PHYS 3313 – Section 001
Lecture #5

Wednesday, Jan. 30, 2019
Dr. Amir Farbin

- Michelson-Morley Experiment
- Einstein's postulates
- Lorentz Transformations
- Time Dilation
- Length Contraction
- Relativistic Velocity Addition

Announcements

- Reminder: Homework #1
  - chapter 2 end of the chapter problems
  - 17, 21, 23, 24, 32, 59, 61, 66, 68, 81 and 96
  - Due is by the beginning of the class, Wednesday, Feb. 8
  - Work in study groups together with other students but PLEASE do write your answer in your own way!

Reminder: Special Project #1

1. Compute the electric force between the two protons separate the farthest in an intact U^{238} nucleus. Use the actual size of the U^{238} nucleus. (10 points)
2. Compute the gravitational force between the two protons separate the farthest in an intact U^{238} nucleus. (10 points)
3. Express the electric force in #1 above in terms of the gravitational force in #2. (5 points)

- You must look up the mass of the proton, actual size of the U^{238} nucleus, etc, and clearly write them on your project report
- You MUST have your own, independent answers to the above three questions even if you worked together with others. All those who share the answers will get 0 credit if copied. Must be handwritten!
- Due for the submission Monday, Feb. 4!

Reminder: Special Project #2

1. Compute the value of the speed of light using the formula (5 points):

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = \lambda f$$

2. Derive the unit of speed from the units specified in the back-side of the front cover of the text book. (5 points)

- Be sure to write down the values and units taken from the back-side of the front cover of the text book.
- You MUST have your own, independent answers to the above three questions even if you worked together with others. All those who share the answers will get 0 credit if copied. Must be handwritten!
- Due for the submission is Wednesday, Feb. 6!

