PHYS 3313 – Section 001
Lecture #11

Wednesday, Feb. 20, 2019

Dr. Jaehoon Yu

- The Photoelectric Effect
- Compton Effect
- Pair production/Pair annihilation
- Rutherford Scattering Experiment and Rutherford Atomic Model

Announcements

- Midterm Exam
  - In class Wednesday, March. 6
  - Covers from CH1.1 through what we learn March 4 plus the math refresher in the appendices
  - Mid-term exam constitutes 20% of the total
  - **Please do NOT miss the exam! You will get an F if you miss it.**
  - BYOF: You may bring a one 8.5x11.5 sheet (front and back) of handwritten formulae and values of constants for the exam
  - No derivations, word definitions or setups or solutions of any problems!
    - No Lorentz velocity addition formula!
    - No Maxwell's equations!
  - No additional formulae or values of constants will be provided!
- Colloquium today
  - Dr. C. Liu of UTA Math Dept.

Physics Department
The University of Texas at Arlington
COLLOQUIUM

Third Generation of Vortex Identification Methods - Omega, Liutex/Rortex, Omega-Liutex

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Wednesday February 20, 2019
4:00 p.m. Room 101 Science Hall

Abstract

Vortex is intuitively recognized as the rotational/swirling motion of the fluids, but a universally accepted definition of vortex is still not available. In thousands of research papers and almost all textbooks, vorticity tube/filament is regarded equivalent to vortex and the magnitude of vorticity is deemed the strength of vortex, which is a misunderstanding of the vortex nature since Helmholtz (1858). These vorticity-based methods can be considered as the first generation of vortex identification methods. During the last three decades, a lot of vortex identification methods, including Q-, Δ-, $\lambda 2$-, λci- criteria, have been developed. Most of these criteria are based on Cauchy-Stokes decomposition and/or eigenvalues of the velocity gradient tensor, which can be considered as the second generation of vortex identification methods. Starting from 2014, the Vortex and Turbulence Research Team at University of Texas at Arlington (UTA Team) focus on the development of a new generation of vortex identification methods. A new Omega vortex identification method, which defined the vortex as a connected region where vorticity overtook deformation, was published in 2016. The Omega method has several advantages: (1) easy to perform, (2) clear on physical meaning, (3) non-dimensional and normalized from 0 to 1, (4) robust to threshold change, (5) able to capture both strong and weak vortices simultaneously. In 2017 and 2018, a Liutex (previously called Rortex) vector was proposed by UTA Team to represent the local rigid rotational part of fluid motion, which is a mathematical definition with its direction as the local rotational axis and its magnitude as the rigid rotation strength. Liutex/Rortex is a new physical quantity with scalar, vector and tensor forms exactly representing the local rigid rotation of fluids. Meanwhile, a decomposition of vorticity to a rotational part namely Liutex/Rortex and an anti-symmetric shear part (RS decomposition) was introduced in 2018 and a velocity gradient tensor decomposition to a rotation part (R) and a non-rotation part (NR) which could be further decomposed as pure shearing (PS) and stretching and compression (SC) was also given in 2018 as a counterpart of Cauchy-Stokes decomposition. Later in early 2019, a Liutex/Rortex based Omega method called Omega-Liutex was developed, which combines the advantages of both Liutex and Omega methods. These breakthroughs in the development of vortex science by UTA Team are classified as the third generation of vortex identification methods. The critical problems for vortex identification concern with the vortex core location, vortex core size, vortex boundary and size, absolute vortex strength, relative vortex strength, mixture of strong and weak vortices, vortex rotation axes, etc. Only the new third generation of vortex identification methods can answer these questions while all the other vortex identification methods fail to answer all questions except for part of the first one (strong vortex cores).

Refreshments will be served at 3:30 p.m. in the Physics Library

Reminder: Special Project #4

- A total of N_i incident projectile particle of atomic number Z_1 kinetic energy KE scatter on a target of thickness t and atomic number Z_2 and has n atoms per volume. What is the total number of scattered projectile particles at an angle θ? (20 points)
- Please be sure to clearly define all the variables used in your derivation! Points will be deducted for missing variable definitions.
- This derivation must be done on your own. Please do not copy the book, internet or your friends'.
- Due next Wednesday, Feb. 27

Photoelectric Effect

Definition: Incident electromagnetic radiation shining on a metal transfers energy to the electrons in the metal, allowing them to escape the surface of the metal. Ejected electrons are called **photoelectrons.**

Heinrich Hertz noticed during his experiment in 1887 that when ultraviolet light falls on metal surface charge gets ejected ➔ Left it to Philip Lenard to study further (1905 Nobel for photoelectric effect)

