PHYS 3313 – Section 001
Lecture #17

Wednesday, March 29, 2017
*Dr. **Jae**hoon **Yu***

- Wave Motion and Properties
- Wave Packets and Packet Envelops
- Superposition of Waves
- Electron Double Slit Experiment
- Wave-Particle Duality
- Uncertainty Principle
- The Copenhagen Interpretation

Announcements

- Reminder: Quadruple extra credit
  - Colloquium on April 19, 2017
  - Speaker: Dr. Nigel Lockyer, Director of Fermilab
  - Make your arrangements to take advantage of this opportunity
  - Seats may be limited
- Colloquium today
  - Dr. Alphonse Sterling, NASA
  - Title: Jets in the Sun’s Corona

Physics Department
The University of Texas at Arlington
Colloquium

Jets in the Sun's Corona

Alphonse Sterling
NASA

Wednesday March 29, 2017
4:00 Room 103 Science Hall

Abstract

Observations from space over the last 25 years revealed that relatively long and narrow jets of plasma shoot out from the Sun's surface into its 2-MK outer atmosphere, the corona. These "coronal jets" are transient (lifetime ~10 min), and very common, with about 60 of them occurring per day in the Sun's polar regions alone, with many more in other solar regions (quiet Sun and active regions). Coronal jets are a consequence of the dynamic magnetic field that permeates the entire solar atmosphere. For a long while the solar community believed that the jets resulted when flux emerged from below the solar surface and reconnected with the base of the coronal field. But in recent years, based on new, high-resolution, high-cadence, multi-wavelength observations, that previously-well-established "emerging-flux" idea for jet production has come into serious question. Instead, an idea (Sterling et al. 2015, Nature, 523, 437) that coronal jets are miniature versions of the large-scale filament eruptions that produce solar flares and coronal mass ejections (CMEs) seems to fit many of the recent observations better. Alas however, close study of some jets, in particular those in active regions, raises the question of whether this "minifilament-eruption" idea truly best explains all coronal jets. Chromospheric jets called "spicules" are 10-100 times smaller than coronal jets, but a million times more numerous. They might work the same way as coronal jets, and might power the corona and the solar wind.

Refreshments will be served at 3:30 p.m. in the Physics Library

Special Project #4

- Prove that the wave function $\Psi=A[\sin(kx-\omega t)+i\cos(kx-\omega t)]$ is a good solution for the time-dependent Schrödinger wave equation. Do NOT use the exponential expression of the wave function. (10 points)
- Determine whether or not the wave function $\Psi=Ae^{-\alpha|x|}$ satisfy the time-dependent Schrödinger wave equation. (10 points)
- Due for this special project is Wednesday, Apr. 5.
- You MUST have your own answers!

Wave Motion

- de Broglie matter waves suggest a further description. The displacement of a typical traveling wave is

$$\Psi(x,t) = A\sin\left[\frac{2\pi}{\lambda}(x - vt)\right]$$

- This is a solution to the wave equation

$$\frac{\partial^2 \Psi}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 \Psi}{\partial t^2}$$

- Define the wave number *k* and the angular frequency ω as:

$$k \equiv \frac{2\pi}{\lambda} \quad \text{and} \quad \omega = \frac{2\pi}{T} \qquad \lambda = vT$$

