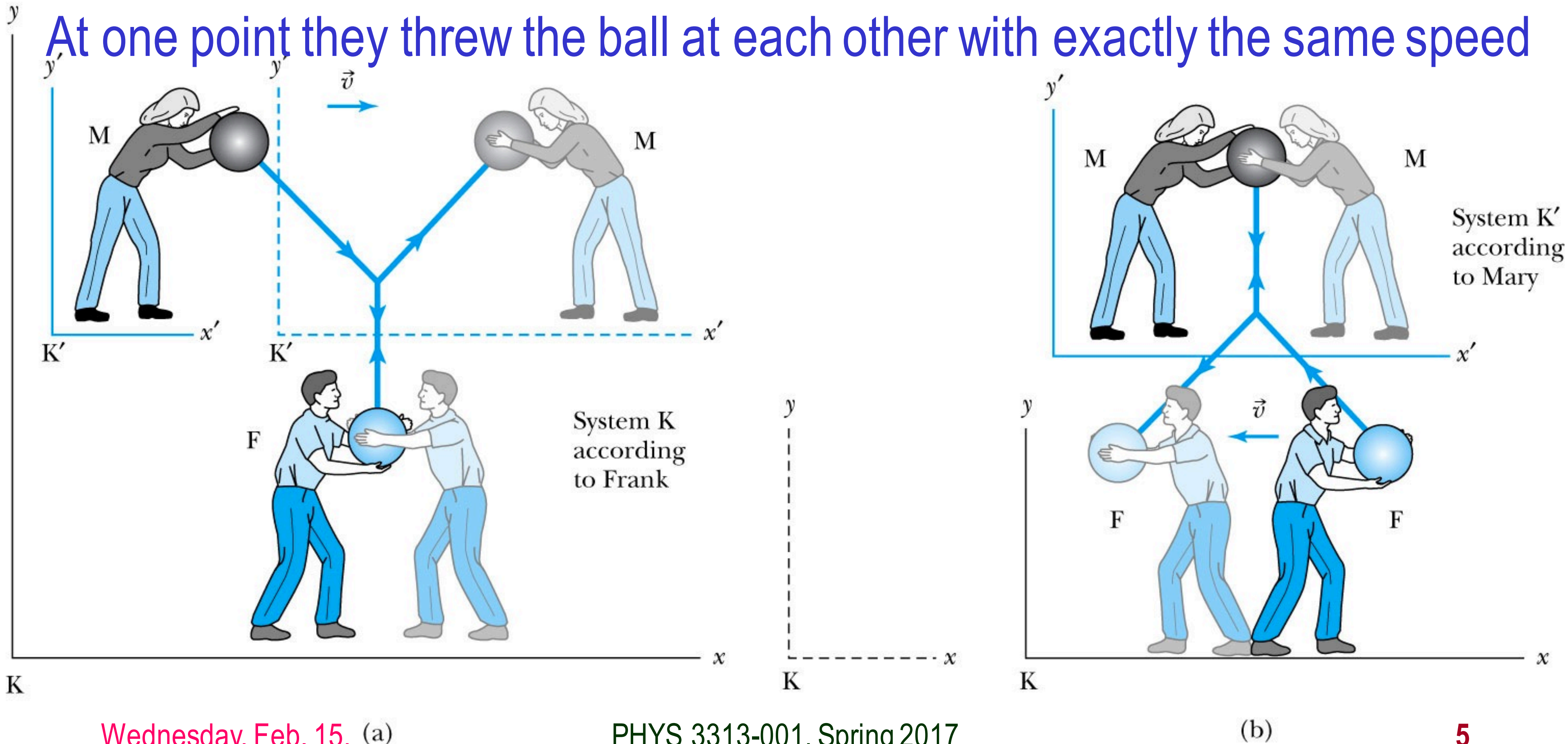
Relativistic Momentum

The most fundamental principle used here is the momentum conservation!

Frank is at rest in system K holding a ball of mass *m*.

Mary holds a similar ball in system K' that is moving in the *x* direction with velocity *v* with respect to system K.