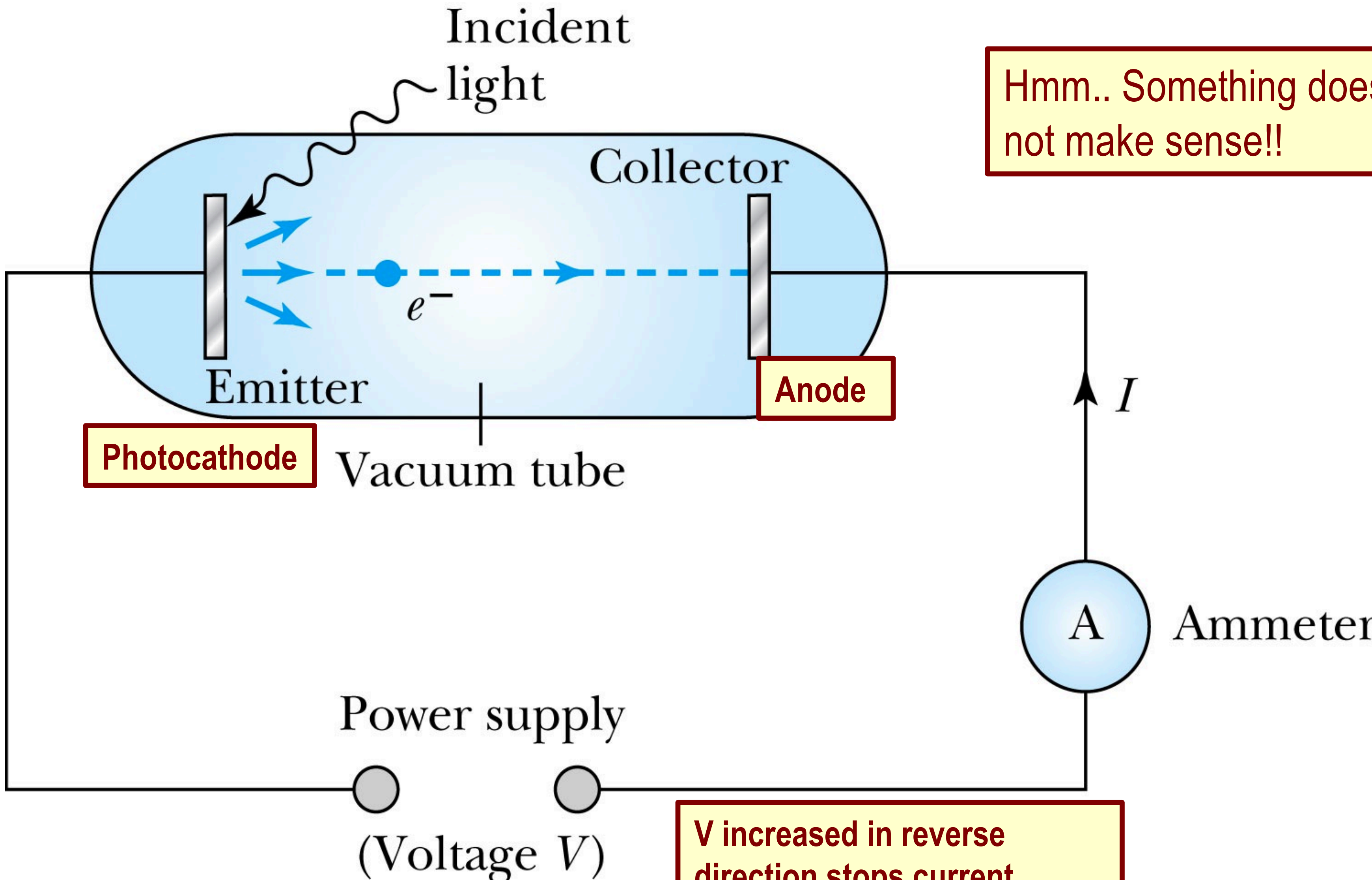
Other methods of electron emission:

- **Thermionic emission**: Application of heat allows electrons to gain enough energy to escape.
- **Secondary emission**: The electron gains enough energy by transfer from another high-speed particle that strikes the material from outside.
- **Field emission**: A strong external electric field pulls the electron out of the material. (an example?)

Classical Interpretation of Photoelectric Effect

- Classical theory allows EM radiation to eject photoelectrons from matter
- Classical theory predicts the energy of the photoelectrons increase in proportion to the radiation intensity
- Thus, the KE of the photoelectrons must be **<u>proportional to the intensity of light</u>** not the electric current
- Time for an experiment!