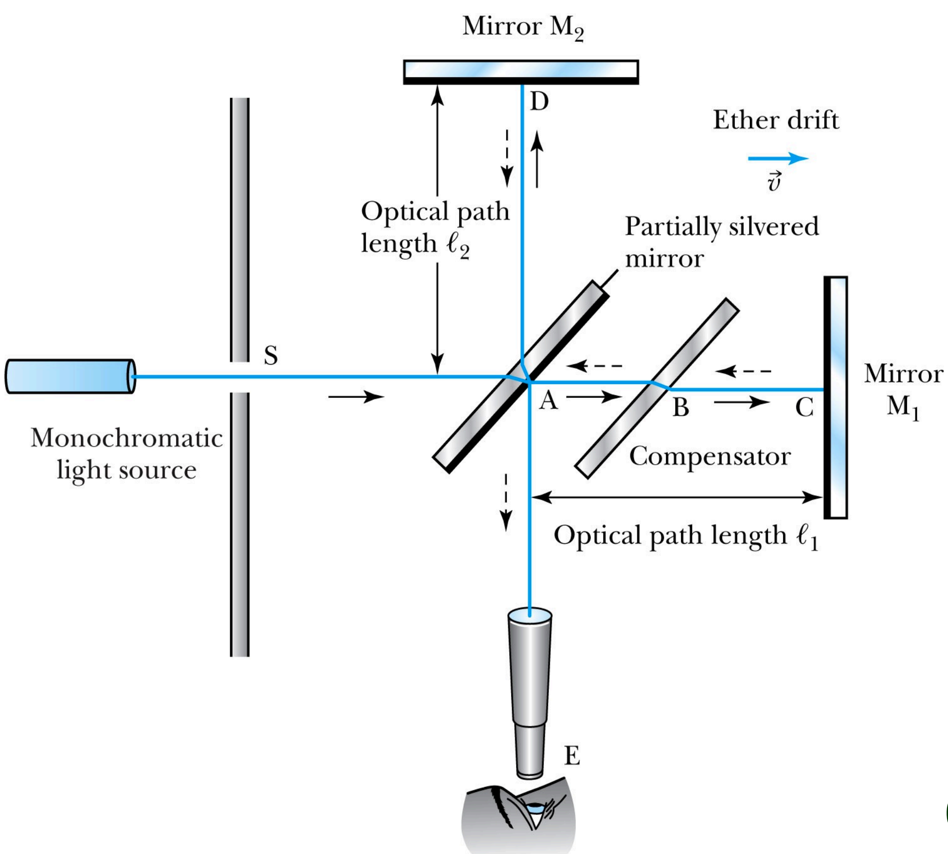
Special Project #3

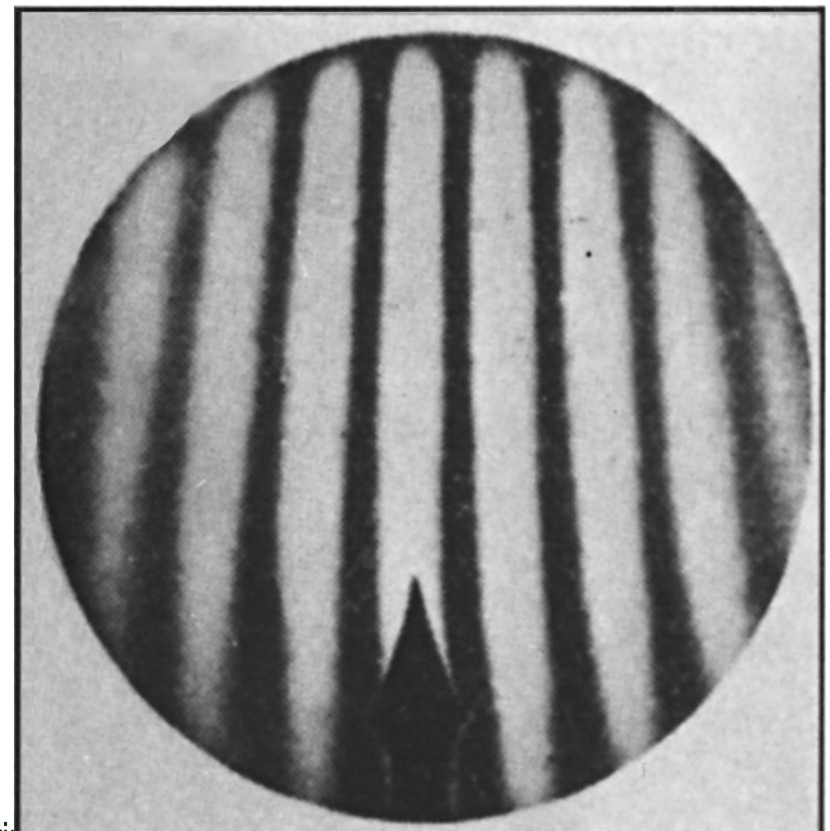
1. Derive the three Lorentz velocity transformation equations, starting from Lorentz coordinate transformation equations. (10 points)
2. Derive the three reverse Lorentz velocity transformation equations, starting from Lorentz coordinate transformation equations. (10 points)
3. Prove that the space-time invariant quantity $s^2=x^2-(ct)^2$ is indeed invariant, i.e. $s^2=s'^2$, in Lorentz Transformation. (5 points)
4. You must derive each one separately starting from the Lorentz spatial coordinate transformation equations to obtain any credit.
   - Just simply switching the signs and primes will NOT be sufficient!
   - Must take the simplest form of the equations, using β and γ.
5. You MUST have your own, independent handwritten answers to the above three questions even if you worked together with others. All those who share the answers will get 0 credit if copied.

- Due for the submission is Wednesday, Feb. 13!

The Michelson-Morley Experiment

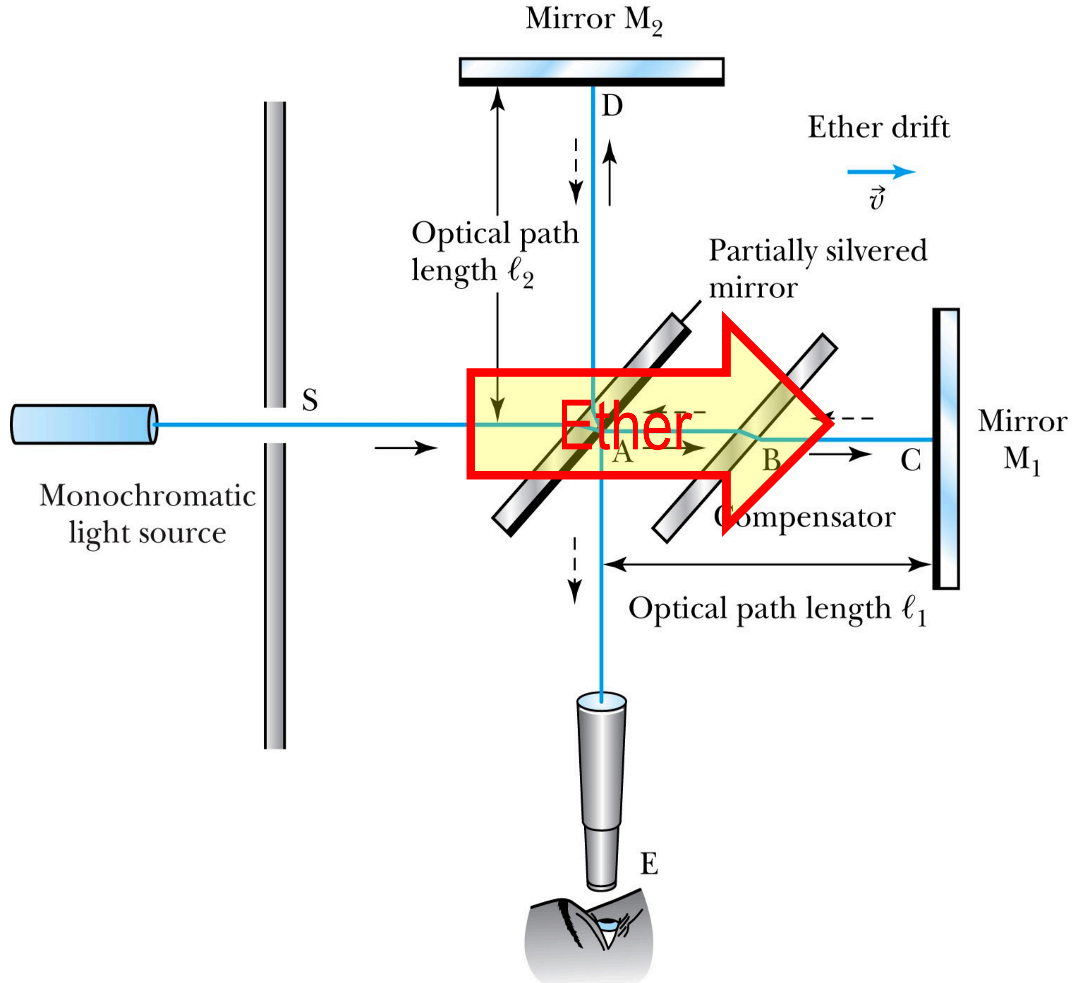
- Albert Michelson (1852–1931) built an extremely precise device called the *interferometer* to measure the phase difference between two light waves traveling in orthogonal directions.