- The wave function can be rewritten: $\Psi(x,t) = A\sin[kx - \omega t]$

Wave Properties

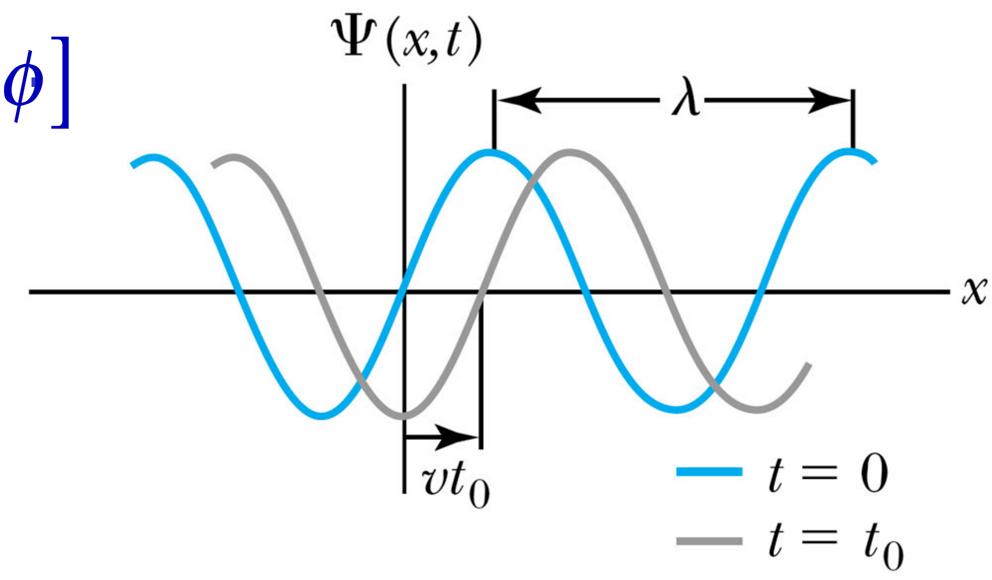
- The phase velocity is the velocity of a point on the wave that has a given phase (for example, the crest) and is given by

$$v_{ph} = \frac{\lambda}{T} = \frac{\lambda}{2\pi}\frac{2\pi}{T} = \frac{\omega}{k}$$

- The phase constant ϕ shifts the wave:

$$\Psi(x,t) = A\sin[kx - \omega t + \phi]$$
$$= A\cos[kx - \omega t]$$

(When $\phi=\pi/2$)



Principle of Superposition

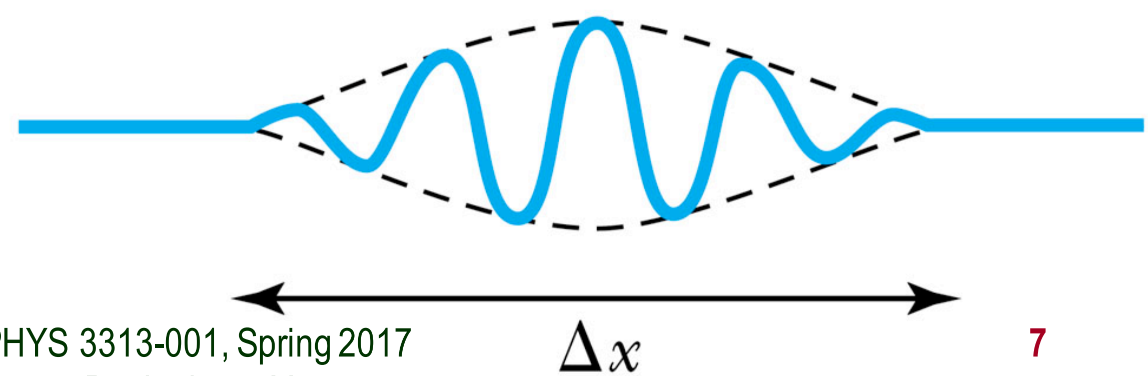
- When two or more waves traverse in the same region, they act independently of each other.
- Combining two waves of the same amplitude yields:

$$\Psi(x,t) = \Psi_1(x,t) + \Psi_2(x,t) = A\cos(k_1 x - \omega_1 t) + A\cos(k_2 x - \omega_2 t) =$$

$$2A\cos\left(\frac{k_1 - k_2}{2}x - \frac{\omega_1 - \omega_2}{2}t\right)\cos\left(\frac{k_1 + k_2}{2}x - \frac{\omega_1 + \omega_2}{2}t\right) = 2A\cos\left(\frac{\Delta k}{2}x - \frac{\Delta\omega}{2}t\right)\cos(k_{av}x - \omega_{av}t)$$

- The combined wave oscillates within an envelope that denotes the maximum displacement of the combined waves.
- When combining many waves with different amplitudes and frequencies, a pulse, or **wave packet**, can be formed, which can move with a **group velocity**:

$$u_{gr} = \frac{\Delta\omega}{\Delta k}$$



Fourier Series

- The sum of many waves that form a wave packet is called a **Fourier series**:

$$\Psi(x,t) = \sum_i A_i \sin\left[k_i x - \omega_i t\right]$$

- Summing an infinite number of waves yields the Fourier integral:

$$\Psi(x,t) = \int \tilde{A}(k) \cos\left[kx - \omega t\right] dk$$

Wave Packet Envelope

- The superposition of two waves yields the wave number and angular frequency of the wave packet envelope.