At one point they threw the ball at each other with exactly the same speed



Relativistic Momentum

- If we use the definition of momentum, the momentum of the ball thrown by Frank is entirely in the *y* direction

$$p_{Fy} = mu_0$$

- The change of momentum as observed by Frank is

$$\Delta p_F = \Delta p_{Fy} = -2mu_0$$

- Mary measures the initial velocity of her own ball to be

$$u'_{Mx} = 0 \text{ and } u'_{My} = -u_0.$$

- In order to determine the velocity of Mary's ball as measured by Frank we use the velocity transformation equations:

Reminder: Lorentz Velocity Transformations

In addition to the previous relations, the **Lorentz velocity transformations** for v'_x, v'_y, and v'_z can be obtained by switching primed and unprimed and changing v to $-v$.(the velocity of the moving frame!!)

$$v'_x = \frac{v_x - v}{1 - \left(v/c^2\right) v_x}$$

$$v'_y = \frac{v_y}{\gamma\left[1 - \left(v/c^2\right) v_x\right]}$$

$$v'_z = \frac{v_z}{\gamma\left[1 - \left(v/c^2\right) v_x\right]}$$

Relativistic Momentum

- If we use the definition of momentum, the momentum of the ball thrown by Frank is entirely in the *y* direction

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- Mary measures the initial velocity of her own ball to be

$$u'_{Mx} = 0 \text{ and } u'_{My} = -u_0.$$

- In order to determine the velocity of Mary's ball as measured by Frank we use the velocity transformation equations:

$$u_{Mx} = v \qquad u_{My} = -u_0\sqrt{1 - v^2/c^2}$$

Relativistic Momentum

Before the collision, the momentum of Mary's ball as measured by Frank (in the **F**ixed frame) with the Lorentz velocity transformation becomes

$$p_{Mx} = mv \qquad p_{My} = -mu_0\sqrt{1 - v^2/c^2}$$

For a perfectly elastic collision, the momentum after the collision is

$$p_{Mx} = mv \qquad p_{My} = +mu_0\sqrt{1 - v^2/c^2}$$

Thus the change in momentum of Mary's ball according to Frank is

$$\Delta p_M = \Delta p_{My} = 2mu_0\sqrt{1-\beta^2} \neq -\Delta p_{Fy}$$

OMG! The linear momentum is not conserved even w/o an external force!!

What do we do?