How does Michelson Interferometer work?

1. AC is parallel to the motion of the Earth inducing an "ether wind"
2. Light from source S is split by mirror A and travels to mirrors C and D in mutually perpendicular directions
3. After reflection the beams recombine at A slightly out of phase due to the "ether wind" as viewed by telescope E.



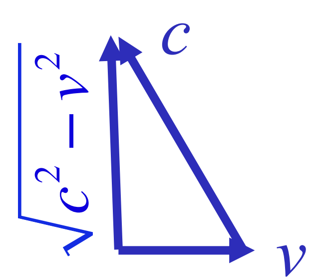
The analysis – Galilean X-formation

- Travel time t_1 for a round trip over AC (the ether direction) is

$$t_1 = \frac{l_1}{c+v} + \frac{l_1}{c-v} = \frac{2l_1 c}{c^2 - v^2} = \frac{2l_1}{c}\frac{1}{1 - v^2/c^2}$$

- Travel time t_2 for a round trip over AD (perpendicular direction to ether) is

$$t_2 = \frac{2l_2}{\sqrt{c^2 - v^2}} = \frac{2l_2}{c}\frac{1}{\sqrt{1 - v^2/c^2}}$$



- The time difference is

$$\Delta t = t_2 - t_1 = \frac{2}{c}\left(\frac{l_2}{\sqrt{1 - v^2/c^2}} - \frac{l_1}{1 - v^2/c^2}\right)$$

The analysis

- After rotating the machine by 90°, the time difference becomes

$$\Delta t' = t_2' - t_1' = \frac{2}{c}\left(\frac{l_2}{1 - v^2/c^2} - \frac{l_1}{\sqrt{1 - v^2/c^2}}\right)$$

- The difference of the time differences

$$\Delta t' - \Delta t = \frac{2}{c}\left(\frac{l_1 + l_2}{1 - v^2/c^2} - \frac{l_1 + l_2}{\sqrt{1 - v^2/c^2}}\right) = \frac{2}{c}(l_1 + l_2)\left(\frac{1}{1 - v^2/c^2} - \frac{1}{\sqrt{1 - v^2/c^2}}\right)$$

- Since v (the Earth's speed) is 10^{-4} of c, we can do binomial expansion of the above

$$\Delta t' - \Delta t = \frac{2}{c}(l_1 + l_2)\left[\left(1 + \frac{v^2}{c^2} + \cdots\right) - \left(1 + \frac{v^2}{2c^2} + \cdots\right)\right] \approx \frac{v^2}{c^3}(l_1 + l_2)$$

The Results

- Using the Earth's orbital speed as:

$$V = 3 \times 10^4 \text{ m/s}$$

together with

$$\ell_1 \approx \ell_2 = 1.2 \text{ m}$$

So that the time difference becomes

$$\Delta t' - \Delta t \approx v^2(\ell_1 + \ell_2)/c^3 = 8 \times 10^{-17} \text{ s}$$