Photoelectric Effect Experimental Setup

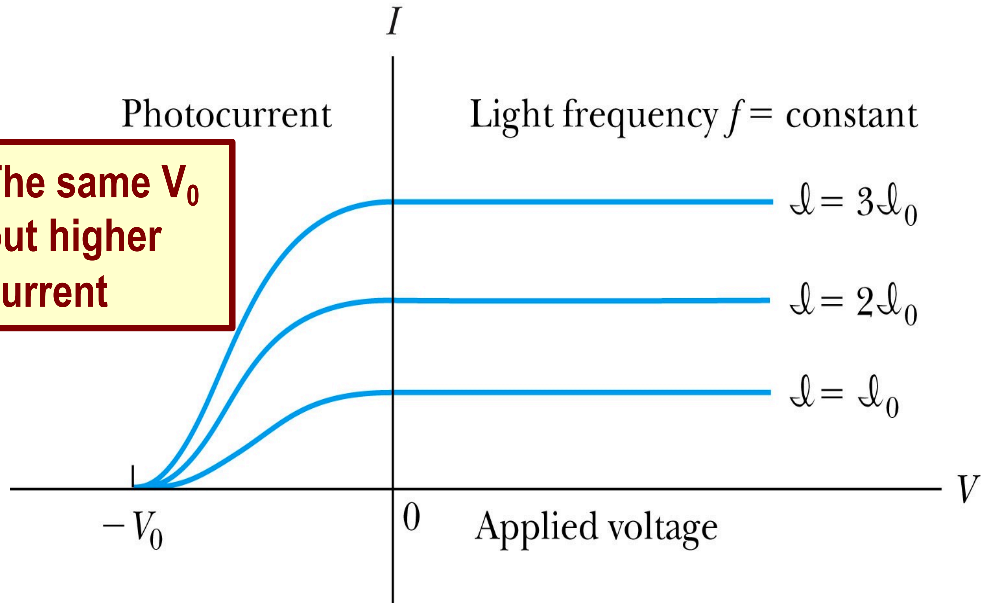


Hmm.. Something does not make sense!!

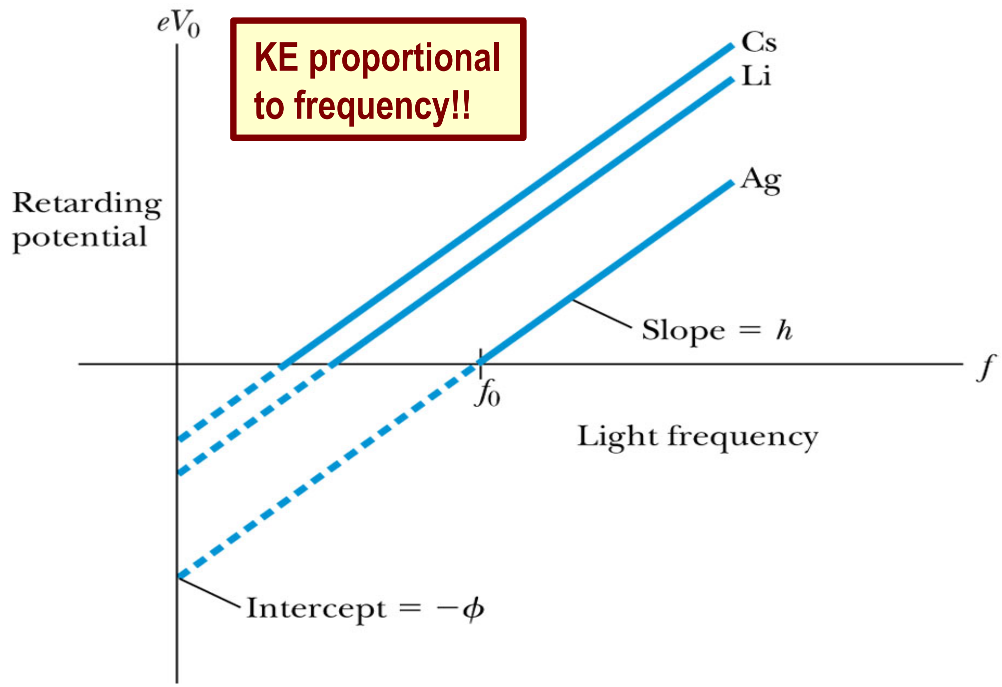
V increased in reverse direction stops current independent of light intensity