$$\cos\left(\frac{\Delta k}{2}x - \frac{\Delta\omega}{2}t\right)$$

- The range of wave numbers and angular frequencies that produce the wave packet have the following relations:

$$\Delta k \Delta x = 2\pi \qquad \Delta\omega\Delta t = 2\pi$$

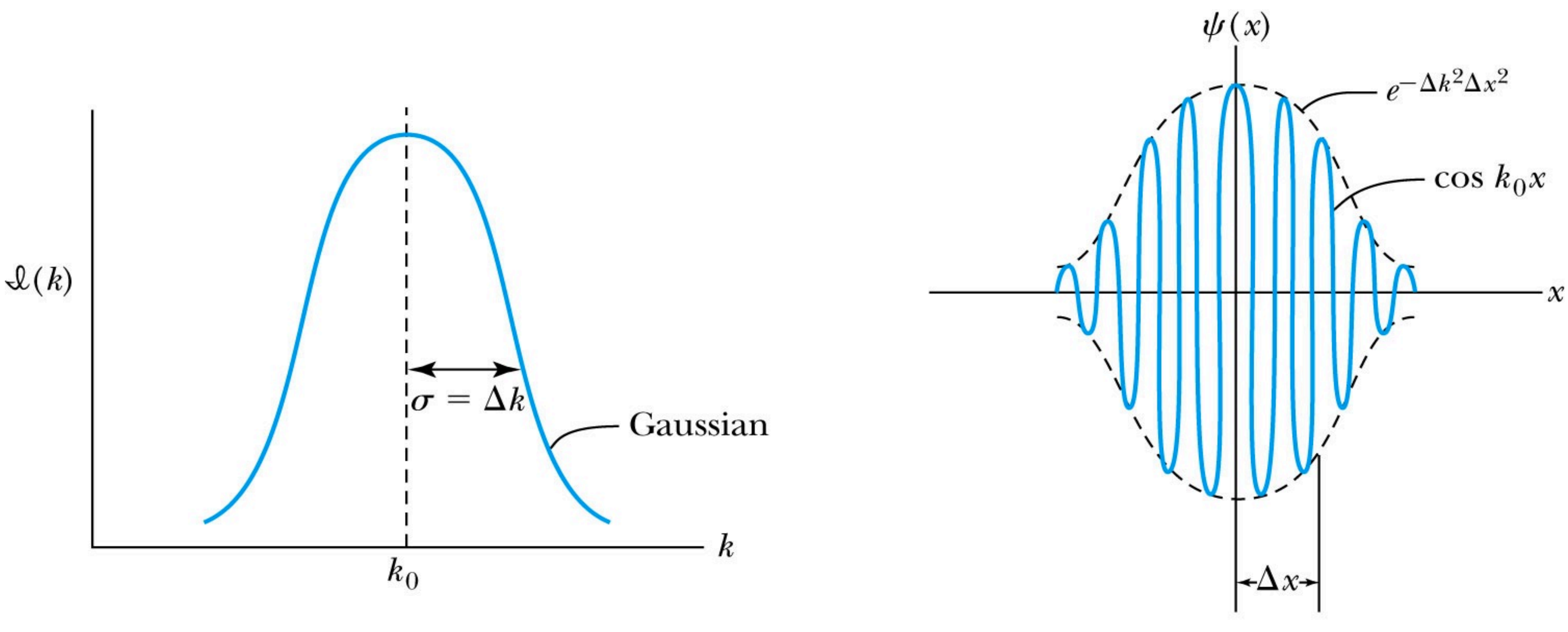
- A **Gaussian wave packet** has similar relations:

$$\Delta k \Delta x = \frac{1}{2} \qquad \Delta\omega\Delta t = \frac{1}{2}$$

- The localization of the wave packet over a small region to describe a particle requires a large range of wave numbers. Conversely, a small range of wave numbers cannot produce a wave packet localized within a small distance.

