PHYS 3313 – Section 001
Lecture #19

Wednesday, April 5, 2017
Dr. Jaehoon Yu

- Time Independent Wave Equation
- Expectation Values
- Momentum Operator
- Position and Energy Operators
- Infinite Square-well Potential

Announcements

- Homework #4
  - End of chapter problems on CH5: 8, 10, 16, 24, 26, 36 and 47
  - Due Wednesday, Apr. 12
- Reminder: Quadruple extra credit
  - Colloquium on April 19, 2017
  - Speaker: Dr. Nigel Lockyer, Director of Fermilab
  - Make your arrangements to take advantage of this opportunity
- Colloquium today
  - Dr. Jodi Cooley of SMU
  - Title: In Pursuit of Dark Matter: Recent Results from SuperCDMS

Physics Department
The University of Texas at Arlington
<u>Colloquium</u>

In Pursuit of Dark Matter: Recent Results from SuperCDMS

Dr. Jodi Cooley
Southern Methodist University

Wednesday April 5, 2017
4:00 Room 103 Science Hall

Abstract

Over the last two decades, astrophysicists and astronomers have produced compelling evidence on galactic and cosmological scales indicates that ~80% of the matter density of the Universe consists of non-luminous, non-baryonic dark matter. Despite this fact, the composition of the dark matter remains unknown. One compelling candidate for particle dark matter is the Weakly Interacting Massive Particle (WIMP). Working in a low-background environment in the Soudan Mine, located in northern Minnesota, the SuperCDMS experiment was designed to directly detect interactions between WIMPs and nuclei in its target Ge crystals. In this talk I will present the latest results from the SuperCDMS experiment. I will also discuss the current status of the SuperCDMS at SNOLAB experiment and plans for a future 50-kg scale experiment which is slated for operation in SNOLAB.

Refreshments will be served at 3:30 p.m. in the Physics Library

Special project #6

- Show that the Schrodinger equation becomes Newton's second law in the classical limit. (15 points)
- Deadline Monday, Apr. 17, 2017
- You MUST have your own answers!

Properties of a Valid Wave Function

Boundary conditions

1) To avoid infinite probabilities, the wave function must be finite everywhere.
2) To avoid multiple values of the probability, the wave function must be single valued.
3) For a finite potential, the wave function and its derivatives must be continuous. This is required because the second-order derivative term in the wave equation must be single valued. (There are exceptions to this rule when V is infinite.)
4) In order to normalize the wave functions, they must approach zero as x approaches infinity.

Solutions that do not satisfy these properties do not generally correspond to physically realizable circumstances.

Time-Independent Schrödinger Wave Equation

- The potential in many cases will not depend explicitly on time.
- The dependence on time and position can then be separated in the Schrödinger wave equation. Let, $\Psi(x,t)=\psi(x)f(t)$

which yields: $i\hbar\psi(x)\frac{\partial f(t)}{\partial t}=-\frac{\hbar^2 f(t)}{2m}\frac{\partial^2\psi(x)}{\partial x^2}+V(x)\psi(x)f(t)$

Now divide by the wave function: $i\hbar\frac{1}{f(t)}\frac{\partial f(t)}{\partial t}=-\frac{\hbar^2 f(t)}{2m}\frac{1}{\psi(x)}\frac{\partial^2\psi(x)}{\partial x^2}+V(x)$

- The *left-hand side* of this last equation depends only on time, and *the right* depends only on spatial coordinates. Hence each side must be equal to a constant. The time dependent side is

$$i\hbar\frac{1}{f}\frac{df}{dt}=B$$

Time-Independent Schrödinger Wave Equation(con't)

- We integrate both sides and find: $i\hbar\int\frac{df}{f}=\int B\,dt \Rightarrow i\hbar\ln f = Bt + C$

  where *C* is an integration constant that we may choose to be 0. Therefore $$\ln f = \frac{Bt}{i\hbar}$$

  This determines *f* to be $f(t)=e^{Bt/i\hbar}=e^{-iBt/\hbar}$. Comparing this to the time dependent portion of the free particle wave function $e^{-i\omega t}=e^{-iBt/\hbar}$

  $$\Rightarrow B=\hbar\omega=E \quad \Rightarrow \quad i\hbar\frac{1}{f(t)}\frac{\partial f(t)}{\partial t}=E$$

- This is known as the **time-independent Schrödinger wave equation**, and it is a fundamental equation in quantum mechanics.