Experimental Observations

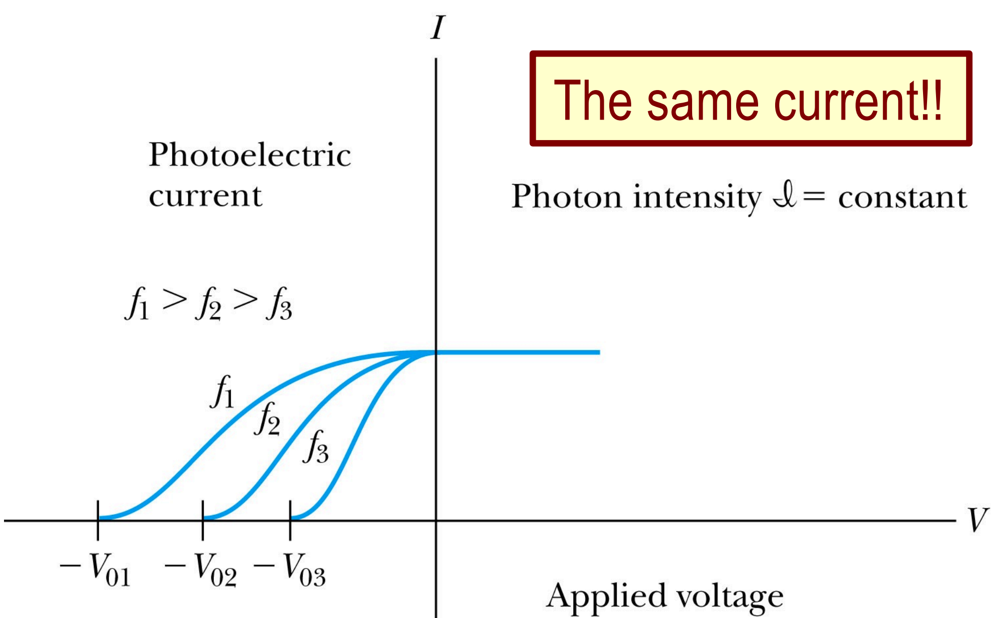
The same V_0 but higher current



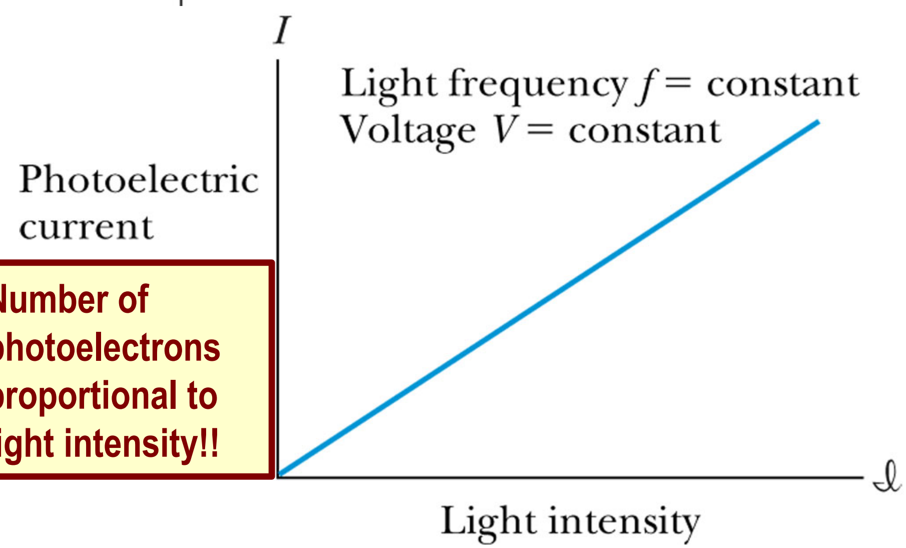
KE proportional to frequency!!



The same current!!



Number of photoelectrons proportional to light intensity!!



Summary of Experimental Observations

- Light intensity does not affect the KE of the photoelectrons
- The max KE of the photoelectrons for a given emitter material depends only on the frequency of the light
- The smaller the work function φ of the emitter material, the smaller is the threshold frequency of the light that can eject photoelectrons.
- When the photoelectrons are produced, their number is proportional to the intensity of light.
- The photoelectrons are emitted almost instantly following the illumination of the photocathode, independent of the intensity of the light. ➔ Totally unexplained by classical physics

Einstein's Theory of Photoelectric Effect

- Einstein suggested that the electromagnetic radiation field of the light is quantized into particles called **photons**. Each photon has the energy quantum:

$$E = hf$$

- where *f* is the frequency of the light and *h* is Planck's constant.
- The photon travels at the speed of light in vacuum, and its wavelength is given by

$$\lambda f = c$$

Einstein's Theory

- Conservation of energy yields:

Energy Before(photon)=Energy After (electron)

$$hf = \phi + KE(photoelectron)$$

where φ is the work function of the metal

The photon energy can then be written

$$hf = \phi + \frac{1}{2}mv_{max}^2$$

- The retarding potentials measure the KE of the most energetic photoelectrons.

$$eV_0 = \frac{1}{2}mv_{max}^2$$

Quantum Interpretation

- The KE of the electron depends only on the light frequency and the work function φ of the material not the light intensity at all