Gaussian Function

- A Gaussian wave packet describes the envelope of a pulse wave. $\Psi(x,0) = \Psi(x) = Ae^{-\Delta k^2 x^2}\cos(k_0 x)$

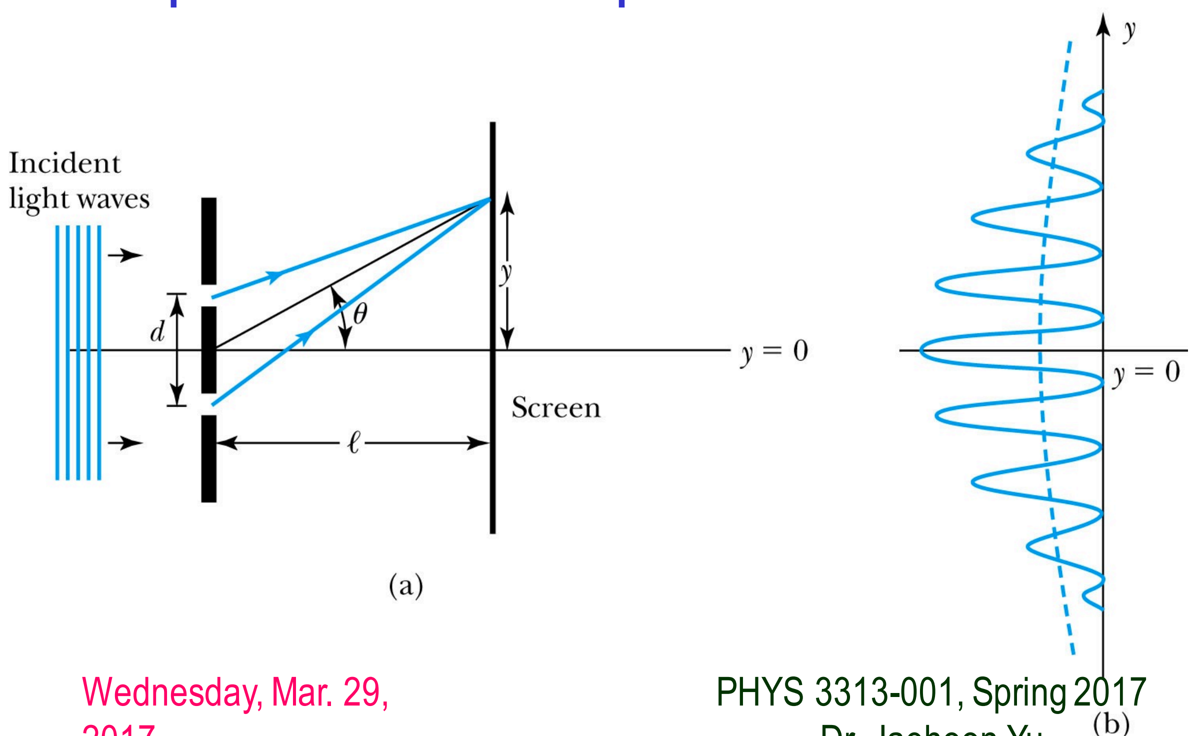


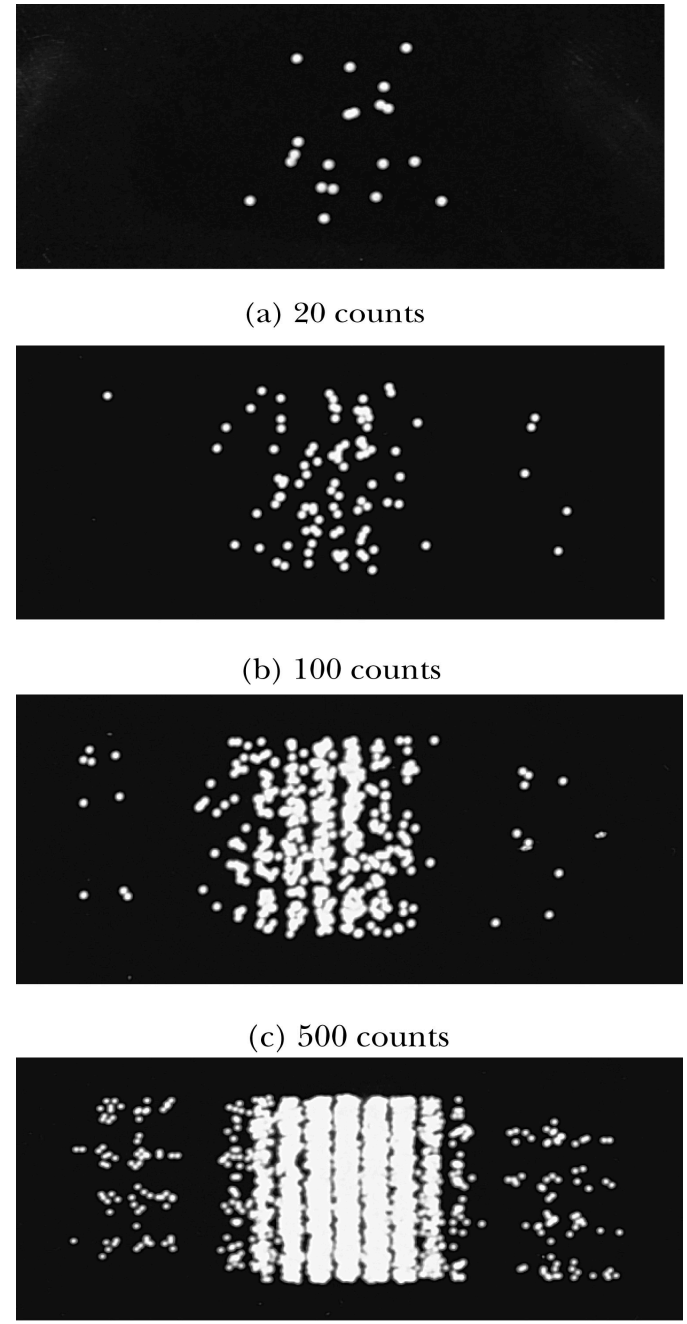
(a) (b)

- The group velocity is $u_{gr} = \dfrac{d\omega}{dk}$

Waves or Particles?

- Young's double-slit diffraction experiment demonstrates the wave property of the light.
- However, dimming the light results in single flashes on the screen representative of particles.