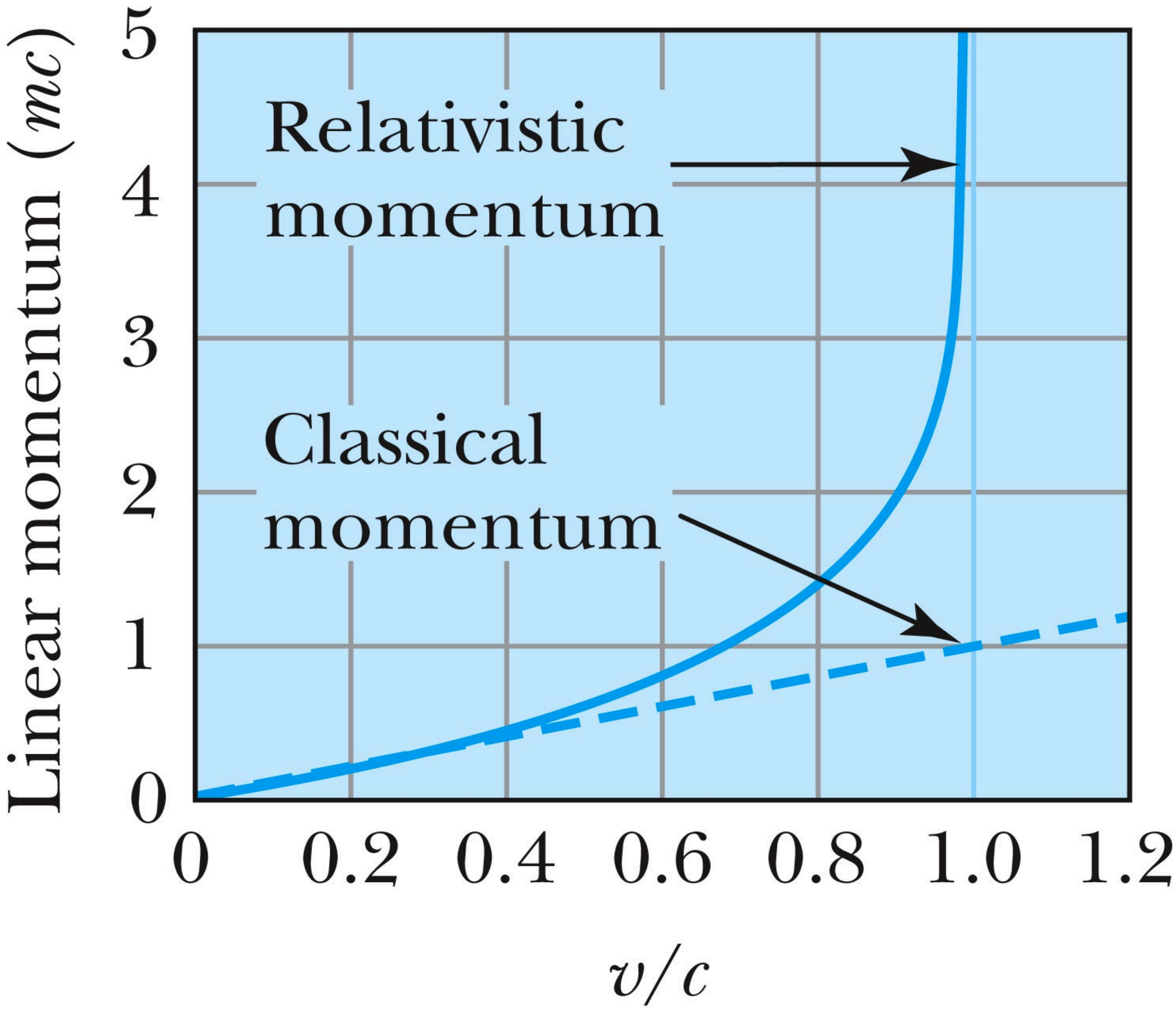
$$\vec{p} = m\frac{d\left(\gamma_u \vec{r}\right)}{dt} = m\gamma_u \vec{u}$$

➔Redefine the momentum in a fashion

➔Something has changed. Mass is now, mγ!! The relativistic mass!!

➔Mass as the fundamental property of matter is called the "rest mass", m_0!

Relativistic and Classical Linear Momentum



How do we keep momentum conserved in a relativistic case?

Redefine the classical momentum in the form:

$$\vec{p} = \Gamma(u) m\vec{u} = \frac{1}{\sqrt{1-u^2/c^2}} m\vec{u}$$

This $\Gamma(u)$ is different than the γ factor since it uses the particle's speed u

➔ What? How does this make sense?

➔ Well the particle itself is moving at a relativistic speed, thus that must impact the measurements by the observer in the rest frame!!

Now, the agreed form of the momentum in all frames is (τ is the proper time):

$$\vec{p} = m\frac{d\vec{r}}{d\tau} = m\frac{d\vec{r}}{dt}\frac{dt}{d\tau} = m\vec{u}\gamma = \frac{1}{\sqrt{1-u^2/c^2}} m\vec{u}$$

Resulting in the new relativistic definition of the momentum: $\vec{p} = m\gamma\vec{u}$

When u➔0, this formula becomes that of the classical.

What can the speed **u** be to maintain the relativistic momentum to 1% of classical momentum?

Relativistic Energy

- Due to the new idea of relativistic mass, we must now redefine the concepts of work and energy.
  - Modify Newton's second law to include our new definition of linear momentum, and the force becomes:

$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d}{dt}\left(\gamma m\vec{u}\right) = \frac{d}{dt}\left(\frac{m\vec{u}}{\sqrt{1-u^2/c^2}}\right)$$

- The work *W* done by a force **F** to move a particle from rest to a certain kinetic energy is $W = K = \int \frac{d}{dt}\left(\gamma m\vec{u}\right)\cdot\vec{u}dt$

- Resulting relativistic kinetic energy becomes

$$K = \int_0^{\gamma u} um \cdot d\left(\gamma u\right) = \gamma mc^2 - mc^2 = \left(\gamma - 1\right)mc^2$$

- Why doesn't this look anything like the classical KE?