- Although a very small number, it was within the experimental range of measurement for light waves.
- Later with Morley, they increased the path lengths to 11m and improved precision better than a factor of 10
- Yet, Michelson FAILED to "see" the expected interference pattern

Conclusions of Michelson Experiment

- Michelson noted that he should be able to detect a phase shift of light due to the time difference between path lengths but found none.
- He thus concluded that the hypothesis of the stationary ether must be incorrect.
- After several repeats and refinements with assistance from Edward Morley (1893-1923), again *a null result.*
- ***Thus, ether does not seem to exist!***
- Many explanations ensued afterward but none worked out!
- This experiment shattered the popular belief of light being waves

The Lorentz-FitzGerald Contraction

- Another hypothesis proposed independently by both H. A. Lorentz and G. F. FitzGerald suggested that the length ℓ_1, in the direction of the motion was *contracted* by a factor of

$$\sqrt{1-v^2/c^2}$$

- Thus making the path lengths equal to account for the zero phase shift.
  - This, however, was an ad hoc assumption that could not be experimentally tested.

Einstein's Postulates

- Fundamental assumption: Maxwell's equations must be valid in all inertial frames
- **The principle of relativity**: The laws of physics are the same in all inertial systems. There is no way to detect absolute motion, and no preferred inertial system exists
  - Published a paper in 1905 at the age 26
  - Believed the principle of relativity to be fundamental
- **The constancy of the speed of light**: Observers in all inertial systems measure the same value for the speed of light in a vacuum.

The Lorentz Transformations

General linear transformation relationship between P=(x, y, z, t) in frame S and P'=(x',y',z',t') in frame S' ➔ these assume measurements are made in S frame and transferred to S' frame

- preserve the constancy of the speed of light between inertial observers
- account for the problem of simultaneity between these observers

$$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}} \quad y' = y \quad z' = z \quad t' = \frac{t - \left(vx/c^2\right)}{\sqrt{1 - v^2/c^2}}$$

- With the definitions $\beta \equiv v/c$ and $\gamma \equiv 1/\sqrt{1-\beta^2}$

$$x' = \gamma\left(x - \beta ct\right) \quad y' = y \quad z' = z \quad t' = \gamma\left(t - \beta x/c\right)$$

Properties of the Relativistic Factor γ

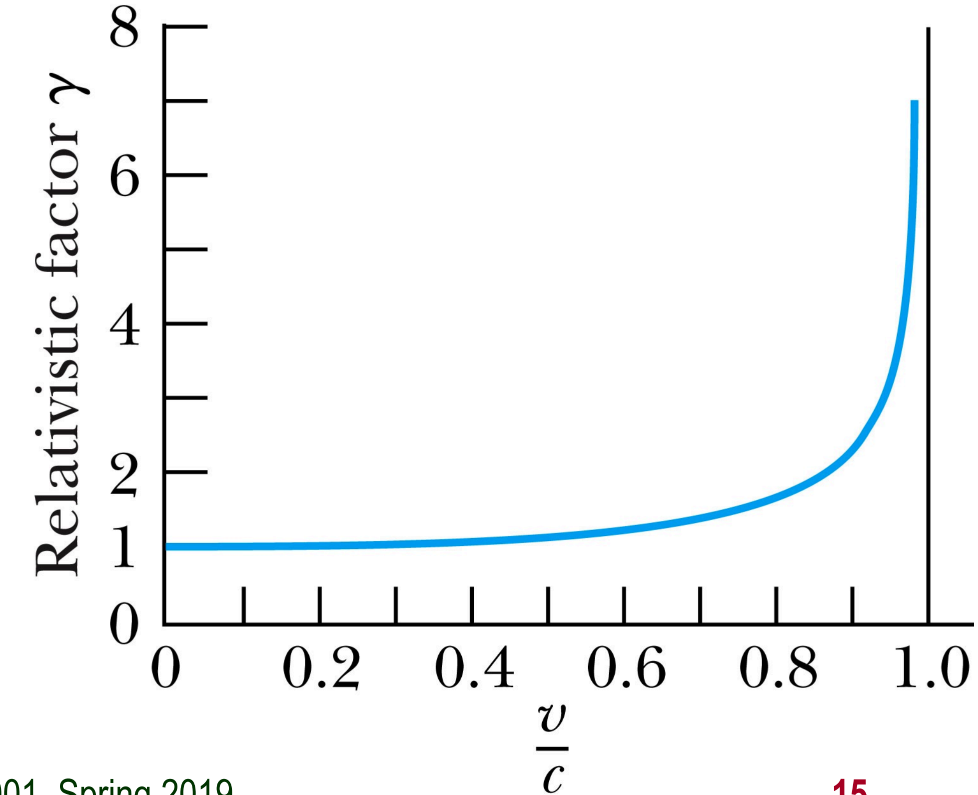
What is the property of the relativistic factor, γ?