(a) 20 counts

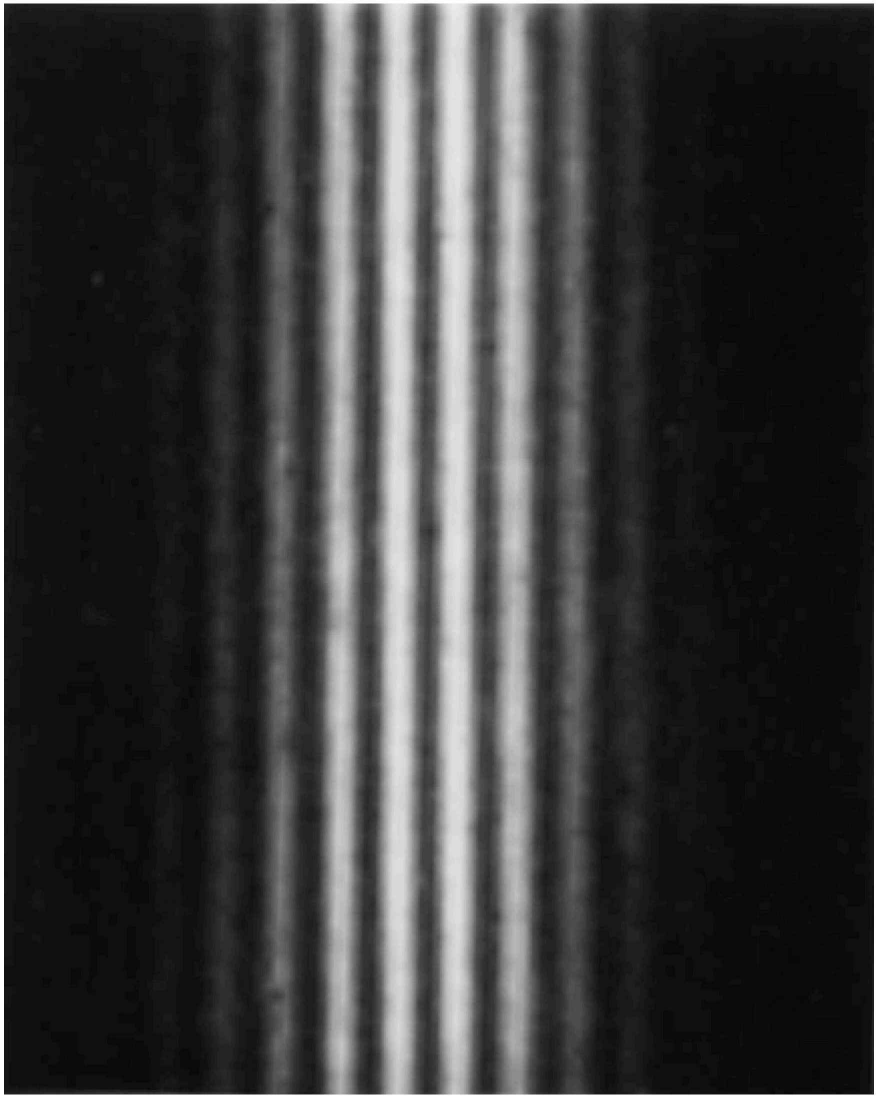
(b) 100 counts

(c) 500 counts

(d) ~4000 counts

Electron Double-Slit Experiment

- C. Jönsson of Tübingen, Germany, succeeded in 1961 in showing double-slit interference effects for **electrons** by constructing very narrow slits and using relatively large distances between the slits and the observation screen.
- This experiment demonstrated that precisely the same behavior occurs for both light (waves) and electrons (particles).



Wave particle duality solution

- The solution to the wave particle duality of an event is given by the following principle.
- **Bohr's principle of complementarity**: It is not possible to describe physical observables simultaneously in terms of both particles and waves.
- **Physical observables** are the quantities such as position, velocity, momentum, and energy that can be experimentally measured. In any given instance we must use either the particle description or the wave description.

Uncertainty Principles

- It is impossible to measure simultaneously, with no uncertainty, the precise values of *k* and *x* for the same particle. The wave number *k* may be rewritten as

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{h/p} = p\frac{2\pi}{h} = \frac{p}{\hbar}$$

- For the case of a Gaussian wave packet we have

$$\Delta k \Delta x = \frac{\Delta p}{\hbar}\Delta x = \frac{1}{2} \quad \Rightarrow \Delta p_x \Delta x = \frac{\hbar}{2}$$

Thus for a single particle, we have Heisenberg's **uncertainty principle**:

$$\Delta p_x \Delta x \geq \frac{\hbar}{2}$$ Momentum-position

$$\Delta E \Delta t \geq \frac{\hbar}{2}$$ Energy-time

Probability, Wave Functions, and the Copenhagen Interpretation