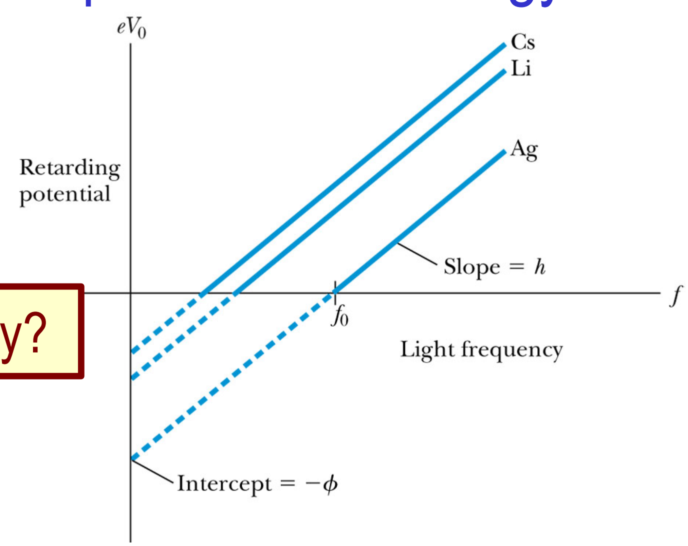
$$\frac{1}{2}mv_{max}^2 = eV_0 = hf - \phi$$

- Einstein in 1905 predicted that the stopping potential was linearly proportional to the light frequency, with a slope *h*, the same constant found by Planck.

$$eV_0 = \frac{1}{2}mv_{max}^2 = hf - hf_0 = h(f - f_0)$$

- From this, Einstein concluded that light is a particle with energy:

$$E = hf = \frac{hc}{\lambda}$$



Was he already thinking about particle/wave duality?

Ex 3.11: Photoelectric Effect

- Light of wavelength 400nm is incident upon lithium (φ=2.93eV). Calculate (a) the photon energy (eV) and (b) the stopping potential V_0.

- Since the wavelength is known, we use plank's formula:

$$E = hf = \frac{hc}{\lambda} = \frac{\left(6.623\times10^{-34} J\cdot s\right)\left(3\times10^{8} m/s\right)}{400\times10^{-9}} = 3.10\left(eV\right)$$

- The stopping potential can be obtained using Einstein's formula for photoelectron energy

$$eV_0 = hf - \phi = E - \phi$$

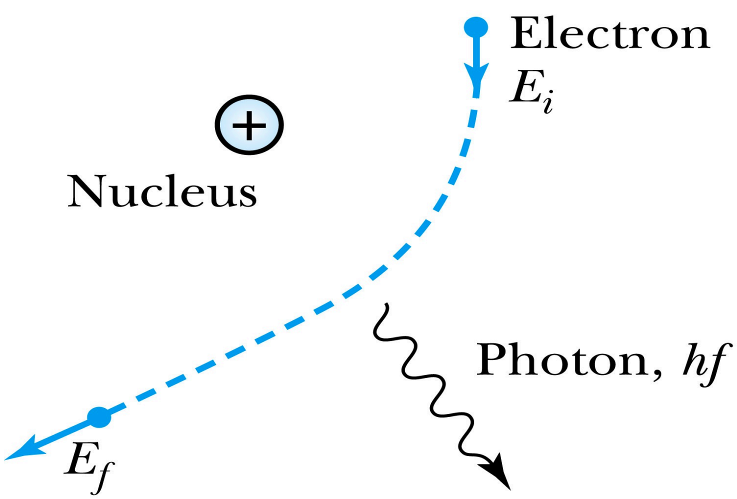
$$V_0 = \frac{E-\phi}{e} = \frac{\left(3.10-2.93\right)eV}{e} = 0.17V$$