Is it bigger or smaller than 1?

Recall Einstein's postulate, $\beta = v/c < 1$ for all observers

- $\gamma = 1$ only when $v = 0$

$$\gamma = 1\Big/\sqrt{1-\beta^2} \geq 1$$



The complete Lorentz Transformations

$$x' = \frac{x - vt}{\sqrt{1-\beta^2}} \qquad x = \frac{x' + vt'}{\sqrt{1-\beta^2}}$$

$$y' = y \qquad y = y'$$

$$z' = z \qquad z = z'$$

$$t' = \frac{t - \left(vx/c^2\right)}{\sqrt{1-\beta^2}} \qquad t = \frac{t' + \left(vx'/c^2\right)}{\sqrt{1-\beta^2}}$$

- Some things to note
  - What happens when β~0 (or v~0)?
    - The Lorentz x-formation becomes Galilean x-formation
  - Space-time are not separated
  - For non-imaginary transformations, the frame speed cannot exceed c!

Time Dilation and Length Contraction

Direct consequences of the Lorentz Transformation:

- **Time Dilation**:

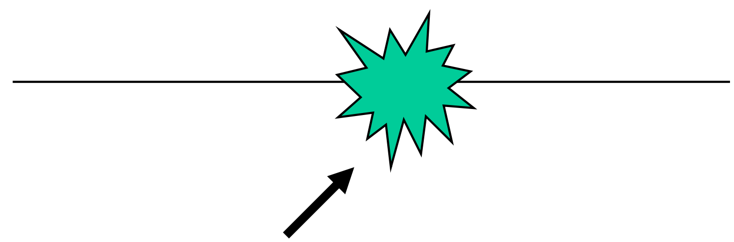
  The clock in a moving inertial reference frame K'
  run slower with respect to the stationary clock in K.
- **Length Contraction**:

  Lengths measured in a moving inertial reference frame K' are shorter with respect to the same lengths stationary in K.

Time Dilation

To understand *time dilation* the idea of **proper time** must be understood:

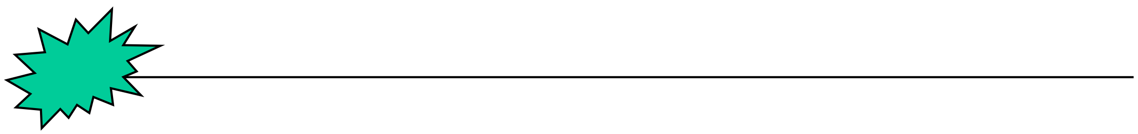
- **proper time**, T_0, is the time difference between two events occurring at the **same** position in a system as measured by **a clock at that position**.



Same location (spark "on" then off")

Time Dilation

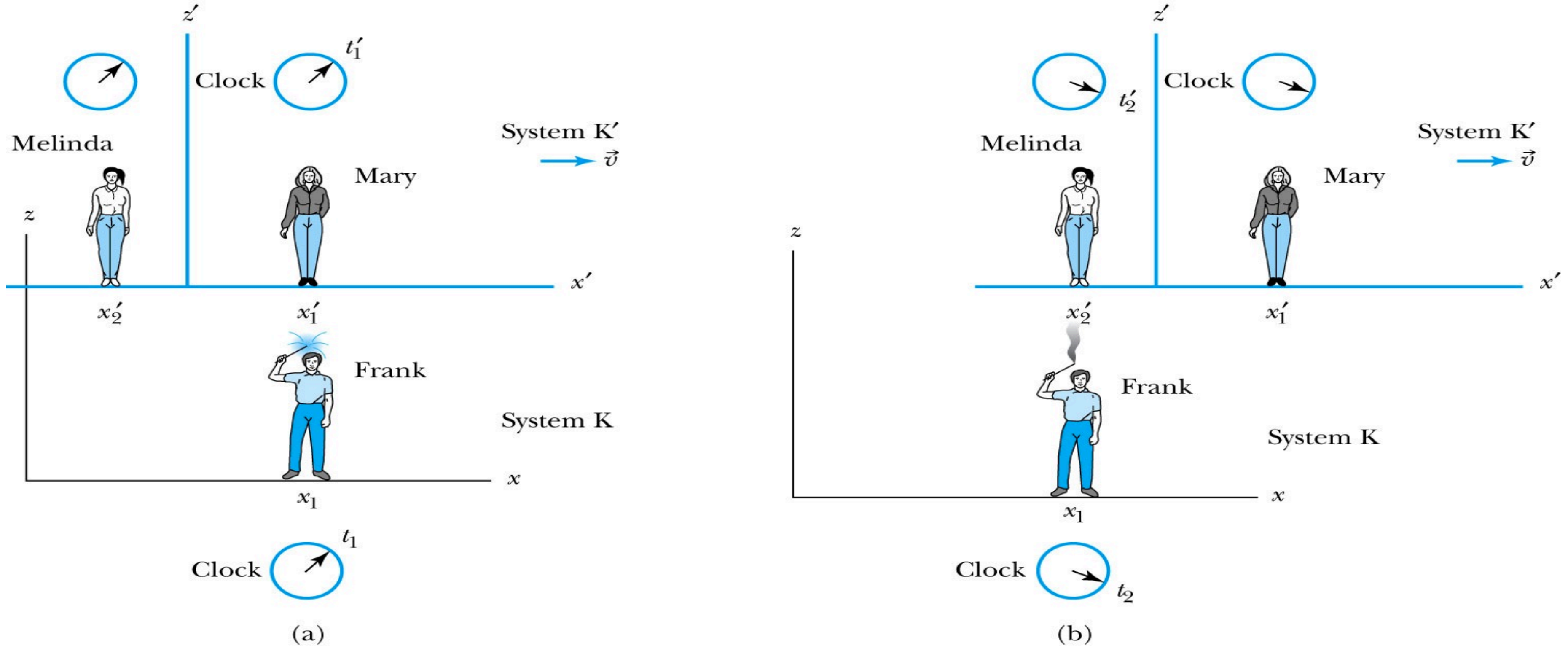
Is this a Proper Time?