Big note on Relativistic KE

- Only $K = (\gamma - 1)mc^2$ is right!
- $K = \frac{1}{2}mu^2$ and $K = \frac{1}{2}\gamma mu^2$ are wrong!

Total Energy and Rest Energy

Rewriting the relativistic kinetic energy:

$$\gamma mc^2 = \frac{mc^2}{\sqrt{1-u^2/c^2}} = K + mc^2$$

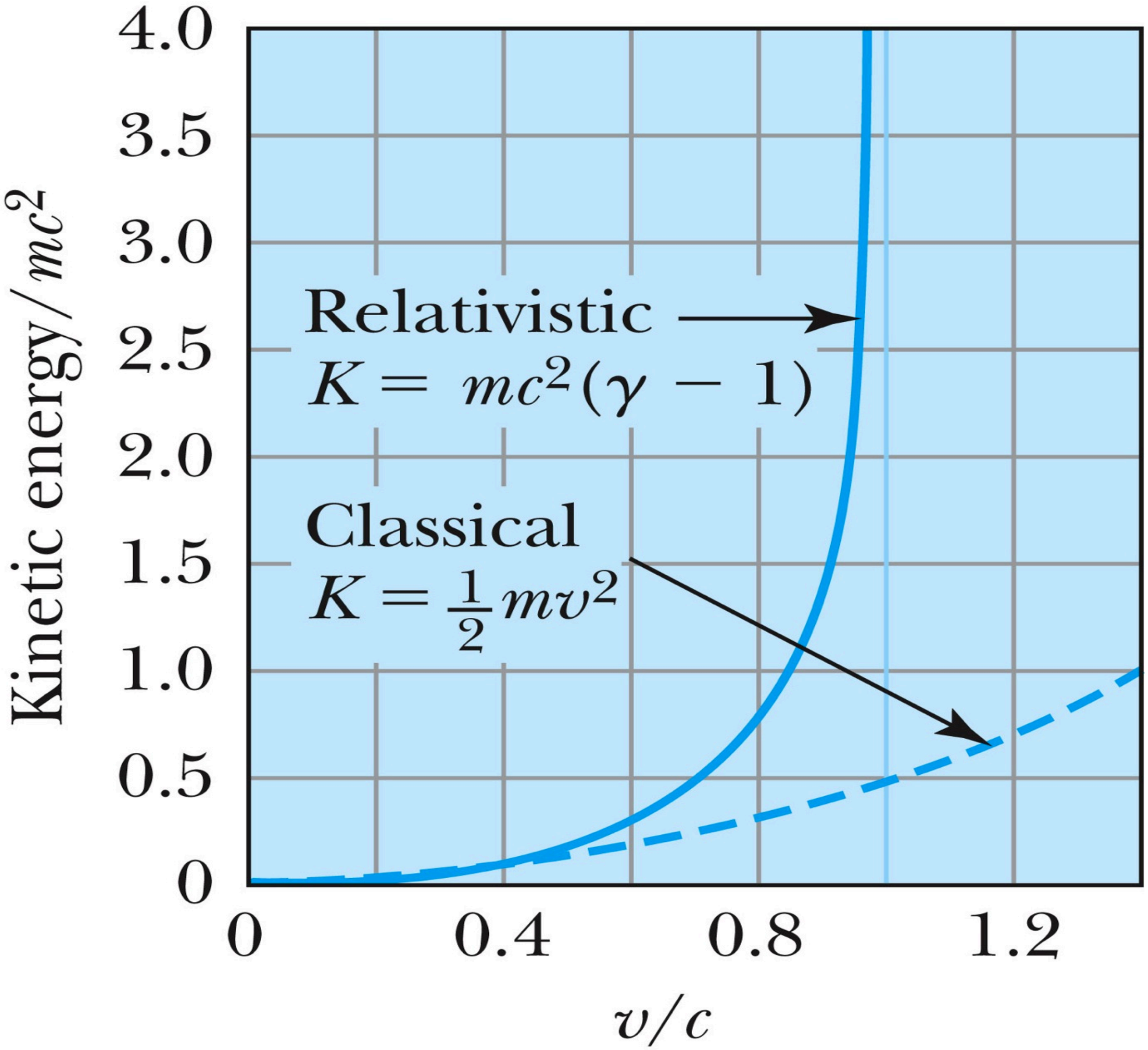
The term mc^2 is called the rest energy and is denoted by E_0.