- The wave function determines the likelihood (or probability) of finding a particle at a particular position in space at a given time.

$$P(y)dy = \left|\Psi(y,t)\right|^2 dy$$

- The total probability of finding the particle is 1. Forcing this condition on the wave function is called the normalization.

$$\int_{-\infty}^{+\infty} P(y)dy = \int_{-\infty}^{+\infty} \left|\Psi(y,t)\right|^2 dy = 1$$

The Copenhagen Interpretation

- Bohr's interpretation of the wave function consisted of 3 principles:
  1) The uncertainty principle of Heisenberg
  2) The complementarity principle of Bohr
  3) The statistical interpretation of Born, based on probabilities determined by the wave function
- Together, these three concepts form a logical interpretation of the physical meaning of quantum theory. According to the Copenhagen interpretation, physics depends on the outcomes of measurement.

Particle in a Box

- A particle of mass m is trapped in a one-dimensional box of width l.
- The particle is treated as a wave.
- The box puts boundary conditions on the wave. The wave function must be zero at the walls of the box and on the outside.
- In order for the probability to vanish at the walls, we must have an integral number of half wavelengths in the box.
  – $$\frac{n\lambda}{2} = \ell \quad \text{or} \quad \lambda_n = \frac{2\ell}{n} \qquad (n = 1,2,3.....)$$
- The energy of the particle is $$E = KE = \frac{1}{2}mv^2 = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2}$$
- The possible wavelengths are quantized which yields the energy:

$$E_n = \frac{h^2}{2m\lambda^2} = \frac{h^2}{2m}\left(\frac{n}{2\ell}\right)^2 = n^2\frac{h^2}{8m\ell^2} \qquad (n = 1,2,3.....)$$

- The possible energies of the particle are quantized.
- Find the quantized energy level of an electron constrained to move in a 1-D atom of size 0.1nm.