spark “on” then spark “off”

Beginning and ending of the event occur at different positions

Time Dilation with Mary, Frank, and Melinda



Frank's clock is at the same position in system K when the sparkler is lit in (a) ($t=t_1$) and when it goes out in (b) ($t=t_2$). ➔ The proper time $T_0=t_2-t_1$

Mary, in the moving system K', is beside the sparkler when it was lit ($t=t_1'$)

Melinda then moves into the position where and when the sparkler extinguishes ($t=t_2'$)

Thus, Melinda, at the new position, measures the time in system K' when the sparkler goes out in (b).

According to Mary and Melinda…

- Mary and Melinda measure the two times for the sparkler to be lit and to go out in system K' as times t_1'and t_2' so that by the Lorentz transformation:

$$t'_2 - t'_1 = \frac{(t_2 - t_1) - (v/c^2)(x_2 - x_1)}{\sqrt{1-\beta^2}}$$

  - Note here that Frank records $x_2 - x_1 = 0$ in K with a proper time: $T_0 = t_2 - t_1$ or

$$T' = t'_2 - t'_1 = \frac{T_0}{\sqrt{1-\beta^2}} = \gamma T_0$$

Time Dilation: Moving Clocks Run Slow

1) $T' > T_0$ or the time measured between two events at *different positions* is greater than the time between the same events at *one position:* ***time dilation.***

The proper time is always the shortest time!!

2) The events do not occur at the same space and time coordinates in the two systems

3) System K requires 1 clock and K' requires 2 clocks.

