PHYS 3313 – Section 001
Lecture #6

Monday, Feb. 4, 2019

Dr. Jaehoon Yu

- Relativistic Velocity Addition
- The Twin Paradox
- Space-time Diagram

Announcements

- Reading assignments: CH 3.3 (special topic – the discovery of Helium) and CH3.7
- Reminder: Homework #1
  - chapter 2 end of the chapter problems
  - 17, 21, 23, 24, 32, 59, 61, 66, 68, 81 and 96
  - Due is by the beginning of the class, Monday, Feb. 11
  - Work in study groups together with other students but PLEASE do write your answer in your own way!
- Quiz 1 results
  - Class average: 29.3/80
    - Equivalent to: 36.7/100
  - Top score: 72/80

Reminder: Special Project #2

1. Compute the value of the speed of light using the formula (5 points):

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = \lambda f$$

2. Derive the unit of speed from the units specified in the back-side of the front cover of the text book. (5 points)

- Be sure to write down the values and units taken from the back-side of the front cover of the text book.
- You MUST have your own, independent answers to the above three questions even if you worked together with others. All those who share the answers will get 0 credit if copied. Must be handwritten!
- Due for the submission is this Wednesday, Feb. 6!

Reminder: Special Project #3

1. Derive the three Lorentz velocity transformation equations, starting from Lorentz coordinate transformation equations. (10 points)
2. Derive the three reverse Lorentz velocity transformation equations, starting from Lorentz coordinate transformation equations. (10 points)
3. Prove that the space-time invariant quantity $s^2=x^2-(ct)^2$ is indeed invariant, i.e. $s^2=s'^2$, in Lorentz Transformation. (5 points)
4. You must derive each one separately starting from the Lorentz spatial coordinate transformation equations to obtain any credit.
   - Just simply switching the signs and primes will NOT be sufficient!
   - Must take the simplest form of the equations, using β and γ.
5. You MUST have your own, independent handwritten answers to the above three questions even if you worked together with others. All those who share the answers will get 0 credit if copied.

- Due for the submission is next Wednesday, Feb. 13!

The complete Lorentz Transformations

$$x' = \frac{x - vt}{\sqrt{1-\beta^2}} \qquad x = \frac{x' + vt'}{\sqrt{1-\beta^2}}$$

$$y' = y \qquad y = y'$$

$$z' = z \qquad z = z'$$

$$t' = \frac{t - \left(vx/c^2\right)}{\sqrt{1-\beta^2}} \qquad t = \frac{t' + \left(vx'/c^2\right)}{\sqrt{1-\beta^2}}$$

- Some things to note
  - What happens when β~0 (or v~0)?
    - The Lorentz x-formation becomes Galilean x-formation
  - Space and time are not separated
  - For non-imaginary transformations, the frame speed cannot exceed c!

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Addition of Velocities

How do we add velocities in a relativistic case?

By taking differentials of the Lorentz transformation, relative velocities may be calculated:

$$dx = \gamma\left(dx' + v\,dt'\right)$$

$$dy = dy'$$

$$dz = dz'$$

$$dt = \gamma\left[dt' + \left(v/c^2\right)dx'\right]$$

So that...

defining velocities as: $v_x = dx/dt$, $v_y = dy/dt$, $v'_x = dx'/dt'$, etc. it can be shown that:

$$\mathrm{v}_x = \frac{dx}{dt} = \frac{\gamma\left[dx' + v\,dt'\right]}{\gamma\left[dt' + \dfrac{v}{c^2}dx'\right]} = \frac{v'_x + v}{1 + \left(v/c^2\right)v'_x}$$

With similar relations for v_y and v_z:

$$\mathrm{v}_y = \frac{dy}{dt} = \frac{v'_y}{\gamma\left[1 + \left(v/c^2\right)v'_x\right]} \qquad \mathrm{v}_z = \frac{dz}{dt} = \frac{v'_z}{\gamma\left[1 + \left(v/c^2\right)v'_x\right]}$$

The Lorentz Velocity Transformations

In addition to the previous relations, the **Lorentz velocity transformations** for v'_x, v'_y, and v'_z can be obtained by switching primed and unprimed and changing *v* to –*v*.(the velocity of the moving frame!!)

$$\mathrm{v}'_x = \frac{v_x - v}{1 - \left(v/c^2\right) v_x}$$

$$\mathrm{v}'_y = \frac{v_y}{\gamma\left[1 - \left(v/c^2\right) v_x\right]}$$

$$\mathrm{v}'_z = \frac{v_z}{\gamma\left[1 - \left(v/c^2\right) v_x\right]}$$

Velocity Addition Summary

- Galilean Velocity addition $v_x = v_x' + v$ where $v_x = \frac{dx}{dt}$ and $v_x' = \frac{dx'}{dt'}$
- From inverse Lorentz transform $dx = \gamma(dx' + vdt')$ and $dt = \gamma(dt' + \frac{v}{c^2}dx')$
- So $$v_x = \frac{dx}{dt} = \frac{\gamma(dx' + vdt')}{\gamma(dt' + \frac{v}{c^2}dx')} \div \frac{dt'}{dt'} = \frac{\frac{dx'}{dt'} + v}{1 + \frac{v}{c^2}\frac{dx'}{dt'}} = \frac{v_x' + v}{1 + \frac{vv_x'}{c^2}}$$
- Thus $$\boxed{v_x = \frac{v_x' + v}{1 + \frac{vv_x'}{c^2}}}$$
- What would be the measured speed of light in S frame?
  - Since $v_x' = c$ we get $$v_x = \frac{c + v}{1 + \frac{v^2}{c^2}} = \frac{c^2(c+v)}{c(c+v)} = c$$

Observer in S frame measures c too! Strange but true!

Velocity Addition Example

- Tom Brady is riding his bus at 0.8c relative to the observer. He throws a ball at 0.7c in the direction of his motion. What speed does the observer see?

$$v_x = \frac{v_x' + v}{1 + \frac{v v_x'}{c^2}}$$

$$v_x = \frac{.7c + .8c}{1 + \frac{.7 \times .8c^2}{c^2}} = 0.962c$$

- What if he threw it just a bit harder?
- Doesn't help—asymptotically approach c but can't exceed (it's not just a postulate it's the law)

A test of Lorentz velocity addition: π^0 decay

- How can one test experimentally the correctness of the Lorentz velocity transformation vs Galilean one?
- In 1964, T. Alvager and company performed a measurements of the arrival time of two photons resulting from the decay of a π^0 in two detectors separated by 30m.
- Each photon has a speed of 0.99975c. What are the speed predicted by Galilean and Lorentz x-mation? The π^0 is moving at the speed of light when it decays.
  - $v_G = c + 0.99975c = 1.99975c$
  - $v_L = \frac{c + 0.99975c}{1 + 0.99975c^2/c^2} \approx c$
- How much time does the photon take to arrive at the detector?

Twin Paradox

The Set-up: Twins Mary and Frank at age 30 decide on two career paths: Mary (the **M**oving twin) decides to become an astronaut and to leave on a trip 8 light-years (ly) from the Earth at a great speed and to return; Frank (the **F**ixed twin) decides to stay on the Earth.

The Problem: Upon Mary's return, Frank reasons, that her clocks measuring her age must run slow. As such, she will return younger. However, Mary claims that it is Frank who is moving and consequently his clocks must run slow.

The Paradox: Who is younger upon Mary's return?