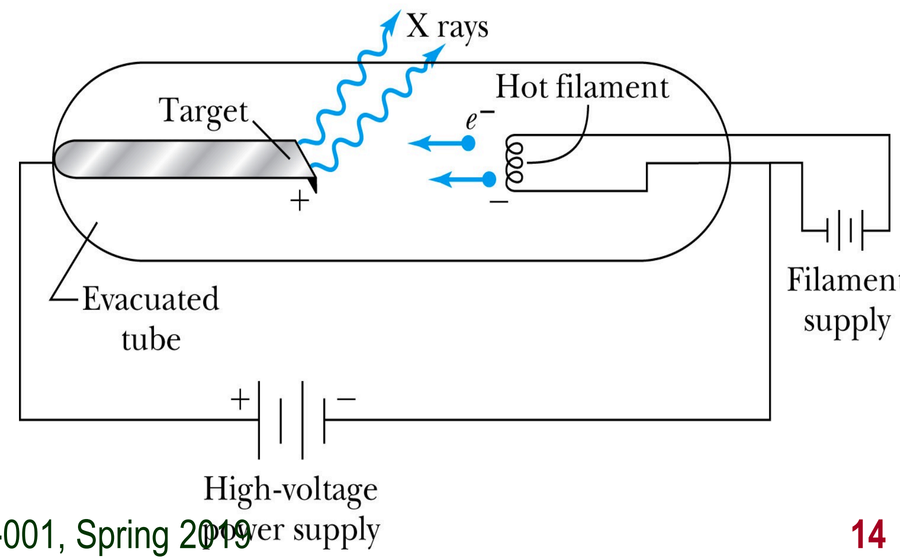
X-Ray Production

- **Bremsstrahlung** (German word for braking radiation): Radiation of a photon from an energetic electron passing through matter due to an acceleration
- Since linear momentum must be conserved, the nucleus absorbs very little energy, and it is ignored. The final energy of the electron is determined from the conservation of energy

$$E_f = E_i - hf$$

- An electron that loses a large amount of energy will produce an X-ray photon.
  - Current passing through a filament produces copious numbers of electrons by thermionic emission.
  - These electrons are focused by the cathode structure into a beam and are accelerated by potential differences of thousands of volts until they impinge on a metal anode surface, producing x rays by bremsstrahlung as they stop in the anode material
  - X-ray wavelengths range 0.01 – 10nm. What is the minimum energy of an electron to produce X-ray?





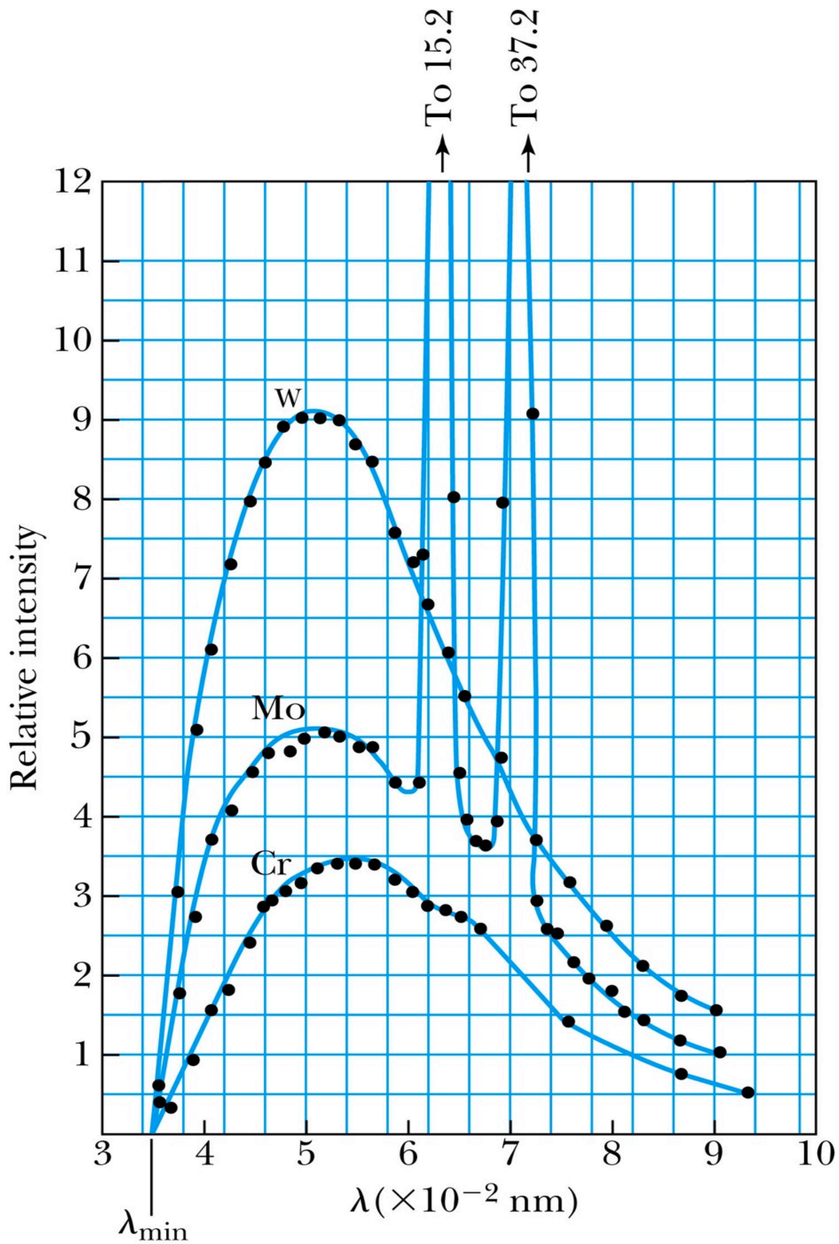
Inverse Photoelectric Effect.