Time Dilation Example: muon lifetime

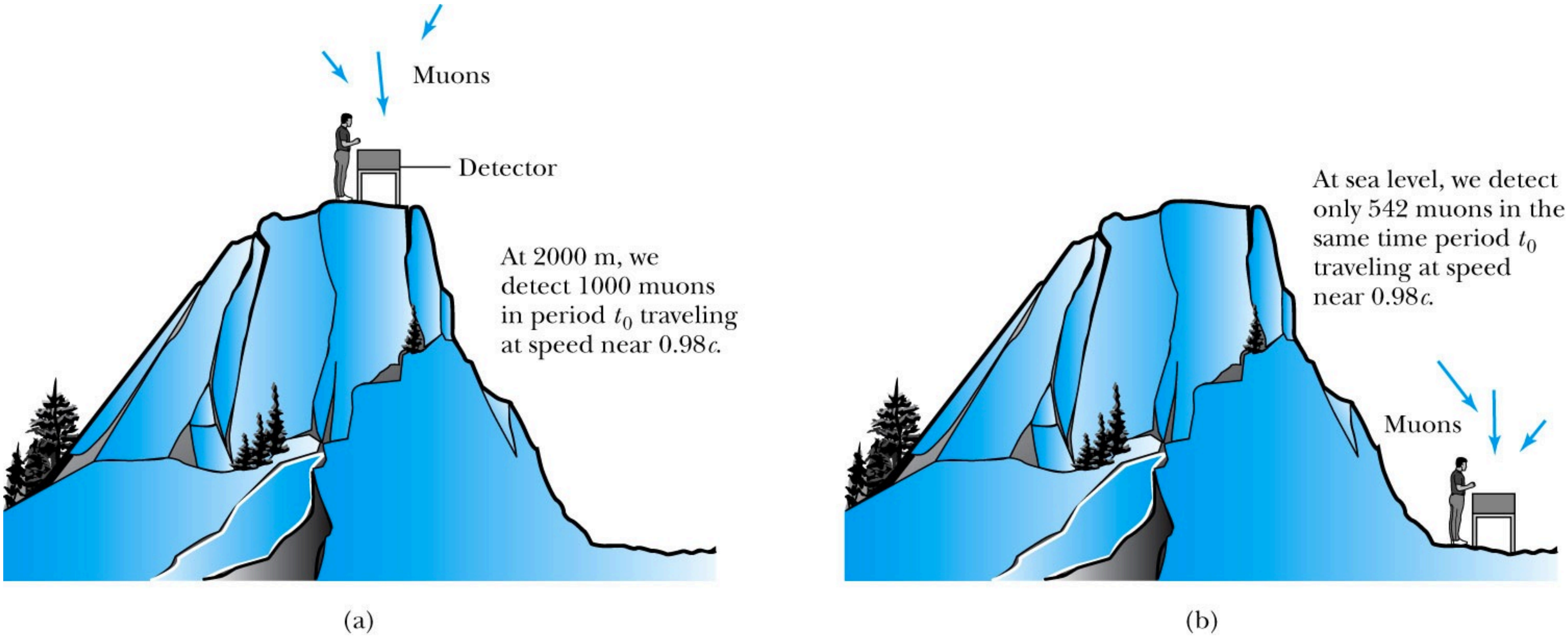
- Muons are essentially heavy electrons (~200 times heavier)
- Muons are typically generated in collisions of cosmic rays in upper atmosphere and, unlike electrons, decay ($t_0 = 2.2$ μsec)
- For a muon incident on Earth with v=0.998c, an observer on Earth would see what lifetime of the muon?
- 2.2 μsec?

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \approx 16$$

- t=35 μsec
- Moving clocks run slow so when an outside observer measures, they see a longer lifetime than the muon itself sees.

Experimental Verification of Time Dilation

Arrival of Muons on the Earth's Surface



The number of muons detected with speeds near 0.98*c* is much different (a) on top of a mountain than

(b) at sea level, because of the muon's decay. The experimental result agrees with our time dilation equation.

Length Contraction

To understand *length contraction* the idea of **proper length** must be understood:

- Let an observer in each system K and K' have a meter stick at rest in ***their own system*** such that each measures the same length at rest.
- The length as measured at rest **<u>at the same time</u>** is called the **proper length**.

Length Contraction cont'd

Each observer lays a stick down along his or her respective *x* axis, putting the left end at x_ℓ (or x'_ℓ) and the right end at x_r (or x'_r).

- Thus, in the rest frame K, Frank measures his stick to be:

$$L_0 = x_r - x_l$$

- Similarly, in the moving frame K', Mary measures her stick at rest to be:

$$L'_0 = x'_r - x'_l$$

- Frank in his rest frame measures the length of the stick in Mary's frame which is moving with speed v.
- Thus using the Lorentz transformations Frank measures the length of the stick in K' as:

$$x'_r - x'_l = \frac{(x_r - x_l) - v(t_r - t_l)}{\sqrt{1-\beta^2}}$$

Where both ends of the stick must be measured simultaneously, i.e, $t_r = t_\ell$