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2}+V(x)\psi(x)=E\psi(x)$$

Stationary State

- Recalling the separation of variables: $\Psi(x,t)=\psi(x)f(t)$

  and with $f(t)=e^{-i\omega t}$ the wave function can be written as: $\Psi(x,t)=\psi(x)e^{-i\omega t}$
- The probability density becomes:

$$\Psi^*\Psi=\psi^2(x)\left(e^{i\omega t}e^{-i\omega t}\right)=\psi^2(x)$$

- The probability distributions are constant in time. This is a standing wave phenomena that is called the stationary state.

Comparison of Classical and Quantum Mechanics

- Newton's second law and Schrödinger's wave equation are both differential equations.
- Newton's second law can be derived from the Schrödinger wave equation, so the latter is the more fundamental.
- Classical mechanics only appears to be more precise because it deals with macroscopic phenomena. The underlying uncertainties in macroscopic measurements are just too small to be significant.

Expectation Values

- The **expectation value** is the expected result of the average of many measurements of a given quantity. The expectation value of *x* is denoted by <*x*>.
- Any measurable quantity for which we can calculate the expectation value is called the **physical observable**. The expectation values of physical observables (for example, position, linear momentum, angular momentum, and energy) must be real, because the experimental results of measurements are real.
- The average value of *x* is $\bar{x} = \frac{N_1 x_1 + N_2 x_2 + N_3 x_3 + N_4 x_4 + \cdots}{N_1 + N_2 + N_3 + N_4 + \cdots} = \frac{\sum_i N_i x_i}{\sum_i N_i}$

Continuous Expectation Values

- We can change from discrete to continuous variables by using the probability *P*(*x*,*t*) of observing the particle at the particular *x*.

$$\bar{x} = \frac{\int_{-\infty}^{+\infty} xP(x)\,dx}{\int_{-\infty}^{+\infty} P(x)\,dx}$$

- Using the wave function, the expectation value is:

$$\bar{x} = \frac{\int_{-\infty}^{+\infty} x\Psi(x,t)^*\Psi(x,t)\,dx}{\int_{-\infty}^{+\infty} \Psi(x,t)^*\Psi(x,t)\,dx}$$

- The expectation value of any function *g*(*x*) for a normalized wave function:

$$\langle g(x)\rangle = \int_{-\infty}^{+\infty} \Psi(x,t)^* g(x)\Psi(x,t)\,dx$$

Momentum Operator

- To find the expectation value of *p*, we first need to represent *p* in terms of *x* and *t*. Consider the derivative of the wave function of a free particle with respect to *x*:

$$\frac{\partial \Psi}{\partial x} = \frac{\partial}{\partial x}\left[e^{i(kx-\omega t)}\right] = ike^{i(kx-\omega t)} = ik\Psi$$

With $k = p/\hbar$ we have $\frac{\partial \Psi}{\partial x} = i\frac{p}{\hbar}\Psi$

This yields $p\left[\Psi(x,t)\right] = -i\hbar\frac{\partial \Psi(x,t)}{\partial x}$

- This suggests we can define the momentum operator as $\hat{p} = -i\hbar\frac{\partial}{\partial x}$.
- The expectation value of the momentum is

$$\langle p \rangle = \int_{-\infty}^{+\infty} \Psi^*(x,t)\hat{p}\Psi(x,t)\,dx = -i\hbar\int_{-\infty}^{+\infty} \Psi^*(x,t)\frac{\partial \Psi(x,t)}{\partial x}dx$$

Position and Energy Operators

- The position *x* is its own operator as seen above.
- The time derivative of the free-particle wave function is $$\frac{\partial \Psi}{\partial t} = \frac{\partial}{\partial t}\left[e^{i(kx-\omega t)}\right] = -i\omega e^{i(kx-\omega t)} = -i\omega\Psi$$

  Substituting $\omega = E/\hbar$ yields $E\left[\Psi(x,t)\right] = i\hbar\frac{\partial \Psi(x,t)}{\partial t}$

- So the energy operator is $\hat{E} = i\hbar\frac{\partial}{\partial t}$
- The expectation value of the energy therefore is

$$\langle E\rangle = \int_{-\infty}^{+\infty}\Psi^*(x,t)\hat{E}\Psi(x,t)\,dx = i\hbar\int_{-\infty}^{+\infty}\Psi^*(x,t)\frac{\partial \Psi(x,t)}{\partial t}dx$$