$$E_0 = mc^2$$

The sum of the kinetic energy and rest energy is interpreted as the total energy of the particle. (note that u is the speed of the particle)

$$E_{Tot} = \gamma mc^2 = \frac{mc^2}{\sqrt{1-u^2/c^2}} = \frac{E_0}{\sqrt{1-u^2/c^2}} = K + E_0$$

Relativistic and Classical Kinetic Energies



Relationship of Energy and Momentum

$$p = \gamma m u = \frac{mu}{\sqrt{1 - u^2/c^2}}$$

We square this formula, multiply by c^2, and rearrange the terms.

$$p^2c^2 = \gamma^2 m^2 u^2 c^2 = \gamma^2 m^2 c^4 \left(\frac{u^2}{c^2}\right) = \gamma^2 m^2 c^4 \beta^2$$

$$\beta^2 = 1 - \frac{1}{\gamma^2} \Rightarrow p^2c^2 = \gamma^2 m^2 c^4 \left(1 - \frac{1}{\gamma^2}\right) = \gamma^2 m^2 c^4 - m^2 c^4$$

Rewrite

$$p^2c^2 = E^2 - E_0^2$$

Rewrite

$$E^2 = p^2c^2 + E_0^2 = p^2c^2 + m^2c^4$$

Massless Particles have a speed equal to the speed of light c

- Recall that a photon has "zero" rest mass and the equation from the last slide reduces to: E = pc and we may conclude that:

$$E = \gamma mc^2 = pc = \gamma muc$$

- Thus the speed, u, of a massless particle must be c since, as $m \to 0, \gamma \to \infty$ and it follows that: u = c.

Units of Work, Energy and Mass

- The work done in accelerating a charge through a potential difference V is $W = qV$.
  - For a proton, with the charge $e = 1.602 \times 10^{-19}$ C being accelerated across a potential difference of 1 V, the work done is

$$1\ \text{eV} = 1.602 \times 10^{-19}\ \text{J}$$

$$W = (1.602 \times 10^{-19})(1\ \text{V}) = 1.602 \times 10^{-19}\ \text{J}$$

- eV is also used as a unit of energy.

Other Units

1) Rest energy of a particle:
   Example: Rest energy, E_0, of proton

$$E_0(proton) = m_p c^2 = \left(1.67 \times 10^{-27} kg\right) \cdot \left(3.00 \times 10^8 m/s\right) = 1.50 \times 10^{-10} J$$

$$= 1.50 \times 10^{-10} J \cdot \frac{1eV}{1.602 \times 10^{-19} J} = 9.38 \times 10^8 eV$$

2) **Atomic mass unit** (amu): Example: carbon-12

$$M\left({}^{12}C \text{ atom}\right) = \frac{12\, g/mole}{6.02 \times 10^{23} atoms/mole}$$

$$= 1.99 \times 10^{-23} g/atom$$

$$M\left({}^{12}C \text{ atom}\right) = 1.99 \times 10^{-26} kg/atom = 12 u/atom$$

What is 1u in eV?

Binding Energy

- The potential energy associated with the force keeping a system together ➔ E_B.
- The difference between the rest energy of the individual particles and the rest energy of the combined bound system.

$$M_{\text{bound system}}c^2 + E_B = \sum_i m_i c^2$$

$$E_B = \sum_i m_i c^2 - M_{\text{bound system}}c^2$$

Examples 2.13 and 2.15

- Ex. 2.13: A proton with 2-GeV kinetic energy hits another proton with 2 GeV KE in a head on collision. (proton rest mass = 938MeV/c^2)
  - Compute v, β, p, K and E for each of the initial protons
  - What happens to the kinetic energy?
- Ex. 2.15: What is the minimum kinetic energy the protons must have in the head-on collision in the reaction $p+p \rightarrow \pi^+ + d$, in order to produce the positively charged pion (139.6MeV/c^2) and a deuteron.(1875.6MeV/c^2).