- Conservation of energy requires that the electron KE equals the maximum photon energy
  - Work function neglected since it's small compared to the potential energy of the electron.
- This is the **Duane-Hunt limit**
  - The minimum photon wavelength depends only on the accelerating voltage
  - It is the same for all targets.

$$eV_0 = hf_{max} = \frac{hc}{\lambda_{min}}$$

$$\lambda_{min} = \frac{hc}{eV_0} = \frac{1.24\times10^{-6} V \cdot m}{V_0}$$

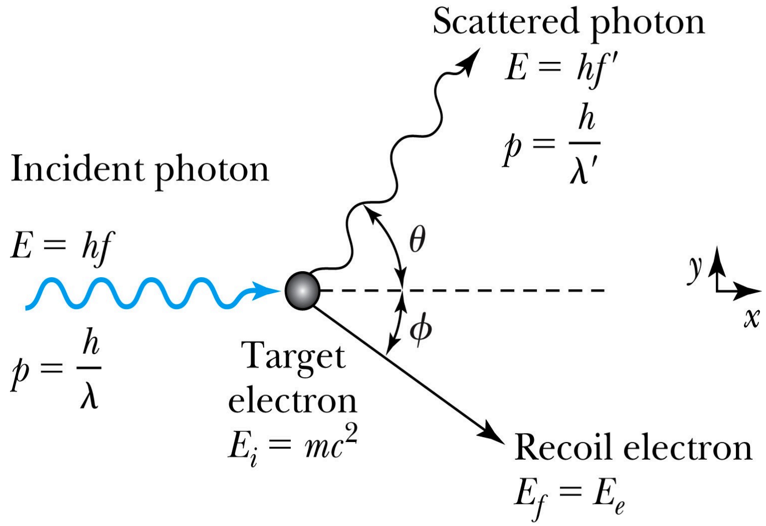
  - Intensity of x-ray depends on the square of the target atomic number and V_0



Compton Effect

- When a photon enters matter, it is likely to interact with one of the atomic electrons.
- The photon is scattered from only one electron
- The laws of conservation of energy and momentum apply as in any elastic collision between two particles. The momentum of a particle moving at the speed of light is

$$p = \frac{E}{c} = \frac{hf}{c} = \frac{h}{\lambda}$$



- The electron energy can be written as

$$E_e^2 = \left(m_e c^2\right)^2 + p_e^2 c^2$$

- Change of the scattered photon wavelength is known as the **Compton effect**:

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_e c}\left(1 - \cos\theta\right)$$

Pair Production and Annihilation

- If a photon can create an electron, it must also create a positive charge to balance charge conservation.
- In 1932, C. D. Anderson observed a positively charged electron (e^+) in cosmic radiation. This particle, called the positron, had been predicted to exist several years earlier by P. A. M. Dirac.
- A photon's energy can be converted entirely into an electron and a positron in a process called the **pair production**.
  - Can only happen inside a material
  - How much energy do you think is needed?

$$\gamma \rightarrow e^{-} + e^{+}$$

Pair Production in Empty Space?

- Energy conservation for pair production in empty space

$$hf = E_+ + E_- + K.E.$$

- Momentum conservation yields

$$hf = p_- c\cos\theta + p_+ c\cos\theta$$

- Thus max momentum exchange $$hf_{\max} = p_- c + p_+ c$$

- Recall that the total energy for a particle can be written as

$$E_\pm^2 = p_\pm^2 c^2 + m_e^2 c^4$$

However this yields a contradiction: $$hf > p_- c + p_+ c$$

Hence the conversion of energy in empty space is impossible and thus pair production cannot happen in empty space

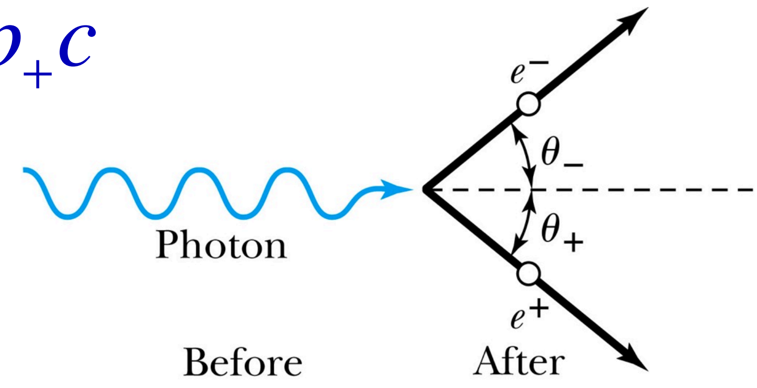
Pair Production in Matter

- Since the relations $hf_{\max} = p_- c + p_+ c$ and $hf > p_- c + p_+ c$ contradict each other, a photon can not produce an electron and a positron in empty space.
- In the presence of matter, however, the nucleus absorbs some energy and momentum.