Here Mary's proper length is $L'_0 = x'_r - x'_\ell$

and Frank's measured length of Mary's stick is $L = x_r - x_\ell$

Measurement in Rest Frame

The observer in the rest frame measures the moving length as *L* given by

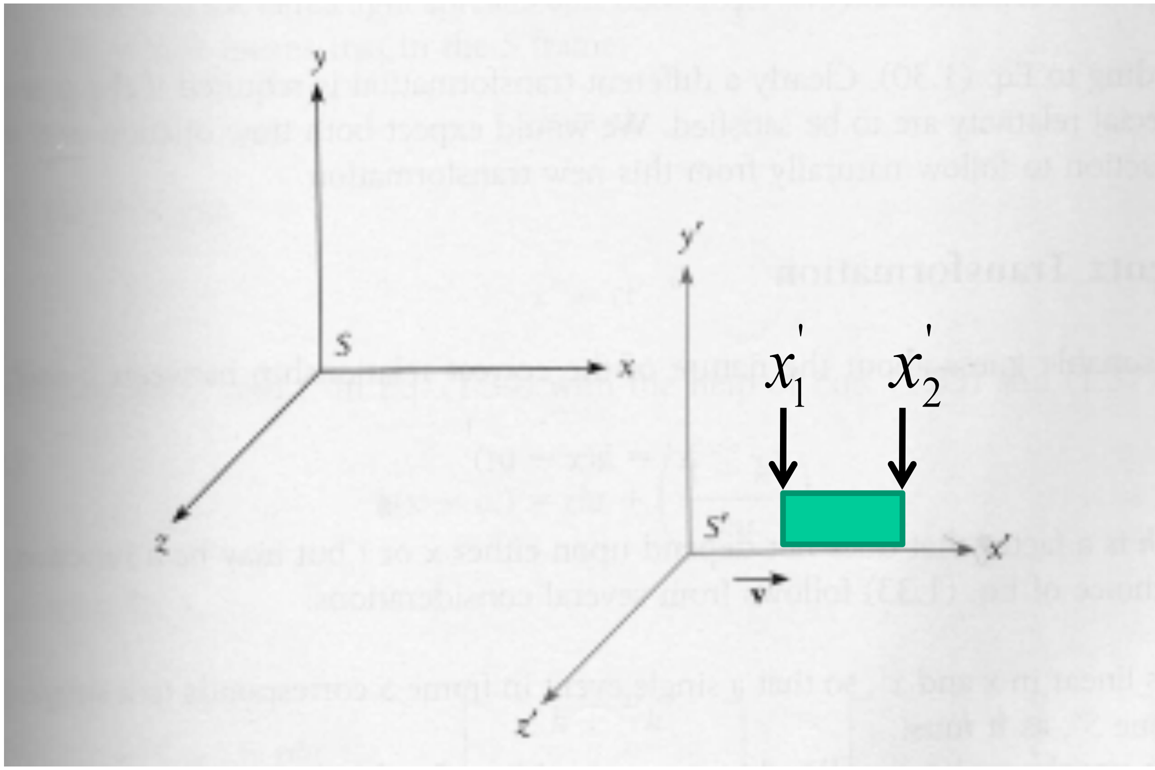
$$L_0' = \frac{L}{\sqrt{1-\beta^2}} = \gamma L$$

but since both Mary and Frank in their respective frames measure $L'_0 = L_0$

$$L = L_0\sqrt{1-\beta^2} = \frac{L_0}{\gamma}$$

and $L_0 > L$, i.e. the moving stick shrinks

Length Contraction Summary



- Proper length (length of object in its own frame:

$$L_0 = x_2' - x_1'$$

- Length of the object in observer's frame:

$$L = x_2 - x_1$$

$$L_0' = L_0 = x_2' - x_1' = \gamma(x_2 - vt) - \gamma(x_1 - vt) = \gamma(x_2 - x_1)$$

$L_0 = \gamma L$ $\quad L = L_0 / \gamma$

Since γ>1, the length is shorter in the direction of motion (length contraction!)

More about Muons

- Rate: 1/cm^2/minute at Earth's surface (so for a person with 600 cm^2 surface area, the rate would be 600/60=10 muons/sec passing through the body!)
- They are typically produced in atmosphere about 6 km above surface of Earth and often have velocities that are a substantial fraction of speed of light, v=.998 c for example and lifetime of 2.2 μsec $vt_0 = 2.994\times 10^8 \frac{m}{\sec}\cdot 2.2\times 10^{-6}\sec = 0.66km$
- How do they reach the Earth if they only go 660 m and not 6000 m?
- The time dilation stretches life time to t=35 μsec not 2.2 μsec, thus they can travel 16 times further, or about 10 km, implying they easily reach the ground
- But riding on a muon, the trip takes only 2.2 μsec, so how do they reach the ground???
- Muon-rider sees the ground moving towards him, so the length he has to travel contracts and is only $L_0/\gamma = 6/16 = 0.38km$
- At 1000 km/sec, it would take 5 seconds to cross U.S., pretty fast, but does it give length contraction? $L = .999994L_0$ {not much contraction}
  (for v=0.9c, the length is reduced by 66%)

The Complete Lorentz Transformations

$$x' = \frac{x - vt}{\sqrt{1-\beta^2}} \qquad x = \frac{x' + vt'}{\sqrt{1-\beta^2}}$$

$$y' = y \qquad y = y'$$

$$z' = z \qquad z = z'$$

$$t' = \frac{t - \left(vx/c^2\right)}{\sqrt{1-\beta^2}} \qquad t = \frac{t' + \left(vx'/c^2\right)}{\sqrt{1-\beta^2}}$$

Addition of Velocities

How do we add velocities in a relativistic case?

Taking differentials of the Lorentz transformation, relative velocities may be calculated:

$$dx = \gamma\left(dx' + v\,dt'\right)$$

$$dy = dy'$$

$$dz = dz'$$

$$dt = \gamma\left[dt' + \left(v/c^2\right)dx'\right]$$