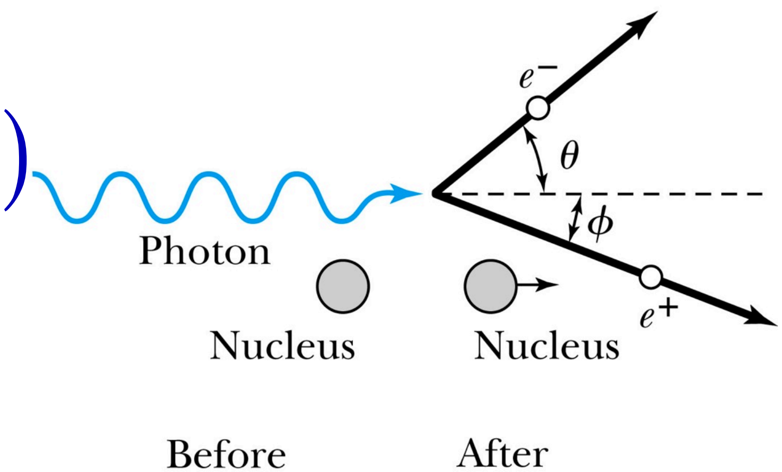
$$hf = E_- + E_+ + K.E.(\text{nucleus})$$

- The photon energy required for pair production in the presence of matter is

$$hf > 2m_e c^2$$



(a) Free space (**cannot occur**)



(b) Beside nucleus

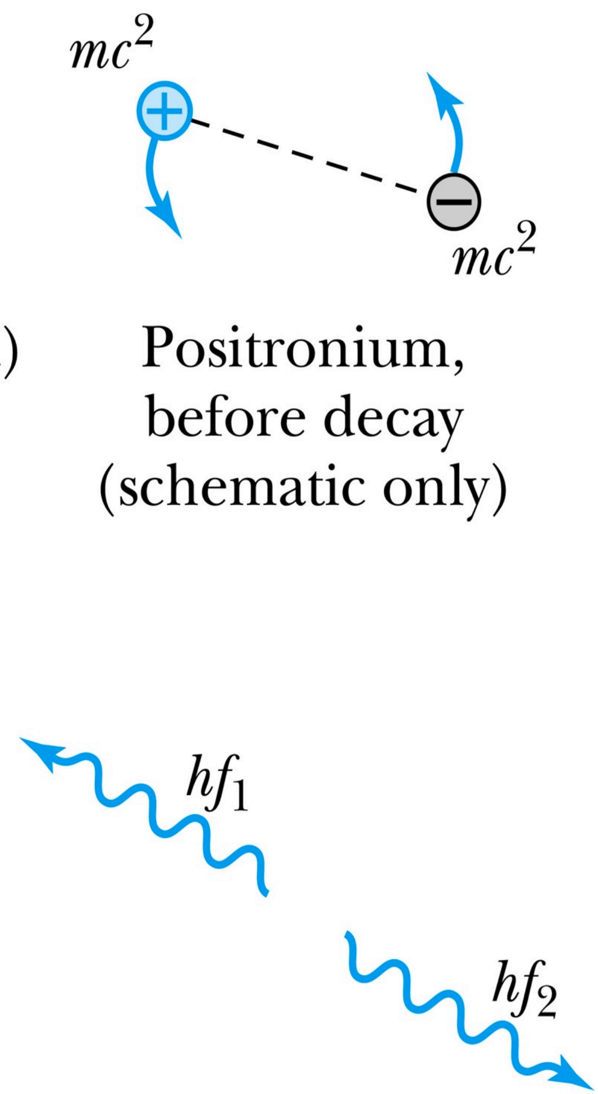
Pair Annihilation

- A positron going through matter will likely **annihilate** with an electron.
- A positron is drawn to an electron and form an atom-like configuration called **positronium**.
- Pair annihilation in empty space will produce two photons to conserve momentum. Annihilation near a nucleus can result in a single photon.
- Conservation of energy: $2m_e c^2 \approx hf_1 + hf_2$
- Conservation of momentum: $0 = \frac{hf_1}{c} - \frac{hf_2}{c}$
- The two photons will be almost identical, so that

$$f_1 = f_2 = f$$

- The two photons from a positronium annihilation will move in the opposite directions with an energy of:

$$hf = m_e c^2 = 0.511\text{MeV}$$



(a) Positronium, before decay (schematic only)

(b) After annihilation